

Binary Classification

$$(x, y) \quad x \in \mathbb{R}^{n \times 1}, y \in \{0, 1\}$$

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数据 标签

$$\{x^{(1)}, y^{(1)}\}, \{x^{(2)}, y^{(2)}\}$$

$$\# \text{ training set} = m / m_{\text{train}} = \dots \{x^{(m)}, y^{(m)}\}$$

$$\# \text{ test set} = m_{\text{test}}$$

矩阵形式:

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix} \begin{matrix} \uparrow \\ n \times \\ \downarrow \end{matrix}$$

k m 1

$n \uparrow$ 矩阵的维度

$$X \in \mathbb{R}^{n \times m}$$

$$X.\text{shape} = (n, m)$$

$$y = [y^{(1)}, y^{(2)}, \dots, y^{(m)}] \quad y \in \mathbb{R}^{1 \times m}$$

$$y.\text{shape} = (1, m)$$

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Logistic Regression.

x 特征 \rightarrow 由像素值组成的特征向量: $x \in \mathbb{R}^{n \times 1}$

\hat{y} 分类概率 $P(y=1|x)$.

在 x 的条件下, $y=1$ 的概率

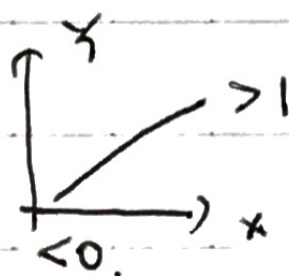
显然, $0 \leq \hat{y} \leq 1$.

参数 $\left\{ \begin{array}{l} \text{权重 } w \in \mathbb{R}^{n \times 1} \\ \text{偏置值 } b \in \mathbb{R} \end{array} \right.$

为何线性回归模型不能解决?

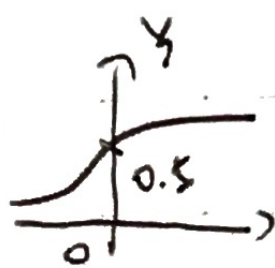
$$\hat{y} = w^T x + b$$

$\because \hat{y} \in [0, 1]$ &



\therefore 激活函数 σ sigmoid.

$$\sigma(z) = \frac{1}{1 + e^{-z}}, \quad z \in \mathbb{R}.$$



$$\lim_{z \rightarrow +\infty} \sigma(z) = \frac{1}{1+0} = 1$$

$$\lim_{z \rightarrow -\infty} \sigma(z) = \frac{1}{+\infty} = 0. \quad \text{符合 } \hat{y} \in [0, 1]$$

LR cost func

$$y^{(i)} = \sigma(w^T x^{(i)} + b), \text{ 其中 } \sigma(z) = \frac{1}{1 + e^{-z}}$$

给一组训练集 $\{(x^1, y^1), \dots, (x^m, y^m)\}$,

期望 $\hat{y}^{(i)} \rightarrow y^{(i)}$ 预测值接近真实值.

定义 Loss func.

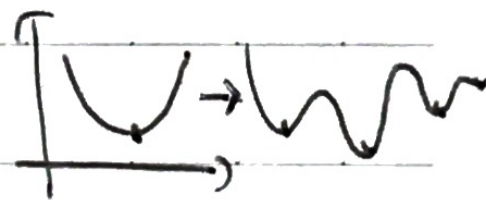
没理解这两个 loss

$$J(\hat{y}, y) = \frac{1}{2} (\hat{y} - y)^2$$

的选取理由.

在LR中不这样定义 Loss func 的原因是?

因为在LR中优化问题是非凸的. 这样一来梯度下降只能得到各个局部最优解, 而非全局最优.



在LR中定义 Loss func:

$$J(\hat{y}, y) = - (y \log \hat{y} + (1-y) \log (1-\hat{y}))$$

$y=1, J = -\log \hat{y} \rightarrow J \downarrow, \log \hat{y} \uparrow, \hat{y} \uparrow \rightarrow \hat{y} \uparrow$ 与 $y=1$ 接近

$y=0, J = -(1-y) \log (1-\hat{y}) \rightarrow J \downarrow, \hat{y} \downarrow \rightarrow \hat{y} \downarrow$ 与 $y=0$ 接近

这是针对一个 $(x^{(i)}, y^{(i)})$ 组合而言的.

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为训练集 (训练样本 $(x^1, y^1), \dots, (x^m, y^m)$).

$$\text{Cost func} = J(w, b) = \frac{1}{m} \sum_{i=1}^m J(\hat{y}^i, y^i).$$

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参数

$$\text{即} - \frac{1}{m} \sum_{i=1}^m [y^i \log \hat{y}^i + (1 - y^i) \log (1 - \hat{y}^i)].$$