# CALC III
# Written Homework 9

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1. Find radius of convergence for the series

$$\sum_{n=0}^{\infty}(-1)^n * \frac{(2n)!}{(1-2n)4^n(n!)^2}x^n$$

Ratio Test:

$$\frac{|(-1)^{n+1} * \frac{(2(n+1))!}{(1-2(n+1))4^{n+1}((n+1)!)^2} * x^{n+1}|}{|(-1)^n * \frac{(2n)!}{(1-2n)4^n(n!)^2} * x^n|}$$

some simplifying, done using the following ideas

1:

$$\frac{x^{n+1}}{x^n} = x$$

2:

$$\begin{aligned}\frac{(n!)^2}{((n+1)!)^2} &= \frac{n^2 * (n-1)^2 * ...}{(n+1)^2 * n^2 * ...} \\ &= \frac{1}{(n+1)^2}\end{aligned}$$

3:

$$\begin{aligned}\frac{(2(n+1))!}{(2n)!} &= \frac{(2n+2)!}{(2n)!} \\ &= \frac{(2n+2)(2n+1)(2n)(2n-1)...}{(2n)(2n-1)...} \\ &= (2n+2)(2n+1)\end{aligned}$$

4:

$$\frac{4^n}{4^{n+1}} = \frac{1}{4}$$

Using these ideas, we get

$$\frac{(2n+2)(2n+1)}{4(n+1)^2} * |x|$$

simplifying more

$$\frac{2(n+1)(2n+1)}{4(n+1)^2} * |x| = \frac{2(2n+1)}{4(n+1)} * |x|$$

$$= \frac{n+\frac{1}{2}}{n+1} * |x| = \frac{1+\frac{\frac{1}{2}}{n}}{1+\frac{1}{n}} * |x|$$

limit to infinity =

$$|x|$$

thus, this converges when |x| $<$ 1

$\boxed{\text{Radius of convergence = 1}}$

2. (a) $(1+x)^{\frac{-1}{2}}$

derivative of $\sqrt{1+x}$ is $\frac{1}{2}(1+x)^{\frac{-1}{2}}$

so we need to derive the sum, and multiply both sides by 2 to offset

$$(\sum_{n=0}^{\infty}(-1)^n * \frac{(2n)!}{(1-2n)4^n(n!)^2} * x^n)' = \sum_{n=0}^{\infty}(-1)^n * \frac{(2n)!}{(1-2n)4^n(n!)^2} * x^n * 2$$

(b) $(1-x^2)^{\frac{-1}{2}}$

This is the same formula as above, but with $-x^2$ substituted for $x$.

$$= \sum_{n=0}^{\infty}(-1)^n * \frac{(2n)!}{(1-2n)4^n(n!)^2} * (-x^2)^n * 2$$

(c) arcsinx

derivative of arcsinx = $(1-x^2)^{-\frac{1}{2}}$

so if we take the integral of the previous series, we will have a formula for arcsin.

$$\int \text{above series} dx$$

$$= \sum_{n=0}^{\infty} (-1)^{2n} * \frac{(2n)! * 2}{(1-2n)4^n (n!)^2} * \frac{x^{2n+1}}{2n+1}$$

3. (a) first few terms: $1, \frac{-2}{(1+x)^3}, \frac{6}{(1+x)^4}$

conclusion: $f^{(n)}(0) = (-1)^n * \frac{n!}{(1+x)^{n+2}}$

(b) series = $\sum_{n=0}^{\infty} (-1)^n * \frac{n!}{(1+x)^{n+2}} x^n$

(c) geometric series

start with default series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

substitute -x

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n * x^n$$

differentiate both sides:

$$\frac{1}{(1+x)^2} = \sum_{n=0}^{\infty} (-1)^n * n x^{n-1}$$

4. (b)

$$ln(x) = \sum_{n=0}^{\infty} (1-x)^n$$

$$\frac{ln(x) + 1 - x}{(1-x)^2} = \frac{(\sum_{n=0}^{\infty} (1-x)^n) + 1 - x}{(1-x)^2}$$

$$= \frac{1 + (1-x)^1 + (1-x)^2 + ... + 1 - x}{(1-x)^2}$$

$$= \frac{1}{(1-x)^2} + \frac{1-x}{(1-x)^2} + \frac{(1-x)^2}{(1-x)^2} + \frac{(1-x)^3}{(1-x)^2} + ... + \frac{1}{(1-x)^2} - \frac{x}{(1-x)^2}$$

the terms become larger and larger in the sum as time goes on, so this series goes to infinity