Math 1953
Written Homework 7

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1. Infinite Series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

    Explain why alternating haromic series converges to a limit:
    This converges fairly easily using the Alternating Series Test:

    1: $\rightarrow$ 0?

    this acts like $\dfrac{1}{n}$ when looking at if it goes to 0, and $\dfrac{1}{n} \rightarrow 0$

    Decreasing?

    $\dfrac{1}{n} > \dfrac{1}{n+1}$, so yes

2. Series within 0.1 of true value
    n=10 = exactly -0.1, so using n=11 for the N+1 in the formula $\sum_{n=1}^{N} x_n$ = within N+1 of true value
    $\sum_{n=1}^{11}(-1)^{n+1}\frac{1}{n}$ = 0.7365

3. ln(2) = 0.6931, ln(2) + 0.1 = 0.7931, ln(2) < 0.7365 < ln(2) + 0.1

4. Positive and Negative Part

    Pos part:

$$1 + \frac{1}{3} + \frac{1}{5} + ... = \sum_{n=1}^{\infty} \frac{1}{2n-1}$$

LCT with $\dfrac{1}{n} = \dfrac{n}{2n-1}$.limit using L'H $= \dfrac{1}{2}$

$\frac{1}{n}$ diverges, so so does the pos part

Neg part:

$$-\frac{1}{2}-\frac{1}{4}-\frac{1}{6}=\sum_{n=1}^{\infty}-\frac{1}{2n}$$

Similar LCT with $\frac{1}{n}$ gives us that $\frac{1}{2n}$ converges similarly to $\frac{1}{n}$, so the neg part also diverges

3 Ratio Test

$$\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!} \qquad = \frac{\frac{((n+1)!)^2}{(2(n+1))!}}{\frac{(n!)^2}{(2n)!}}$$

$$= \frac{(2n)!((n+1)(n!))^2}{(2n+2)(2n+1)(2n)!(n!)^2}$$

$$= \frac{((n+1)(n!))^2}{(2n+1)(2n+2)(n!)^2}$$

$$= \frac{(n+1)^2(n!)^2}{(4n^2+6n+2)(n!)^2}$$

$$= \frac{n^2+2n+1}{4n^2+6n+2}$$

This is less than 1, so series converges