CALC III Notes Day 30

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Homework help

Written Homework 3

$$\sum_{n=2}^{\infty} \frac{1}{n!}.$$

Show converges with comparison test
Idea: show that the above sum is less than or equal to something that converges

What is $\frac{1}{n!}$ less than?

$$\frac{1}{n!} \leq \frac{1}{2}$$

n! is greater than or equal to 2
Unfortunately, that isn't gonna help (that sum is still diverging)

$$\frac{1}{n!} \leq \frac{1}{n}$$

this STILL doesn't converge lmao

$$\frac{1}{n!} \leq \frac{1}{n^2}$$

This IS true, but how do we prove it?

prove n! greater than n sqared

$$n! \geq n(n-1)$$
$$= n^2 - n$$

Limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

11.5 Alternating Series

Alternating Series: Terms swap from pos, neg, pos, neg, etc

EX

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$$

Question: Do the alternating signs cause enough 'cancelling" to make this converge?

$$0 \leq series \leq 1$$

can "group" every two pairs and they're always greater than 0
If we group them in pairs IGNORING 1, every single thing inside those groups are positive, and 1 minus a bunch of positive things is less than 1
What if we grouped things after a point?

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + (\frac{1}{5} - \frac{1}{6}) + (\frac{1}{7} - \frac{1}{8}) ... > 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

Grouping after a different number

$$1 - \frac{1}{2} + \frac{1}{3} - (\frac{1}{4} - \frac{1}{5}) - (\frac{1}{6} - \frac{1}{7}) ... < 1 - \frac{1}{2} + \frac{1}{3}$$

Trapped series between two numbers $\frac{1}{4}$ apart: if we keep going, it turns out you can do this forever until the series converges

Lets talk about why this works
Alternating series should converge because of the reasoning above
say series is

$$\sum_{n=1}^{\infty} (-1)^n x_n x_n \geq 0.$$

or

$$\sum_{n=1}^{\infty}(-1)^{n+1}.$$

need some things:

1. $x_n \to 0$
2. x_n is decreasing

Then series converges!!

Thats all you need for the Alternating Series Test (AST)
If a series is alternating, just ceck if it goes to 0 and is decreasing
Much easier than limit or integral or comparison test, because if it follows these guidelines then it converges and you're done

EX

Does series

$$\sum_{n=2}^{\infty}(-1)^n * \frac{n^2}{n^3+1}.$$

Converge or Diverge?

$$x_n = \frac{n^2}{n^3+1}$$

just checking 2 things: does it go to 0, and is it decreasing?
turn into a function, take derivative

$$\begin{aligned}\frac{n^2}{n^3+1} &\to \frac{x^2}{x^3+1} \\ &\to L'H \lim_{x\to\infty}\frac{2x}{3x^2} \\ &= \lim_{x\to\infty}\frac{2}{3x} = 0\end{aligned}$$

now check if decreasing

$$\begin{aligned}(\frac{x^2}{x^3+1})' &= \frac{2x(x^3+1)-x^2(3x^2)}{(x^3+1)^2} \\ &= \frac{x(2-x^3}{(x^3+1)^2}\end{aligned}$$

Less than 0, so original function is decreasing (plug in numbers from sum)

Its decreasing and going to 0, so its converging

EX

$\sum_{n=2}^{\infty}(-1)^n * \frac{n}{2n+1}$

1. $x_n \to 0?$

$$\frac{2 * \frac{1}{n}}{(2n+1) * \frac{1}{n}} = \frac{1}{2 + \frac{1}{n}}$$
$$\to \frac{1}{2}$$

Terms don't go to zero, so it doesn't converge (this is a rule of all series, not just the Alternating Series)