Poisson Scale Mixture Model

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Abstract

In this paper, we introduce Poisson Scale Mixture (PSM) distribution and develop a process based on this distribution. Option pricing model under the process is developed to incorporate the asymmetry of information and the corresponding implied volatility surface is numerically investigated. This model provides a good fit to observed option prices. To demonstrate the advantage of the new process, we conduct empirical studies to compare its performance to other processes that have been used in the literature. This work was supported with funding from Undergraduate Research & Creative Activities (URECA).

Keywords— Poisson mixture model, Informed traders, Option pricing

1 Introduction

For a long time in academics, we have studied rational finance to describe the phenomena in finance. Rational finance is based on rational and logical theories, such as the capital asset pricing model and the efficient market hypothesis. Although these theories have done a respectable job of predicting and explaining certain events, there has been found many anomalies that could not been answered by this rational finance. These anomalies include excess volatility, calendar effects, equity premium puzzle, and more. The anomalies promoted academics to look to psychology to explain the irrational and illogical behaviors that rational finance had failed to describe. This new branch of finance that endeavors to bridge between traditional finance and psychology is called the behavioral finance.

Unlike the rational finance, the behavioral finance uses psychological theory with little mathematics to explain the phenomena in finance. Due to the gap, there are people who have a positive view to the emergence of the behavioral finance ([Markus Glaser and Noeth(2004)] and [Werner De Bondt and Staikouras(2015)]), people who have a negative view to the emergence of the behavioral finance ([Rubinstein(2006)]), and people who attempt to bridge the gap ([Dayala(2012)]).

The ground-breaking work of [Black and Scholes(1973)] and [Merton(1973)] -typically referred to as the Black-Scholes model- option pricing has been based on the frame of rational finance and the assumption that asset returns are normally distributed. Due to the frame of rational finance, the Black-Scholes model assumes the efficient market in which all players have equal access to available information, symmetric information. This assumption is too simplified as we can easily see that any more than one person would hold different information, however. Also, not only has the normal distribution assumption been rejected by numerous empirical studies, it is a well-documented fact that asset returns exhibit asymmetry and heavy tails.

In the model we propose in this article, we incorporate the asymmetry of information which is the view of the behavioral finance to deal with the nonnormality. The larger class of our model is the scale mixture models and they are well studied in finance, cf. [Andrews and Mallows(1974)], [Mittnik and Rachev(1993)], [Madan and Seneta(1990)], [Eberlein and Keller(1995)], and [Soltyk and Gupta(2011)]. Existing models, however, are typically unnecessarily complex with restrictions on the parameters space when calibrated to option data. Thus, in this paper, we introduce a simple Poisson Scale Mixture (PSM) model in which the information is captured by Poisson distribution and present an approach to pricing European options under the assumptions that the financial instruments involved are governed by a PSM process.

The main contribution of this article is an attempt to bridge the gap between the rational finance and the behavioral finance by capturing the asymmetry of information by Poisson distribution. Providing a mathematically closed option pricing model under the assumption from behavioral finance will help to fill the gap. Given that PSM model tends to provide a better description of empirically observed processes, as it permits fat-tailedness and skewness, it is also expected to price options more realistically. In the following, we present a closed-form solution for European option pricing under the PSM model. An empirical application to European option on S&P 500 index illustrates that the PSM model clearly dominates the Black-Scholes model.

The remainder of this paper is organized as follows. In section 2, we discuss the PSM distribution and the properties of PSM distribution. We introduce the asset pricing model using PSM distribution in section 3. In section 4 and 5, we discuss the discretization of the asset pricing model and the measure transformation to risk-neutral measure using the discretization. The option pricing model on the PSM asset pricing model is introduced in section 6. We empirically evaluate the performance of our proposed model in Section 7 and offer some concluding remarks in Section 8.

2 The Poisson scale Mixture (PSM) distribution

Consider the Poisson Scale Mixture (PSM) random variable (rv) $X^{(\mu,\rho,\lambda,\sigma)} = \mu + \rho(T^{(\lambda)}) + \sqrt{T^{(\lambda)}}Z^{(\sigma)}$, where $Z^{(\sigma)} \sim \mathcal{N}(0,\sigma^2)$ and $T^{(\lambda)} \sim Poisson(\lambda), \lambda > 0, \mu \in \mathbb{R}, \rho \in \mathbb{R}$, and $\sigma > 0$. We denote $X^{(\mu,\rho,\lambda,\sigma)} \sim PSM(\mu,\rho,\lambda,\sigma)$.

The characteristic function (ch.f.) of X is given by

$$\varphi_X^{(\mu,\rho,\lambda,\sigma)}(u) = \mathbb{E}e^{iuX^{(\mu,\rho,\lambda,\sigma)}} = \exp\left(iu\mu - \lambda\left(1 - e^{iu\rho - \frac{\sigma^2 u^2}{2}}\right)\right), \tag{2.1}$$

and thus

$$\mathbb{E}e^{aX^{(\mu,\rho,\lambda,\sigma)}} = \exp\left(\mu a - \lambda\left(1 - e^{\rho a + \frac{\sigma^2 a^2}{2}}\right)\right), a > 0, \tag{2.2}$$

$$\mathbb{E}X^{(\mu,\rho,\lambda,\sigma)} = \mu + \rho\lambda, \tag{2.3}$$

and the variance

$$var\left(X^{(\mu,\rho,\lambda,\sigma)}\right) = \lambda\left(\rho^2 + \sigma^2\right), \tag{2.4}$$

and the skewness and excess kurtosis

$$Skew\left(X^{(\mu,\rho,\lambda,\sigma)}\right) = \frac{\lambda(\rho^3 + 3\sigma^2\rho)}{(\lambda(\rho^2+\sigma^2))^{\frac{3}{2}}}, \tag{2.5}$$

$$ExcessKurt\left(X^{(\mu,\rho,\lambda,\sigma)}\right) = \frac{\lambda(\rho^4 + 6\sigma^2\rho^2 + 3\sigma^4)}{(\lambda(\rho^2+\sigma^2))^2} \tag{2.6}$$

We call $X^{(\rho,\sigma)} \sim PSM\left(-\lambda\rho, \rho, \lambda, \sqrt{\frac{1-\lambda\rho^2}{\lambda}}\right)$ standard PSM random variable as it has zero mean and unit variance.

The distribution function (df) $F_{X^{(\mu,\rho,\lambda,\sigma)}}$ of $X^{(\mu,\rho,\lambda,\sigma)}$ is given by

$$F_{X^{(\mu,\rho,\lambda,\sigma)}}(x) = \sum_{j=1}^{\infty} \frac{\lambda^j}{j!}e^{-\lambda} \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2 j}} e^{-\frac{(u-\mu-\rho j)^2}{2\sigma^2 j}} du, x \in \mathbb{R}, \tag{2.7}$$

and the probability density function (pdf)

$$f_{X^{(\mu,\rho,\lambda,\sigma)}}(x) = \sum_{j=1}^{\infty} \frac{\lambda^j}{j!} \frac{e^{-\left(\lambda + \frac{(x-\mu-\rho j)^2}{2\sigma^2 j}\right)}}{\sqrt{2\pi\sigma^2 j}}, x \in \mathbb{R}. \tag{2.8}$$

Also, $X^{(\mu,\rho,\lambda,\sigma)}$ is infinitely divisible. If $a \in \mathbb{R}, b \in \mathbb{R}, X^{(\mu,\rho,\lambda,\sigma)} \triangleq PSM(\mu,\rho,\lambda,\sigma)$, and $aX^{(\mu,\rho,\lambda,\sigma)}+b \triangleq PSM(a\mu+b, a\rho, \lambda, |a|\sigma)$. If $X^{(\mu^{(1)},\rho,\lambda^{(1)},\sigma)} \triangleq PSM(\mu^{(1)},\rho,\lambda^{(1)},\sigma)$ and $X^{(\mu^{(2)},\rho,\lambda^{(2)},\sigma)} \triangleq PSM(\mu^{(2)},\rho,\lambda^{(2)},\sigma)$ and they are independent, then $X^{(\mu^{(1)},\rho,\lambda^{(1)},\sigma)} + X^{(\mu^{(2)},\rho,\lambda^{(2)},\sigma)} \triangleq PSM(\mu^{(1)}+\mu^{(2)},\rho,\lambda^{(1)}+\lambda^{(2)},\sigma)$.

3 The Asset Price Model

$T^{(\lambda)}(t), t \geq 0$ is $Poisson(\lambda)$-process on the probability space $(\Omega_0, \mathcal{F}_0, P_0)$ and generates the filtration $\mathcal{F}^T = \{\mathcal{F}_t^T = \sigma\{T(s), 0 \leq s \leq t\}, t \geq 0\}$. $W(t), t \geq 0$ is a standard Brownian motion on another probability space $(\Omega_1, \mathcal{F}_1, P_1)$, and thus, independent of T. It generates the filtration $\mathcal{F}^W = \{\mathcal{F}_u^W = \sigma\{W(v), 0 \leq v \leq u\}, u \geq 0\}$. The process subordinated to the standard Brownian motion W by the independent process T is denoted by $M = \{M(t) = W(T(t)), t \geq 0\}$. It is used as the driving process in equation (3.1) below and is defined on the product probability space $(\Omega, \mathcal{F}, P) = (\Omega_0, \mathcal{F}_0, P_0) \otimes (\Omega_1, \mathcal{F}_1, P_1)$. Furthermore, it is adapted to the joint filtration $\mathcal{F}^M = \{\mathcal{F}_t^M = \mathcal{F}_t^T \otimes \mathcal{F}_{T(t)}^W, t \geq 0\}$. We note that the Poisson process T captures the asymmetry of information that is independent from the noise process W.

The stock process $S(t), t \geq 0$, satisfies the SDE of the following subordinated type:

$$S(t) = S(t_0)exp\{\mu(t-t_0) + \rho(T(t) - T(t_0)) + \sigma(W(T(t)) - W(T(t_0)))\}, \tag{3.1}$$

where μ, ρ, and σ are assumed to be constant for $0 \leq t < \infty$. We also introduce the short term interest rate r as a constant in the same manner as the classical Black and Scholes option pricing. The money account is then defined by the function

$$B = B\{B(t) = \exp(rt), t \geq 0\}. \tag{3.2}$$

The discounted asset price is then

$$\bar{S}(t) = \frac{S(t)}{B(t)} = S(t)\exp(-rt), \tag{3.3}$$

for $0 \leq t_0 \leq t < \infty$, which by equation (3.1) can be written as

$$\bar{S}(t) = S(0)\exp((\mu - r)t + \rho(T^{(\lambda)}(t))) + \sigma(W(T^{(\lambda)}(t)))). \tag{3.4}$$

In a more general set-up, one could choose μ, ρ, σ, and r to be time dependent function or ever stochastic processes.

4 The Discrete Time Model

We realize that the continuous time subordinated asset price model introduced above an only be interpreted as an idealization of the naturally discrete price formation as it occurs in the market.We therefore introduce a time discretisation of the interval $[t_0, \mathcal{T}]$ with discretisation points $\tau_i, i = 0, 1, 2, \dots$, in the form

$$t_0 = \tau_0 < \tau_1 < \cdots < \tau_N = \mathcal{T}, \tag{4.1}$$

with $N \in \{1, 2, 3, \dots\}$ and $0 \leq t_0 \leq \mathcal{T} < \infty$. The discretisation points may be random, and we introduce for a given $t \geq t_0$, the integer k_t as the largest integer k such that τ_k is less than or equal to t, that is,

$$k_t = max\{k \in \{0, 1, 2, \dots\}, \tau_k \leq t\}. \tag{4.2}$$

We assume for all $t \geq t_0$ that the random variable t_{k_t} is $\mathcal{F}_t^T$-measurable.

With our given time discretisation, we obtain from equation (3.4) the following recursive relation for the discounted asset price process:

$$\bar{S}(\tau_k) = \bar{S}(\tau_{k-1}) exp\{(\mu - r)(\tau_k - \tau_{k-1}) + \rho(T(\tau_k) - T(\tau_{k-1})) + \sigma\left(W(T(\tau_k)) - W(T(\tau_{k-1}))\right)\}, \tag{4.3}$$

for all $k \in 1, 2, 3, \dots, N$ with $\bar{S}(\tau_0) = S(t_0)$. For times t which do not coincide with any discretisation point, the value of the discounted asset price $\bar{S}(t)$ given in equation (3.4) appears as a natural interpolation of the values given by equation (4.3) in the form

$$\bar{S}(t) = \bar{S}(\tau_{k_t}) exp\{(\mu - r)(t - \tau_{k_t}) + \rho\left(T(t) - T(\tau_{k_t})\right) + \sigma\left(W(T(t)) - W(T(\tau_{k_t}))\right)\} \tag{4.4}$$

for all $t \in [t_0, \mathcal{T}]$.

5 MEASURE TRANSFORMATION

At this point, we assume that $\mu = r$, which will be justified later. We are now going to introduce an equivalent probability measure $\widetilde{P} = P_0 \otimes \widetilde{P}_1$ such that the discounted asset price process $\bar{S} = \{\bar{S}(t), t \in [t_0, \mathcal{T}]\}$ represents a $(\widetilde{P}, \mathcal{F}^Z)$-martingale. Then, $\widetilde{P}$ will be interpreted as the risk neutral pricing measure. This allows us to link our approach to the no-arbitrage concept. The measure $\widetilde{P}$ will be characterized by a measure transformation with Radon-Nikodym derivative

$$\Psi = \frac{d\widetilde{P}_1}{dP_1}, \tag{5.1}$$

such that the process $\widetilde{W} = \{\widetilde{W}(u), u \geq 0\}$ with

$$\widetilde{W}(u) = W(u) - \int_0^u \psi(v) dv \tag{5.2}$$

is a $\widetilde{P}_1$-Wiener process and the discounted asset price process $\bar{S} = \{\bar{S}(t), t \in [t_0, \mathcal{T}]\}$ becomes

$$\bar{S}(t) = \bar{S}(s) exp\left\{\sigma\left(\widetilde{W}(T(t)) - \widetilde{W}(T(s))\right) - \frac{1}{2}\sigma^2(T(t) - T(s))\right\}, \tag{5.3}$$

for all $t_0 \leq s \leq t \leq T$, where we will specify the market price for risk ψ below.

Let $\widehat{\mathcal{F}}^M$ be the joint filtration $\{\widehat{\mathcal{F}}_t^M = \mathcal{F}_{\mathcal{T}}^T \otimes \mathcal{F}_{T(t)}^W, t \in [t_0, \mathcal{T}]\}$. Note that this filtration is generated by the σ-algebra $\mathcal{F}_{\mathcal{T}}^T$ and the family of σ-algebra $\mathcal{F}_{T(\cdot)}^W$, and therefore, contains more information than the filtration $\mathcal{F}^M$, that is

$$\mathcal{F}_t^M \subset \widehat{\mathcal{F}}_t^M, \tag{5.4}$$

for $t \in [t_o, \mathcal{T}]$. We now observe that if equation (5.2) holds, then with $\widetilde{E}$ being the expectation with respect to the probability measure $\widetilde{P}$, we have for the discounted asset price the martingale property

$$\begin{aligned}\widetilde{E}(\bar{S}(t) \mid \mathcal{F}_s^M) &= \widetilde{E}\left(\widetilde{E}\left(\bar{S}(t) \mid \widehat{\mathcal{F}}_s^M\right) \mid \mathcal{F}_s^M\right) \\ &= \bar{S}(s)\widetilde{E}\left(\widetilde{E}\left(exp\left\{\sigma\left(\widetilde{W}((t)) - \widetilde{W}(T(s))\right) - \frac{1}{2}\sigma^2(T(t) - T(s))\right\} \middle| \widehat{\mathcal{F}}_s^M\right) \middle| \mathcal{F}_s^M\right) \\ &= \bar{S}(s)\widetilde{E}\left(1 \mid \mathcal{F}_s^M\right) \\ &= \bar{S}(s),\end{aligned} \tag{5.5}$$

for all $t_0 \leq s \leq t \leq \mathcal{T}$.

Let us now identify the Radon-Nikodym derivative Ψ, which is providing this martingale property, by not assuming from the beginning that we have $\mu = r$. According to our time discretisation, we will look for a piece-wise constant function ψ, that is

$$\int_{T(\tau_{k-1})}^{T(\tau_k)} \psi(v)dv = \psi\left(T(\tau_{k-1})\right)\left(T(\tau_k) - T(\tau_{k-1})\right), \tag{5.6}$$

which fulfills relation (5.2) and (5.3). Thus, equating the right-hand sides of equations (4.3) and (5.3) with $s = \tau_{k-1}$ and $t = \tau_k$, we have

$$\begin{aligned}(\mu - r)(\tau_k - \tau_{k-1}) + \rho\left(T(\tau_k) - T(\tau_{k-1})\right) + \sigma\left(W(T(\tau_k)) - W(T(\tau_{k-1}))\right) \\ = \sigma\left(\widetilde{W}(T(\tau_k)) - \widetilde{W}(T(\tau_{k-1}))\right) - \frac{1}{2}\sigma^2\left(T(\tau_k) - T(\tau_{k-1})\right)\end{aligned} \tag{5.7}$$

and by equations (5.2) and (5.6), we obtain

$$\begin{aligned}(\mu - r)(\tau_k - \tau_{k-1}) + \rho\left(T(\tau_k) - T(\tau_{k-1})\right) + \sigma\left(W(T(\tau_k)) - W(T(\tau_{k-1}))\right) \\ = \sigma\left(W(T(\tau_k)) - W(T(\tau_{k-1}))\right) - \left(\sigma\psi\left(T(\tau_{k-1})\right) + \frac{1}{2}\sigma^2\right)\left(T(\tau_k) - T(\tau_{k-1})\right)\end{aligned} \tag{5.8}$$

Solving this relation for the market price for risk ψ, we get

$$\psi\left(T(\tau_{k-1})\right) = \left(\frac{r-\mu}{\sigma}\right)\frac{\tau_k - \tau_{k-1}}{T(\tau_k) - T(\tau_{k-1})} - \frac{\rho + (1/2)\sigma^2}{\sigma}. \tag{5.9}$$

Let us discuss the asymptotic behaviour of ψ for finer and finer time discretisations. We observe that for $t \in [t_0, \mathcal{T}]$, the random ratio $(t_{k_t} - t_{k_t-1})/(T(\tau_{k_t}) - T(\tau_{k_t-1}))$ does not approach a well-defined limit as $max_{t\in\{t_0,T\}}\{\tau_{k_t} - \tau_{k_t-1}\} \to 0$. Consequently, when $\mu \neq r$, the market price for risk ψ does not approach a well-defined limit as the time discretisation is taken finer. Therefore, when $\mu \neq r$, we cannot expect existence of an equivalent martingale measure in the continuous time limit of finer and finer time discretisation. However, as we assume that $\mu = r$, we can derive an equivalent martingale measure as described by equation (5.9), which also applies for the continuous time limit. In this case, the continuous time martingale measure $\widetilde{P} = P_0 \otimes \widetilde{P}_1$ is defined by the Radon-Nykodym derivative

$$\Psi(u) = \frac{d\widetilde{P}_1}{dP_1}(u) = exp\Big\{\psi(W(u) - W(T(t_0))) - \frac{1}{2}\psi^2(u - T(t_0))\Big\}, \tag{5.10}$$

where the market price for risk is

$$\psi = -\frac{\rho + (1/2)\sigma^2}{\sigma}, \tag{5.11}$$

for $u \in [T(t_0), T(\mathcal{T})]$.

6 Option pricing

Consider an European call option with exercise price K and time to maturity $\mathcal{T}$, and denote its price by $C_t, t \geq 0, C_{\mathcal{T}} = max\,(0, S(\mathcal{T}) - K)$. The risk neutral dynamics is determined by (5.11) the market price for risk, (5.10) the Radon-Nikodym derivative , and (5.2) a Standard Brownian motion on the risk neutral world. Therefore, on the risk neutral world, the stock price has the following dynamics:

$$\tilde{S}(t) = \tilde{S}(t_0)e^{\sigma\left(\tilde{B}\left(T^{(\lambda)}(t)\right) - \tilde{B}\left(T^{(\lambda)}(t_0)\right) - \frac{1}{2}\sigma^2\left(T^{(\lambda)}(t) - T^{(\lambda)}(t_0)\right)\right)}, 0 \leq t_0 < t < \infty. \tag{6.1}$$

as explained in section 5.
Denote then $Y^{(t,\mathcal{T},\sigma,\lambda)} = \sigma^2\left(T^{(\lambda)}(\mathcal{T}) - T^{(\lambda)}(t)\right)$ and

$$G^{(+)}(x) = \int_0^\infty \Phi\left(\frac{x - \frac{y}{2}}{\sqrt{y}}\right) dF_Y(t,\mathcal{T},\sigma,\lambda)(y) = \sum_{k=0}^{\infty}\frac{\lambda^k}{k!}e^{-\lambda}\Phi\left(\frac{x + \frac{\sigma^2 k}{2}}{\sigma\sqrt{k}}\right), \tag{6.2}$$

$$G^{(-)}(x) = \int_0^\infty \Phi\left(\frac{x + \frac{y}{2}}{\sqrt{y}}\right) dF_Y(t,\mathcal{T},\sigma,\lambda)(y) = \sum_{k=0}^{\infty}\frac{\lambda^k}{k!}e^{-\lambda}\Phi\left(\frac{x - \frac{\sigma^2 k}{2}}{\sigma\sqrt{k}}\right), \tag{6.3}$$

where $\Phi(x) = F_{\mathcal{N}(0,\sigma^2)}(x), x \in R$ is the standard normal density function. Let $K^{(t,\mathcal{T},r)}$ be the discounted exercise price of the option, $K^{(t,\mathcal{T},r)} = Ke^{-r(\mathcal{T}-t)}$. Then the price of the European call at $t \in [0, \mathcal{T})$ is given by[1]

$$C_t = S(t)G^{(-)}\left(\log\left\{\frac{S(t)}{K^{(t,\mathcal{T},r)}}\right\}\right) - K^{(t,\mathcal{T},r)}G^{(+)}\left(\log\left\{\frac{S(t)}{K^{(t,\mathcal{T},r)}}\right\}\right). \tag{6.4}$$

7 Empirical Study

Probability Density Function

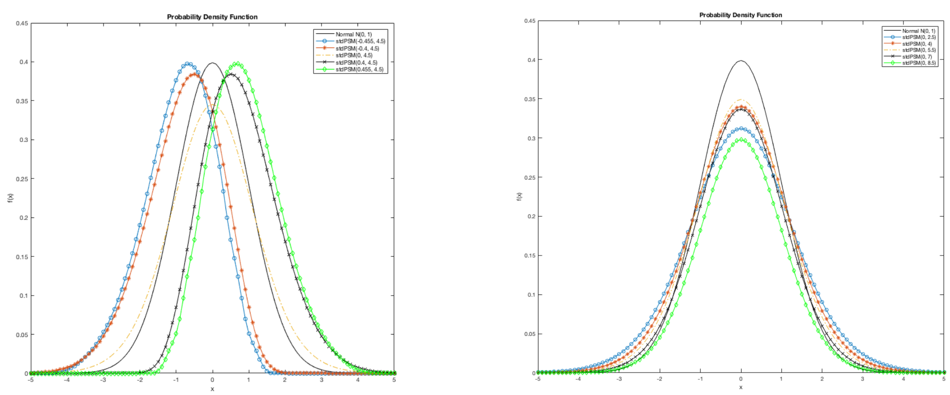

Figure 1: The probability density functions $f(\cdot)$ corresponding to the standard PSM distribution and standard Normal distribution.

As seen in Figure 1, the probability density function of standard PSM is more flexible than the probability density function of standard Normal for capturing skewness and fatter tail.

Goodness of Fit Measures

For comparative purpose, we compute four different measures which give an estimate of the goodness of fit. They are the average absolute error as a percentage of the mean price(APE), the average absolute error (AAE), the average relative percentage error (ARPE) and the root-mean-square error (RMSE):

$$APE = \frac{1}{mean\ option\ price} \sum_{options} \frac{|market\ price - model\ price|}{number\ of\ options}. \tag{7.1}$$

$$AAE = \sum_{options} \frac{|market\ price - model\ price|}{number\ of\ options}, \tag{7.2}$$

[1]The proof follows the one in Section 6 of [Hurst *et al.*(1999)Hurst, Platen, and Rachev].

$$ARPE = \frac{1}{number\ of\ options} \sum_{options} \frac{|market\ price - model\ price|}{market\ price}, \tag{7.3}$$

$$RMSE = \sqrt{\sum_{options} \frac{(market\ price - model\ price)^2}{number\ of\ options}}. \tag{7.4}$$

Typically, we estimate the model parameters by minimizing the root-mean-square error between the market and model prices.

Date	Model	Risk neutral parameters	AAE	APE	ARPE	RMSE
September 16	B-S	$\sigma = 0.2793$	3.0261	0.0789	0.1971	3.4307
	PSM	$\mu = 0.0197$, $\rho = 0.0007$, $\lambda = 70.1788$, $\sigma = 0.0287$	1.3096	0.0341	0.1131	1.4771

Table 1: Calibrated risk neutral parameters under Black-Scholes and PSM for S&P 500 index European Call options together with goodness-of-fit measures for selected trading days in 2008.

Date	Model	Risk neutral parameters	AAE	APE	ARPE	RMSE
September 16	B-S	$\sigma = 0.2855$	3.1551	0.0974	0.1707	3.7539
	PSM	$\mu = 0.5147$, $\rho =$ -0.0133, $\lambda = 30.4363$, $\sigma = 0.0621$	0.3525	0.0109	0.0129	0.5285

Table 2: Calibrated risk neutral parameters under Black-Scholes and PSM for S&P 500 index European Put options together with goodness-of-fit measures for selected trading days in 2008.

As indicated in the tables 1 and 2, the PSM option pricing model has a better fit than the Black-Scholes model for the European option data.

Prices and Volatility

The implied volatility of an option contract is that value of the volatility of the underlying instrument which, when input in an option pricing model will return a theoretical value equal to the current market price of the option.
(The implied volatility of an option contract is that value of the volatility of the underlying instrument when the value minimizes the squares of the difference between the option pricing model and the Black-Scholes model.)
We denote the Black-Scholes and PSM pricing models for call option by functions $C_{B-S}(\cdot)$ and $C_{PSM}(\cdot)$ respectively. We find the implied volatility of PSM call option pricing model by

$$\underset{\sigma(K_n,T_n)}{\arg\min}\left(C_{B-S}(S_0, K_n, T_n, r, \sigma(K_n, T_n)) - C_{PSM}(K_n, T_n)\right)^2 \tag{7.5}$$

,where S_0 denotes the current price of the underlying instrument, K_n denotes the strike price, T_n denotes the time to maturity, r denotes the risk-free interest rate, $\sigma(K_n, T_n)$ denotes the volatility of the underlying instrument, and n denotes the number of option contracts. The implied volatility of PSM put option pricing model can be found by replacing Black-Scholes call option pricing model to Black-Scholes put option pricing model and PSM call option pricing model to PSM put option pricing model in 7.5. Unlike historical volatility, implied volatility is derived from an option's price and shows what the market implies about the stock's volatility in the future. Thus, implied volatility is particularly important for options traders because it offers a way to test forecasts and identify entry and exit points. The implied volatility of the Balck-Scholes model is a constant as the moneyness varies which is not reasonable. However, the implied volatility of the PSM options pricing model changes as the moneyness varies which is reasonable. They are compared in figure 3.

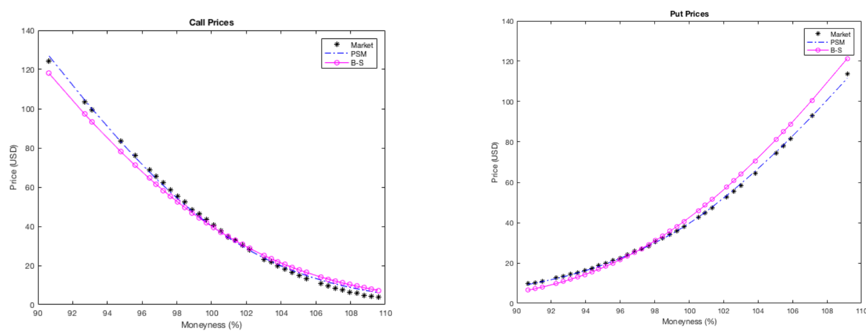

Figure 2: European call prices (left) and European put prices (right) implied by the Market, Black-Scholes (B-S) and the Poisson Scale Mixture (PSM) model as a function of moneyness, $K/S(0)$, where K denotes the strike price and $S(0)$ denotes the current underlying index level.

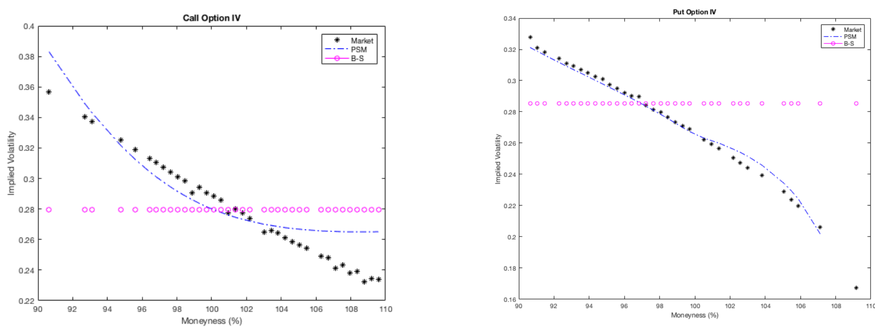

Figure 3: Volatility on European call prices (left) and Volatility on European put prices (right) implied by the Market, Black-Scholes (B-S) and the Poisson Scale Mixture (PSM) model as a function of moneyness, $K/S(0)$, where K denotes the strike price and $S(0)$ denotes the current underlying index level.

8 Conclusion

We propose the PSM distribution which captures the asymmetric information among traders in an attempt to bridge the rational finance and behavioral finance. Our model is more flexible for explaining the return distribution such as skewness and non-Gaussian distributional tails. The option pricing model based on PSM distribution has an improved fit for European options, compared to the Black-Scholes model. Also, the implied volatility looks much more reasonable than the constant volatility of Black-Scholes model.

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