Math 1953 Written Homework 4

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HELPFUL
$\int_0^1 \frac{1}{x^p} dx$ Converges if $p < 1$
$\int_1^\infty \frac{1}{x^p} dx$ Converges if $p > 1$

1. For which values of p does the improper integral $\int_e^\infty \frac{1}{x(ln(x))^p} dx$ converge, and what is its value (in terms of p)?

$$\int_e^\infty \frac{1}{x(lnx)^p} dx = \lim_{t\to\infty} \int_e^t \frac{1}{x(lnx)^p} dx \tag{1}$$

Assuming $\lim_{t\to\infty}$ for next steps

$$= \int_e^t \frac{(ln(x))^{-p}}{x} dx \tag{2}$$

$$\tag{3}$$

2. $\int_0^\infty \frac{1}{x^3+\sqrt[2]{x}} dx$

$$= \int_0^1 \frac{1}{x^3+\sqrt[2]{x}} dx + \int_1^\infty \frac{1}{x^3+\sqrt[2]{x}} dx \tag{4}$$

Prove that both of these converge

$$= \frac{1}{x^3+\sqrt[2]{x}} \leq \frac{1}{x^3} \tag{5}$$

Because of the information in the box, $\int_1^\infty \frac{1}{x^3+\sqrt[2]{x}} dx$ converges

Also

$$= \frac{1}{x^3+\sqrt[2]{x}} \leq \frac{1}{x^{\frac{1}{2}}} \tag{6}$$