# Analysis of a Homemade Pendulum

**Yizhou Mi**

**October 4, 2023**

## 1 Introduction

The structure of a pendulum is quite simple, consisting of a small but heavy mass that is hung from a protruding edge and a sturdy structure that supports the entire system. The general motion of a pendulum will be explored in this lab report, and the information extracted through experimentation will include the general trend of the period compared to the angle at the maxima, the trend of the amplitude over time, and the Q factor(Quality of the pendulum). Some equations that will be used within this analysis will include the power series:

$$T = T_0(1 + B\theta_0 + C\theta_0^2 + \dots) \tag{1}$$

, which will provide $T_0$.

$$Q = \pi\frac{\tau}{T} \tag{2}$$

, for the numerical calculation of Q.

$$Ae^{-x/\tau} \tag{3}$$

provides the decay of amplitude trend.

## 2 Methods and Procedures

For the frame of the pendulum, a 10.5 by 6.5 by 29-inch (26.7 by 16.5 by 73.7-centimeter) toilet paper holder was purchased and used. This was thought to make for an excellent mount for the pendulum string as it had a flat metal base, hypothesized to resist the motion of the swinging object. Thus, the motion data collected in later steps would only consider the movement of the pendulum's weight, and the movement of the pendulum's frame would be negligible. The toilet paper holder also did not confine the angle at which the weight could be extended (considering 0 degrees lies perpendicular to the ground), a marble of radius (1.2±0.05) cm and mass (0.019±0.0005) kg, was permitted to be dropped. To attach these two components together, a braided jute rope with a length of (52.4±0.05) cm and mass (0.003±0.0005) kg was used due to its resistance to stretching. Figure 1 shows the setup of the pendulum. A Pixel 7 back camera with 4k resolution and 60fps was placed (158.0±0.05) cm away from the pendulum, which was in turn placed (24.6±0.05) from the wall. The pendulum was aligned with the center of the camera using the approximation provided by the tiled flooring.

Figure 1: A marble is tied to a light string hung on the arm of the toilet paper holder. The marble extends down so that it is above the intersection of the tiled flooring. This arrangement allowed for more precise procedures

marble was positioned directly above a tile line perpendicular to that of the camera-pendulum alignment, and its angular displacement was slowly increased while maintaining a plane of oscillation as parallel to this line as possible. This ensured that the pendulum swung perpendicular to the camera's line of sight. To obtain the position of the pendulum, the videos were uploaded to the Tracker App, a video analysis and modeling tool that can estimate the location of an object via pixel comparison, and the maxima were plotted by going through the frames one by one. The *fit_example.py* script was edited so that periods were considered to begin at each maximum; in the same oscillation, the period was tracked starting at a positive $\theta$ and again at a negative $\theta$. This way, this one set of data would provide almost twice as many period measurements as it would originally have. The collected raw data was then plotted into an Angle vs. Period graph and a custom Python script was used to plot an Amplitude vs. Time graph.

# 3 Results and Data Analysis

The experiment included 3 videos taken, each prolonging for around one and a half minutes. In total, 9145 frames were analyzed using the Tracker application. The results were two graphs: a period versus angle graph and an amplitude versus time graph.

## 3.1 Angle Versus Period Analysis

The Angle Versus Period data is shown in figure 3.1.

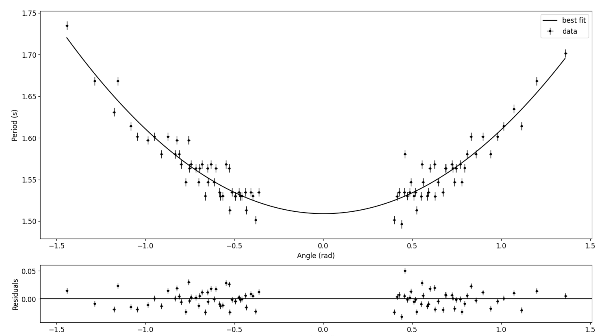


Figure 2: Data comparison between the angle of the pendulum at its highest positions and its equilibrium position compared to the period of motion. The angular displacement to the left of the equilibrium position has been considered negative, while the right has been considered positive. The best-fit line is a quadratic, represented by the power series equation $T = 1.51(1 + (0.000649 \pm 0.001)\theta_0 + (0.07 \pm 0.003)\theta_0^2 + \ldots)$. The red lines represent the error bars, and these variations of data from the line of best fit have been plotted in the "Residuals"

the data points are scattered with large differentials, and the line of fit does not fall within the uncertainties. The B value is also smaller than its uncertainty and thus could be interpreted as zero. Value C cannot as its uncertainty is noticeably smaller. In this experiment, $T_0$ is 1.508582139669913. Some of the sources of uncertainty that influenced the data divergence include the measurement uncertainties that arise from the frame rate and the motion of the imperfect pendulum paired with the Tracker App's ability to measure. Since the frame rate is 60fps, the motion of the pendulum between these frames is undefined and thus unable to be considered in the data. The consecutive maxima should be measured when the velocity of the pendulum is zero, but this requires at least three frames with the pendulum at the exact same position to be true, and this does not occur. The actual maxima lie $\pm 0.5$ frames from the documented maxima, and so the period of oscillation has an uncertainty of twice this value (0.0167s). Another source of uncertainty arises from the presence of air resistance. The motion of the pendulum is subject to damping, thus smaller angles arise with time, and the period decreases with this angle decrease. Though we tried to drop the pendulum perpendicular to the line of sight of the camera, the oscillation path was inevitably imperfect. This would result in theta measurements that are not fully correct. The Tracker App only measures angles up to one decimal place, so the angle uncertainty lies between $\pm 0.05$ degrees. The friction between the pendulum's rope and the toilet paper holder's arm is also a considerable source of uncertainty that results in damping.This friction is inconsistent and depends greatly on the motion of the pendulum at a given time. Lastly, the perpendicularity of the camera to the ground is also an issue that arises. Though the phone camera was supported and intended to be straight, there will be slight inaccuracies in the position.

### 3.2 Amplitude Versus Time Analysis

The line of fit in this graph generally lies within the uncertainty value of the data points. As with the previous experiment, the largest source of uncertainty for this was due

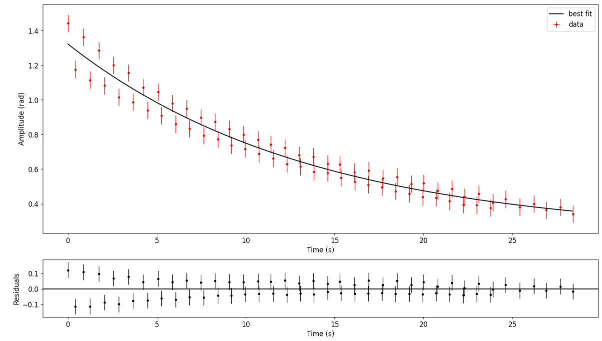


Figure 3: Time series data of the maximum amplitude of the pendulum after it is released. The best fit takes on the equation of $1.28 \pm 0.04e^{-t/19.9\pm 1}$. The red lines denote the error bars of the data.

to the Tracker App's ability to measure angles. This uncertainty had a magnitude of 0.05. Finally, the Q factor was determined in two different methods through this experiment. The counting of oscillations for Q/4 by finding when the amplitude becomes around 46% provided the Q factor value of $41 \pm 2$. The analysis of Q factor via $(\pi * \tau)/T$ provided the value of $42.2 \pm 3$. These Q factors agree with each other as they fall within the uncertainties of one another; The Q factors acquired from the two methods respectively range from 39-43 and 39.2-45.2. However, they are unusually small in value. This could be due to the length of my video and also frictional influences within the pendulum. These include the previously mentioned friction between the string and pendulum arm and possibly the instability of the pendulum itself.

## 4 Conclusion

In conclusion, the period of oscillation is not independent of the angle, though by equation, it should be. This is because, in reality, there are many more factors that are involved in the period of motion in a non-perfect system, mainly frictional components. Considering these sources of uncertainties, however, it is reasonable to say that the current model is acceptable. For the purpose of this experiment, the relation between the amplitude and the time is in the form of an exponential decay, and the plotting of this graph allowed the Q factor to be revealed in two different ways. The resulting Q factor from these two methods is $41.5 \pm 4$.

To minimize uncertainties for future experiments, the Tracker Application's auto-tracking ability should be deployed and the oscillation path of the pendulum could be improved by using two strings instead one.