CALC III Notes day 38

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May 23, 2019

11.8 Power series

Remember: General form of power series

$$\sum_{n=0}^{\infty} c_n * (x-a)^n$$

EX

$$\sum_{n=0}^{\infty} x^n \text{conv if between -1,1, otherwise not}$$

Power series are basically polynomials that are getting closer and closer to the curve of a graph, within a given interval
Reminder from last class

$$\sum_{n=0}^{\infty} \frac{2^n}{n} * x^n$$

to find out where a power series conv:

Root / Ratio test:

Root:

$$\sqrt[n]{|\frac{2^n}{n} * x^n}$$

$$= \frac{2|x|}{n^{\frac{1}{n}}} \to 2|x|$$

Conv if 2|x| $<$ 1, div if greater, inconclusive if 1

What happens at endpoints?

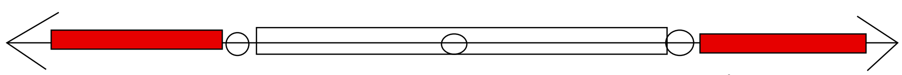

Figure 1: figure 1

Does it converge at x = 1 / 2? does it converge at -1 / 2?
For these values, just plug in the value of x and see which converge test to use

$$\sum_{n=1}^{\infty} \frac{2^n}{n} * (\frac{1}{2})^n$$

we already tried root test earlier, so lets try something else

$$2^n * (\frac{1}{2})^n = 1^n$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ this diverges!}$$

so x = $\frac{1}{2}$ diverges!

Plugging in x = $-\frac{1}{2}$:

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{ (see above)}$$

AST:

$$x_n = \frac{1}{n} \to 0$$

decreasing because n increasing

so negative -0.5 IS converging, but positive 0.5 diverges

For any power series, the interval of convergence is the interval of all x values where series converges

For the above problem, interval of convergence is $[\frac{-1}{2}, \frac{1}{2})$
This will ALWAYS be an interval with center x = a

Example:

$$\sum_{n=1}^{\infty} \frac{(4x-8)^n}{3^n n^2}$$

is this a power series? doesn't look like

can rewrite as

$$\sum_{n=1}^{\infty} \frac{1}{3^n n^2} * (4x-8)^n$$

still a problem, theres a 4 infront of the x

first thing: factor 4 out

$$... * (4(x-2))^n$$

$$\sum_{n=1}^{\infty} \frac{1}{3^n n^2} * 4^n (n-2)^n$$

$$= \sum_{n=1}^{\infty} \frac{4^n}{3^n n^2} * (x-2)^n$$

now its in the proper form

what is the interval of convergence?

but first, on why there is always one interval and its centered on a

when the x-a part is 0, it will ALWAYS converge

so when x =2, this will converge

what about if we plug in 2.5? $(\frac{1}{2})^n$

the two exponential parts "fight", and as long as we're closeish to 2, the part going to 0 will win

so the area that converges is the area closeish to 2 (a)

NOW BACK TO THE REAL PROBLEM

Step 1: root / ratio test

$$[|(\frac{4}{3})^n * \frac{1}{n^2} * (x-2)^n|]^{\frac{1}{n}}$$

only put abs value over where its needed

$$[(\frac{4}{3})^n * \frac{1}{n^2} * |x-2|^n]^{\frac{1}{n}}$$

Can split up the exponent

$$[(\frac{4}{3})^n]^{\frac{1}{n}} * [\frac{1}{n^2}]^{\frac{1}{n}} * [|x-2|^n]^{\frac{1}{n}}$$

the parts with n to the 1 / n cancel

and the n to the 2 / n part goes to 1

$$= \frac{4}{3} * |x-2|$$

CONVERGE if the above is less than 1, diverge if greater than 1, inconclusive if 1

Which x values satisfy $\frac{4}{3}|x-2| < 1$?

$$|x-2| < \frac{1}{\frac{4}{3}}$$

$$|x-2| < \frac{3}{4}$$

if $x = 2\frac{3}{4}$, then we hit a limit

negatively, if $x = 2 - \frac{3}{4}$, then we hit another limit

basically, for any of these the range is gonna be the distance to the left and right from the center value a

Step 2: Find out what happens at the end points (we know it converges INSIDE and diverges OUTSIDE, but what about DIRECTLY AT THEM?

try $2+\frac{3}{4}$

$$\sum_{n=1}^{\infty}(\frac{4}{3})^n * \frac{1}{n^2} * (2+\frac{3}{4}-2)^n$$

2s cancel
so do the two fractions

$$(\frac{4}{3})^n * (\frac{3}{4})^n = 1^n = 1$$

going back to the sum:

$$\sum_{n=1}^{\infty}\frac{1}{n^2} \text{ converges!}$$

trying negative value:

$$\sum_{n=1}^{\infty}...(2-\frac{3}{4}-2)^n$$

everything still cancels, but instead

$$\sum_{n=1}^{\infty}\frac{1}{n^2} * (-1)^n$$

AST: goes to 0, decreasing
so converges!

Example:

$$\sum_{n=1}^{\infty}\frac{x^n}{n!}$$

Root Test:

$$\sqrt[n]{|\frac{x^n}{n!}|}$$

Some simplification:

$$= \frac{|x|}{\sqrt[n]{n!}}$$

bottom goes to infinity (just a rule)
so whole thing goes to 0,

so it DOESNT MATTER what x is, interval is negative infinity to infinity