

Mobius 8

$$4) z - z_0 = \frac{df}{dx}(x - x_0) + \frac{df}{dy}(y - y_0) + \frac{df}{dz}(z - z_0)$$

$$5) z - z_0 = \frac{df}{dx}(x - x_0) + \frac{df}{dy}(y - y_0)$$

$$z_0 =$$

6)

$$L(x, y, z) = f(x, y, z) + \frac{df}{dx}(x - x_0) + \frac{df}{dy}(y - y_0) + \frac{df}{dz}(z - z_0)$$

$$7) \left(\frac{dg}{ds}, \frac{dy}{dt} \right) \cdot \frac{v}{|v|}$$

9)

$$\frac{dT}{dt} = \frac{dT}{dx} \cdot \frac{dx}{dt} + \frac{dT}{dy} \cdot \frac{dy}{dt}$$

$$10) z = f(x, y) \quad x = g(t) \quad \frac{dx}{dt} = \frac{dz}{dt} = g'(t)$$

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$$

$$12) 6xy^2 = 3x^2 + y^2 + 3z^2 + 5$$

$$\frac{dz}{dx} = ? \quad \frac{dz}{dy} = ?$$

$$\frac{d}{dx}(6xy^2) = \frac{d}{dx}(3x^2 + y^2 + 3z^2 + 5)$$

$$6y(1)2 + 6yx \frac{dz}{dx} = \dots$$

Mobius 7

$$2) \frac{2 \cdot 7^n \cdot x^{3n}}{n}$$

$$f(x) = \frac{2 \cdot 7^n}{n} \cdot \frac{d}{dx}(x^{3n})$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \dots = \frac{2 \cdot 7 \cdot x^2}{x^1} < 1$$

$$2 \cdot 7 \cdot x^3 < 1$$

$$x^3 < \frac{1}{2 \cdot 7}$$

$$3) \frac{x}{x^3} \text{ what's } C$$

$$\text{in } C \cdot x^6?$$

$$\frac{a}{1-r} \quad |r| < 1$$

$$\frac{x}{3(1-\frac{x}{3})} = \left(\frac{x}{3} \right) \cdot \frac{1}{1-\frac{x}{3}} \quad |1-\frac{x}{3}| < 1$$

$$\frac{x}{3} \left[\left(\frac{x}{3} \right)^1 + \dots + \left(\frac{x}{3} \right)^5 \right]$$

$$-\frac{x}{3} \left(\frac{x}{3} \right)^5 = -\frac{x^6}{3 \cdot 3^5} = -\frac{1}{3^6} \cdot x^6$$

$$4) \ln(1+9x^2) \text{ radius of convergence?}$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

$$\ln(1+9x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (9x^2)^{n+1}}{n+1}$$

$$5) e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{2x^2} = \sum_{n=0}^{\infty} \frac{(2x^2)^n}{n!}$$

$$(1+3x) e^{2x^2} = \sum_{n=0}^{\infty} \frac{(1+3x)(2x^2)^n}{n!}$$

Mobius 4

$$1) A = A_0 \left(\frac{1}{2} \right)^{\frac{t}{C}}$$

3) Bacteria (Not half life)

$$A = A_0 e^{kt}$$

4) Find growth rate of population after 4 hours given

$$P(t) = 1000 e^{\frac{\ln 2}{2} t}$$

$$P'(4) = ?$$

10) Find a, r, S of geometric series

$$\sum_{n=0}^{\infty} \frac{2^{n+1}}{(2^n)^3}$$

$$\dots = 2^{1-n}$$

$$\text{form: } a_n = a_1 \cdot r^{n-1}$$

$$= 1 \cdot 2^{-n+1}$$

$$S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$$

$$= 1 \cdot (2^{-1})^{n-1}$$

$$a_1 = 1 \quad r = \frac{1}{2}$$

11) Sum of series: $36 + \frac{36}{7} + \frac{36}{49} \dots$

$$36 \left(1 + \frac{1}{7} + \frac{1}{49} \dots \right)$$

$$36 \left(\frac{1}{1-\frac{1}{7}} \right) = 42$$

12) geo series may be sum so separate them. Identify it by sigma & sign.

Mobius 3

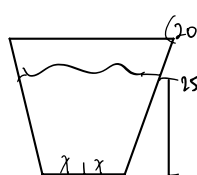
1) Avg value formula:

$$\frac{1}{b-a} \int_a^b f(x) dx$$

2) Arc Length:

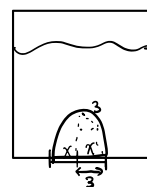
$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

3) What's hydrostatic Force?



$$F = \int_0^{25} 9800 \left(\frac{y+30}{3} \right) (25-y) dy$$

4)



What's F_H ?

$$x = \sqrt{9-y^2}$$

$$V = 2x = 2(\sqrt{9-y^2})$$

$$F_H = \int_0^3 9800 (2\sqrt{9-y^2}) (15-y) dy$$

$n=0$ } with give
 $n=1$ } coefficients
 $n=2$ }

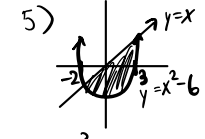
1. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
2. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
3. $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
4. $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
5. $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$
6. Taylor series: $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$
7. Maclaurin Series: $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!}$

Mobius 6

- 1) NST $\lim_{n \rightarrow \infty} a_n = 0$
 $a_{n+1} \leq a_n$
- Ratio Test
 $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ conv.
- 3) Integral Test
 $x \geq 1$ continuous, $f'(x) < 0$ true.
 Plug in $x=1$
 $\sum_{n=1}^{\infty} \frac{1}{n^2+5} = \int_1^{\infty} \frac{1}{x^2+5} dx = \frac{1}{\sqrt{5}} \arctan\left(\frac{x}{\sqrt{5}}\right)$
- 8) Error Approximation for AST at most 10^{-2} .
 $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+27}$ $|b_{n+1}| < \frac{1}{100}$
 $\frac{1}{(n+1)^2+27} < \frac{1}{100}$
 $n \approx 8$
- 13) $\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n \sqrt{n+1}}$ find interval of convergence.
 Ratio Test. $|x-3| < 2$ $[1, 5)$
 Plug in $x=1, 5$
 use AST and p series $n \leq 1$ div, then conv is C.

Mobius 5

- 2) Telescopic Sum
 $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+4}$
 $\frac{1}{n+1} + \frac{1}{n+4} \Rightarrow \sum_{n=1}^{\infty} \left(\frac{-3}{n+4} + \frac{3}{n+1} \right)$
 $\begin{matrix} -\frac{3}{5} & +\frac{3}{5} \\ -\frac{3}{6} & +\frac{3}{3} \\ -\frac{3}{7} & +\frac{3}{4} \\ -\frac{3}{8} & +\frac{3}{5} \end{matrix}$
 AST
 $|s_n - s_{n+1}| = |b_{n+1}| < \frac{1}{100}$

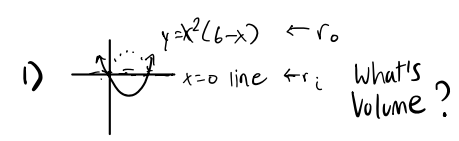


$$M_x = \int_{-2}^3 x(x - (x^2 - 6)) dx$$

$$M_y = \int_{-2}^3 \frac{1}{2} (x^2 - (x^2 - 6)^2) dx$$

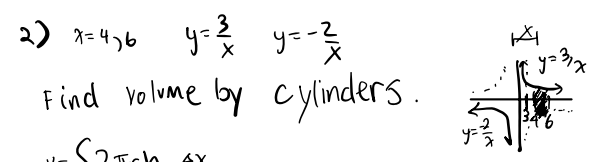
$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{A}, \frac{M_x}{A} \right)$$

Mobius 2



$$V = \int_a^b A(x) dx$$

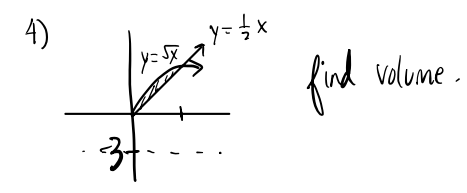
$$V = \pi \int_a^b (r_o^2 - r_i^2) dx$$



Find volume by cylinders.

$$V = \int_4^6 2\pi r h dx$$

$$= 2\pi \int_4^6 (x-3) \left(\frac{3}{x} + \left(\frac{2}{x} \right) \right) dx$$

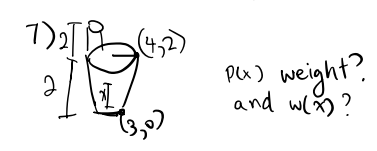


$$V = \int_0^3 \left((\sqrt{x}+3)^2 - \left(\frac{1}{2}x+3 \right)^2 \right) dx$$

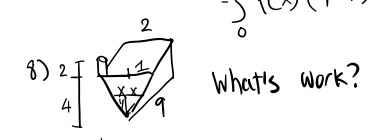
5) Area Squares = l^2

6) Area SemiCircles = $\frac{\pi r^2}{2}$

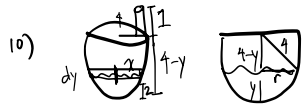
$$r = \frac{d}{2} = \frac{(2+e^x)}{2}$$



7) $m=2$
 $x = \frac{1}{2}y+3$
 $P(x) = \int_0^2 9800 \pi r^2 dy$
 $P(x) = \int_0^2 9800 \pi \left(\frac{1}{2}y+3 \right)^2 dy$
 $w(x) = P(x) \text{ dist}$
 $= \int_0^2 P(x) (4-x) dx$



$$W = Fd = \int_0^4 9800 (2x \cdot dy \cdot 9) (6-y) \quad y = \frac{4}{3}x \quad x = \frac{y}{4}$$



$$W = \int \text{density} \cdot \text{dist} \cdot \text{area} \, dy \quad x^2 + y^2 = 16 \quad x = \sqrt{16 - y^2}$$

$$= \int 9800 (5-y) \pi (\sqrt{\quad})^2 \, dy$$