Calc III Notes Day 22

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Homework help

Question 6 HW 3: $\lim_{x\to 0} \frac{cos}{sin} - \frac{1}{x}$

Last Time:

Prof said "If $(x_n) \to L$, then (x_n) is bounded from above AND below
This makes sense, after some number of terms (say 5 as an example), all of the terms must be within some interval sufficiently close to L
When the sequence is at this point, the sequence is **DEFINITELY** bounded (it has to be within that interval)
If we add in the terms before that point, its **STILL** bounded, just by larger bounds.. But because all points PAST a certain n are bounded, later terms will never be "farther" outside of bounds than the ones before n.

Basically, just know that if x(n) $\to$ L, then x(n) is bounded from above AND below

11.2: Infinite Series!

An infinite series is the sum of an infinite list of numbers

Starting here

EX 1: all zeros:

$$0, 0, 0, 0, ... = 0$$

EX 2

$$1 + 1 + 1 + 1... = \infty.$$

EX 3

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... = 1.$$

We SAY this adds up to 1. See Xeno's Paradox for more information (taking steps that are halfway to the next point each time)
What does this mean for infinite series? None of the items in the series are approaching 1 by themselves, but the SUM is approaching a limit at 1.

For any infinite series $x_1 + x_2 + x_3 + ...$ the nth partial sum $S_n = x_1 + x_2 + ...x_n$ We say the infinite series converges if the seq of partial sums converges.

Part 2: Electric Boogaloo

EX: Take infinite series

$$0 + 0 + 0 + 0... = .$$

Adding up n zeros is 0, no matter how many

$$
\begin{aligned}
S_1 &= 0 && (1)\\
S_2 &= 0 + 0 && (2)\\
S_3 &= 0 + 0 + 0 && (3)
\end{aligned}
$$

The S_n converges to 0, so we say the inf. series converges / adds up to 0

EX 3

$$1 + 1 + 1 + 1 + ...$$

Don't care about the **TERMS**, but about the **SUMS**

$$
\begin{aligned}
S_1 &= 1\\
S_2 &= 1 + 1\\
S_3 &= 1 + 1 + 1\\
S_4 &= 4...
\end{aligned}
$$

The sums ARE moving towards infinity.

EX 4: A more interesting one

$$\frac{1}{10} + (\frac{1}{10})^2 + (\frac{1}{10})^3.....$$

This is the start of **GEOMETRIC SERIES!!**

Geometric series def: Adding up powers of a single number r

$$1 + r + r^2 + r^3 + ...$$

$$
\begin{aligned}
S_1 &= 1\\
S_2 &= 1 + r\\
S_3 &= 1 + r + r^2\\
S_n &= 1 + r + ... + r^{n-1}
\end{aligned}
$$

Question: do the S_n approach limit? Converge?
Try multiplying $S_n * (1 - r)$

$$
\begin{aligned}
(1 + r + ... + r^{n-1})(1 - r) &= 1(1 + r + ...) - r(1 + r + ...)\\
&= 1 + r + ... + r^{n-1} - r - r^2 - ....r^n\\
&= 1 - r^n
\end{aligned}
$$

Multiplying partial sum by (1 - r) gives us a final equation of $1 - r^{n)}$

So now we get: $S_n = \frac{1-r^n}{1-r}$

QUESTION: What happens to r^n as $r \to \infty$?
Yes if $-1 < r < 1$
No if $r > 1 or r \leq -1$
Converges when $-1 < r < 1$