CALC III NOTES DAY 23

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Infinite Series

Geometric Series

Started with $1 + r + r^2 + r^3 ...$

$\boxed{\text{We decided that the nth partial sum } S_n = \frac{1r^n}{1-r} \text{ – Want to know if S converges to a limit or not}}$

What does r^n do as $n \to \infty$?

If $-1 < r < 1$, $r^n \to 0 \to$ Series converges to $S_n = \frac{1-0}{1-r} = \frac{1}{1-r}$

If $r > 1$ or $r < -1$, then r^n diverges. $\to$ Series diverges

EX

$(r^n) = 1, -2, 4, -8, 16$

this is terrible, it **DEFINITELY** diverges (doesn't even go to neg or positive infinity, it just..goes)

back to $1 + r + r^2 + r^3 + ...$

$$\begin{cases} div & r < -1 or r > 1 \\ conv \frac{1}{1-r} & -1 < r < 1 \end{cases}$$

EX for geometric series

$$1 + 0.1 + 0.01 + 0.001 + 0.0001....$$

$$= 1 + \frac{1}{10} + (\frac{1}{10})^2...$$

r is between -1 and 1

$$= \frac{1}{1 - \frac{1}{10}} = 1.11111..$$

Another example

$$5 + \frac{15}{2} + \frac{45}{4} + \frac{135}{8}....$$

factor out a 5:

$$5[1+\frac{3}{2}+\frac{9}{4}+\frac{27}{8}]$$
$$=5[1+\frac{3}{2}+(\frac{3}{2)^2+(\frac{3}{2)^3}}]$$

r $=\frac{3}{2}$ which is not in -1 to 1, so it diverges

Notation for infinite series

$$\sum_{n=1}^{\infty} x_n = Sumofx_nasngoesfrom1to\infty.$$

Example using notation

$$\sum_{n=1}^{\infty} \frac{3^n+1}{5^n} = \sum_{n=1}^{\infty} \frac{3^n}{5^n}+\frac{1}{5^n} = \sum_{n=1}^{\infty} \frac{3}{5}^n + (\frac{1}{5})^n.$$

Write out some of the numbers

$$\frac{3}{5}+\frac{1}{5}+(\frac{3}{5})^2+(\frac{1}{5})^2$$
$$=[\frac{3}{5}+(\frac{3}{5})^2+(\frac{3}{5})^3+...]+[\frac{1}{5}+(\frac{1}{5})^2+(\frac{1}{5})^3]$$

pull things out

$$=\frac{3}{5}[1+\frac{3}{5}+\frac{3}{5}^2+(\frac{3}{5})^3]+\frac{1}{5}[1+\frac{1}{5}+(\frac{1}{5})^2+...]$$

r $=\frac{3}{5}$ and r $=\frac{1}{5}$, so

$$=\frac{3}{5}[\frac{1}{1-\frac{3}{5}}]+\frac{1}{5}[\frac{1}{1-\frac{1}{5}}]$$

That final type of geometric series that looks nothing like the others

2.1481481481 48...as a fraction.

Any repeating fraction can be written as an integer over an integer

$$=2+\frac{148}{1000}+\frac{148}{10^6}+\frac{148}{10^9}+...$$

The two isn't part of the series, so it just kinda... stays out of things for a bit.

$$=2+[\frac{148}{10^3}[1+\frac{1}{10^3+\frac{1}{10^6}+\frac{1}{10^9}}]]$$
$$=\frac{148}{10^3}[1+\frac{1}{10^3}+(\frac{1}{10^3})^2+(\frac{1}{10^3})^3+...]$$

r is $\frac{1}{10^3}$

$$= 2 + \frac{148}{1000}[\frac{1}{1 - \frac{1}{10^3}}]$$

One or two tiny things we missed

If given an infinite list of numbers and they add up to some finite number (say 7)
Then, the numbers in the sum have to be getting closer to 0 (because if the sum is finite, if the numbers all have value then they can't add up to a finite number
Say $x_1 + x_2 + x_3 + x_4 + x_5$ converges $\rightarrow$ L
$S_n - S_{n-1} = x_n$
$x_1 + ... + x_n - (x_1 + x_2 + ...x_{n-1})$
$S_n \rightarrow 0, S_{n-1} \rightarrow 0, so x_n \rightarrow 0$

IF Infinite Series converges, then the terms x_n goes to 0.