Calc III Notes Day 24

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Main topics for test

- Improper Integrals
- Sequences

  Convergent?

  inc dec neither?

  Bounded above below both neither?
- Series

  Def of a series

  geometric

  is series converge, then terms $\to 0$

Reivew for test

3.c from practice exam

$$x_n = \frac{n}{n^2+1}.$$

- Does x n converge?
  (x_n) can be "turned into a function of x" $\frac{x}{x^2+1}$

$$\lim_{x\to\infty} \frac{x}{x^2+1}$$
$$= \frac{\infty}{\infty}$$

L'H

$$= \frac{1}{2x}$$
$$= \lim_{x\to\infty} \frac{1}{2x} = 0$$

(x_n) converges to zero

- is xn inc / dec?
  Take derivative
  $$\frac{x}{x^2+1}$$
  $$= (x)^{'}(x^2+1) - x(x^2+1)^{'}/(x^2+1)^2$$
  $$= \frac{1-x^2}{\text{Some positive number, its squared}}$$
  We're really looking at a sequence, so x is always bigger than one
  $$= 1 - x^2 \leq 0$$
  $$= Derivative is negaitve, so function is dec, so sequence is decreasing$$

- is xn bounded above / below ?
  Seq is convergent, so the sequence is bounded

Practice Exam Question 4

$$\sum_{n=0}^{\infty} \frac{3^{n+1}}{5^{n-1}}.$$

plug in n = 1

$$= \frac{3^2}{5^0}$$

Next term

$$= \frac{3^3}{5^1}$$

next term

$$= \frac{3^4}{5^2}$$
$$= \frac{3^2}{5^0}(1 + \frac{3}{5} + \frac{3^2}{5^2} + ...)$$

This turns into our formula with r between -1 and 1

$$= \frac{3^2}{5^0} * (\frac{1}{1-\frac{3}{5}})$$

WebAssign 3?

$$\lim_{x\to-\infty} x^2 * e^{2x}$$
$$= \infty * 0$$

this is an indeterminate form

$$\begin{aligned}
&= \frac{x^2}{e^{-2x}} \\
&= \frac{2x}{-2e^{-2x}} \\
&= \frac{2}{4e^{-2x}} \\
&= \frac{2}{\infty} = 0
\end{aligned}$$

WebAssign 3 q 15

$$\int_0^3 \frac{18}{x^2 - 6x + 5} dx.$$

Can plug in both end points, and its fine
However, there ARE bad points
DNE if $x^2 - 6x + 5 = 0$
(x -5)(x -1) = 0
5 is not in the integral but 1 is

$$\int_0^1 dx + \int_1^3 dx$$

this is an ugly one so

Comparison TEST

$$\int_1^\infty \frac{2x^2}{x^5 + 10} dx.$$

The only improper integral we have been taught are $\int_1^\infty \frac{1}{x^p} dx$

$$\begin{cases} conv & p > 1 \\ diverge & p \leq 1 \end{cases}$$