CALC III Notes Day 37

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May 22, 2019

Online homework

$$\sum_{n=1}^{\infty} \frac{n^4 9^n}{n!} \text{check conv / div}$$

Ratio Test

$$\frac{\frac{(n+1)^4 9^{n+1}}{(n+1)!}}{\frac{n^4 9^n}{n!}}$$

$$= \frac{(n+1)^4 9^{n+1}}{(n+1)!} * \frac{n!}{n^4 9^n}$$

Things cancel

$$\frac{(n+1)^4}{n^4}$$

$$= (\frac{n+1}{n})^4$$

$$= (1 + \frac{1}{n})^4$$

4:

if we want first 4 decimal place to be correct, we could try to estimate to within 0.00001

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{3^n n!}$$

implementation:

set $x_{N+!} < 0.00001$

$$\frac{1}{3^{N+1}(N+1)!} < 0.00001$$
$$= \frac{1}{3^{N+1}(N+1)!} < \frac{1}{100000}$$
$$3^{N+1}(N+1)! > 100000$$

Could just plug in things until it works

or, solve $3^{N+1} > 100000$ and when you add in the factorial, its still bigger

log both sides

11.8: Power Series

$\boxed{\text{Rest of class is power series!}}$ what if we did this:

$$\sum_{n=0}^{\infty} x^n$$
$$= 1 + x + x^2 + x^3 + ... = \frac{1}{1-x} \text{if x is in (-1, 1)}$$

Diverges all other x

Look at this not as a single infinite series which converges, but as a series of functions which converge for all x in an interval

As we take more and more terms: Picture these partial sums as curves which approach a limit function for some interal of x's

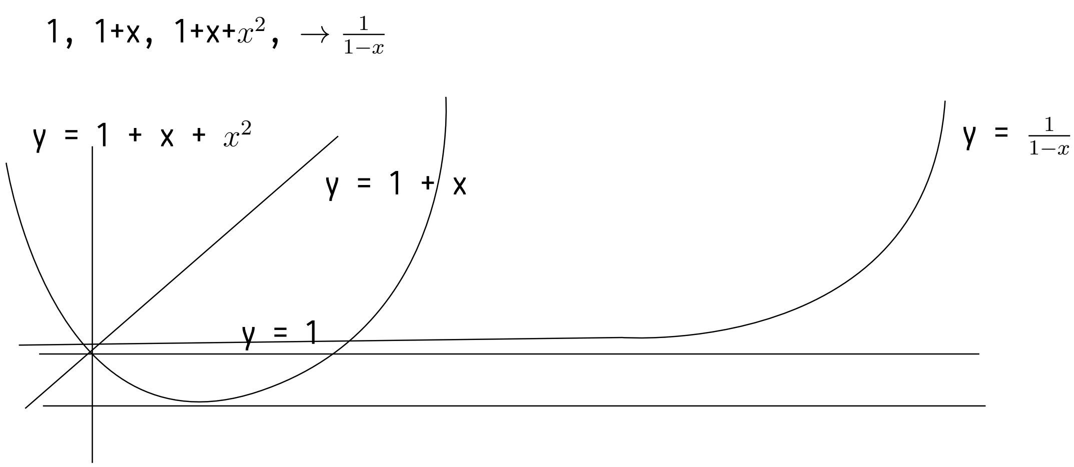

Figure 1: partial sums

what about $\sum_{n=0}^{\infty} 2^n x^n$? Converges / diverges for each value of x? How do we decide if series converges or diverges for each x?

$\sum_{n=0}^{\infty}(2x)^n$ is geometric and converges if 2x is in (-1,1), so if x is (-0.5, 0.5)

Answer: converges if x is $(\frac{-1}{2}, \frac{1}{2})$

IDEA: POWER Series is an infinite series which looks like $\sum_{n=0}^{\infty} c_n * (x-a)^n$
c_n : coefficient
a: center
Question we want the answer to is always for which values of x does the power series converge?

Some more examples to make a little more sense

EX

$$\sum_{n=0}^{\infty} \frac{1}{4^n} * (x-3)^n$$

For which x does this converge?

$$= \sum_{n=0}^{\infty} (\frac{x-3}{4})^n$$

geometric as well, converge if $\frac{x-3}{4}$ is in (-1, 1)

x-3 is in (-4, 4)

Answer: converges for x in interval (-1, 7)

Every answer is an interval

A power series always converges for an interval of x values with the midpoint equal to a (the center)

EX

Harder one:

$$\sum_{n=0}^{\infty} \frac{2^n}{n} * x^n$$

can't just "make it geometric" because not everything is to the same power
IDEA: lets use the root test on it (x is just some number)

$$\sqrt[n]{\frac{2^n}{n} * x^n}$$

$$= (|\frac{(2x)^n}{n}|)^{\frac{1}{n}}$$

$$(\frac{|(2x)^n|^{\frac{1}{n}}}{n^{\frac{1}{n}}})$$

$$= \frac{|2x|}{n^{\frac{1}{n}}}$$

$$= |2x|$$

conv if |2x| less than 1, div if greater than 1, inc if 1