Calc III Notes Day 32

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Exam Friday Material after exam 2 up to 11.5 (ALT Series Test)

One last thing about AST

$$1 - 0 + \frac{1}{2} - 0 + \frac{1}{3} - 0 + \frac{1}{4}$$

This is alternating

The terms $\to 0$

So it should converge, but it actually diverges

basically just $\sum \frac{1}{n}$

It shoudn't actually converge, x_n is NOT decreasing

If the "x_n " decreasing hypothesis fails, then alt series might diverge!

11.6 Absolute Convergence, Ratio Test, and other things

Look at series

$$1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} ...$$

This series converges by AST, but also by telescoping (everything cancels except 1

$$\text{"Positive" part of series: } 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} ...$$

$$\text{"negative part": } -\frac{1}{2} - \frac{1}{3} - \frac{1}{4}$$

Diverges, going to negative ∞

This series "barely converges" because the pos ∞ and neg ∞ barely "canel out"

KIND of like an indeterminate form

Contrasting with

$$1 - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} - ...$$

Both positive and negative parts converge seperately, $\sum \frac{1}{n^2}$ converges

> DEFINITION:
> A series $\sum_{n=1}^{\infty} x_n$ **absolutely** converges if
>
> - pos part and neg part both converge
>
> or can be written as $\sum_{n=1}^{\infty} |x_n|$ conv $< - - -$ making both parts pos, and still conv

Ex

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

Does this converge?

$$\text{AST...?} \frac{1}{\sqrt{n}}$$

going to 0? yes

decreasing? yes, $\sqrt{n}$ is increasing, so decreasing

So this entire thing does converge

Does it absolutely converge?

$$\sum_{n=1}^{\infty} |\frac{(-1)^n}{\sqrt{n}}|$$

$$= \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

p is not bigger than 1, so it diverges

DEFINITION:
Conditionally Convergent means:

- convergent but NOT absolutely convergent

EX

$$\sum_{n=1}^{\infty} \frac{cos(n)}{n^2} \text{ Converges?}$$

- $\cos 1 = +$
- $\cos 2 =$ -
- $\cos 3 =$ -
- $\cos 4 =$ -

- $\cos 5 = +$

Not positive all the time, and also doesn't alternate, sign oscillates "randomly"
Can't use integral test, alternating test, etc
Let's try... absolute convergence?

$$\sum_{n=1}^{\infty} |\frac{cos n}{n^2}| = \sum_{n=1}^{\infty} \frac{|cos n|}{n^2} \text{because } n^2 \text{ is always pos}$$

$$\frac{|cos n|}{n^2} \leq \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} converges$$

due to the comparison test, the ABS series converges (always postive) converges due to comparison test with $\frac{1}{n^2}$.

Absolute Convergence is even **STRONGER** than convergence!

So original series converges, because absolute converges! cool!

This chapter is really about

RATIO / ROOT TESTS

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n} \text{ Does this converge or diverge?}$$

Which wins? 3^n. Exponential growth CRUSHES polynomial growth

The 3^n should make n^3 irrelevant, so that makes this "like a geometric series"

Idea: How to tell if $\sum_{n=1}^{\infty} x_n$ is "basically geometric"?

If this is geometric, then x_n is basically $r^n \rightarrow \sqrt[n]{x_n} == r$

ROOT TEST:
If $\sum_{n=1}^{\infty} x_n$ series and $\sqrt[n]{x_n} \to L$,
Then

1. if $L < 1$ originial series (absolutely) converges

2. if $1 < L$, then series diverges

3. if $L = 1$ then test was inconclusive and you have to use another test