Calc III Written Homework 5

Joseph Brooksbank

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1 Squeeze Theorm on $\frac{2^n}{n!}$

Idea: Trap $\frac{2^n}{n!}$ between two other sequences x_n, z_n such that those both $\to 0$.
We can write $\frac{2^n}{n!}$ as:

$$\frac{2*2*2*2*2...(ntimes)}{n*(n-1)*(n-2)*...*2*1}.$$

This can be "split up":

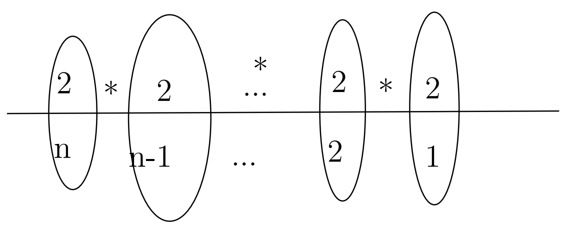

Figure 1: split

This is greater than or equal to 0 (which goes to 0) because n starts at 1 However, from the splitting up above, we can also see that it is equal to $2 * \frac{2}{2} * \frac{2}{3} * ...$ all the way to $\frac{2}{n-1} * \frac{2}{n}$, which means that the first bit (before $\frac{2}{n}$) is always less than 4 because the numbers are decreasing:

$$2 * 1 * (something less than 1) * (something less than that) \leq 2.$$

If we include the last part of the "split", $\frac{2}{n}$, we get that the entire thing is less than $\frac{2}{n} * 2 = \frac{4}{n}$. n $\to \infty$, so $\frac{4}{n} \to 0$. Therefore: the sequence $\frac{2^n}{n!}$ is bounded on both sides by 0, so it also goes to 0.

2 The Sierpinski Carpet

Step 1-2: We removed the middle area of $\frac{1}{9}$. Next, we remove $\frac{1}{9}$ of the remaining 8 squares, each of which already contains $\frac{1}{9}$ th of the original area. If we removed the center of one of the squares, that would represent a loss of $\frac{1}{9}$ of $\frac{1}{9}$, or $\frac{1}{81}$ of the entire area. doing this 8 times gives $\frac{8}{81}$ more area removed, for a total of $\frac{17}{81}$ area removed.

In the next step, we remove $\frac{1}{9} * \frac{1}{9} * \frac{1}{9}$ th area for each of the new holes, or $\frac{1}{729}$ or $\frac{1}{9^3}$. We do this 8 times for each of the 8 squares from step 2, or $8 * 8$ times. This gives us a reduction in area of $64 * \frac{1}{9^3}$ or $\frac{64}{729}$.

This seems to be the sequence $\frac{8^{n-1}}{9^n}$, So the series of the entire carpet's removed area is

$$\sum_{n=1}^{\infty} \frac{8^{n-1}}{9^n}$$

two options: Geometric series or Integral test..Integral test of this doesn't seem super fun, so lets try geometric series:

$$= \frac{1}{9} * (1 + \frac{8}{9} + (\frac{8}{9})^2 + (\frac{8}{9})^3)$$

It works as a geometric series! r = $\frac{8}{9}$, which is between -1 and 1. Thus:

$$= \frac{1}{9} * (\frac{1}{1 - \frac{8}{9}})$$

Series converges to $\frac{1}{9} * (\frac{1}{1-\frac{8}{9}})$, or 1