Calc III Notes Day 21

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Sequences

$$(x_n) = x_1, x_2, x_3....$$

How to check if (x_n) converges to limit as $(n \to \infty)$?

Simplification $\boxed{\text{EX 1:}}$

$$x_n = \frac{n^2}{\sqrt[2]{n^6+1}}.$$

Next: put largest n power on the bottom

$$\frac{n^2}{\sqrt[2]{n^6+1}} * \frac{\frac{1}{n^3}}{\frac{1}{n^3}} \tag{1}$$

$$= \frac{\frac{1}{n}}{\sqrt[2]{\frac{n^6+1}{n^6}}} \tag{2}$$

$\frac{1}{n} \to 0$

$$= \frac{0}{\sqrt[2]{1+\frac{1}{n^6}}} \tag{3}$$

$$= \frac{0}{\sqrt[2]{1+0}} \tag{4}$$

$$= \frac{0}{1} \tag{5}$$

$$= 0 \tag{6}$$

L'H Rule
We already did this so

Squeeze Theorem
Suppose we had 3 sequences, $x_n \le y_n \le z_n$
and
$x_n \to L$
$z_n \to L$
then $y_n \to L$

$\boxed{\text{Example of Squeeze Theorem}}$

$$y_n = \frac{sin(n)}{n}.$$

This equation is totally unpredictable (What is sin(11)?)... but **BETWEEN** -1 and 1

IDEA: squeeze the equation between other things

$$-1 \le \frac{sin(n)}{n} \le 1.$$

The squeeze theorem needs to have both sides of the squeeze go to the same point
so instead:

$$-\frac{1}{n} \leq \frac{sin(n)}{n} \leq \frac{1}{n}.$$

Both $-\frac{1}{n} and \frac{1}{n} \to 0$, so by squeeze theorem, so does $\frac{sin(n)}{n}$!

How to check if $(x_n) inc/dec$?

1. Compare x_n and x_{n+1}
   EX: Is $x_n = \frac{n}{n+1}$ inc, dec, or neither?

   - $x_1 = \frac{1}{2}$
   - $x_2 = \frac{2}{3}$
   - $x_3 = \frac{3}{4}$

   Guess: Increasing!
   $x_n < x_{n+1}$
   $\frac{n}{n+1} < \frac{n+1}{n+2}$ Cross Multiply:
   $n(n+2) < (n+1)(n+1)$
   Check to make sure that this is true, which it is

2. Check if derivative is increasing or decreasing
   Went over this last time so we're not doing it this time, but its pretty self explanatory

(x_n) bounded from above / below?

We say x_n is bounded from above IF:

There's a number M so that $x_n \leq$ M for all n
Same idea for below, but..from...below

How to check if x_n is bounded from above / below?

1. "common sense" / functions known to be bounded

   - **EX 1**

     $$x_n = sin(n).$$

     Bound by -1 and 1, so **BOUND ABOVE AND BELOW**
   - **EX 2**

     $$x_n = 4cos(n+5) + 3.$$

     Also bound from both sides

- **EX 3**

$$x_n = n^2.$$

  Bound from below
  x is always bigger than 0, so its bounded on the bottom but not the top

2. If sequence converges, then its bounded from both sides!

UNBOUND THINGS:
$x_n = n * (-1)^n$ Goes to both -infinity and positive infinity