CALC III Day 31

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Alternate Series Test

if series $\sum_{n=1}^{\infty}(-1)^n x_n$ or $\sum_{n=1}^{\infty}(-1)n+1x_n$ and $x_n \geq 0$, series is alternating.
If:

1. $x_n \to$ 0
2. x_n is decreasing

then series is converging!
if 1 fails, series diverges
if 2 fails, ???

EX

1:

$$\sum_{n=2}^{\infty} \frac{1}{lnn} * (-1)^n$$

$$= \lim_{x\to\infty} \frac{1}{lnx} = \frac{1}{\infty} = 0$$

2:Decreasing?

$$\frac{1}{lnx} = \frac{0 * lnx - 1 * \frac{1}{x}}{(lnx)^2}$$
$$= \frac{\frac{-1}{x}}{(lnx)^2}$$

-1 is negative, so we need to make sure rest is positive
x is always positive, and ln x is squared so its always positive so we good

Other way to tell if its decreasing:

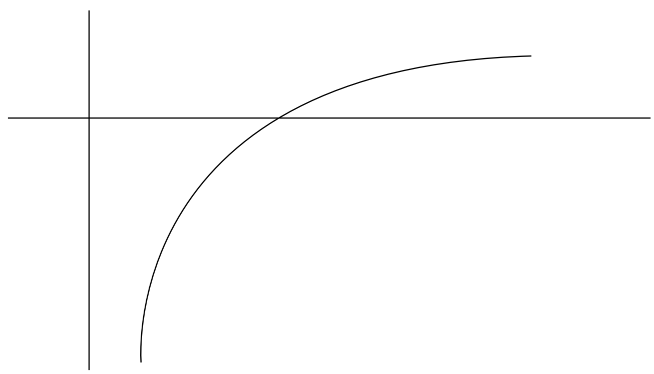

Figure 1: graph of ln x

ln x is increasing, so $\frac{1}{increasing}$ is decreasing

$\boxed{\text{Going to 0 and decreasing, so by AST its converging :)}}$

EX

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

converges to a number due to the AST

could we approximate to within 0.05?

the only way to get a guess / estimate for infinite series is to add up a bunch of terms
How many terms N do we need to add up to be certain that we're within $\frac{1}{N^2}$ of true value?
if add up the firt N turns, how far off am I from th eactual infinite series?

lets try 15 terms, how close am I to the infinite series?

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-....+\frac{1}{15^2}$$

how far apart is this from the infinite series? Take the series

$$(\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-....+\frac{1}{15^2})-(\frac{1}{1^2}-\frac{1}{2^2}..)$$

everything from the first term cancels, so we're left with

$$--\frac{1}{16^2}+\frac{1}{17^2}-...$$
$$=\frac{1}{16^2}-\frac{1}{17^2}+\frac{1}{18^2}...$$

this is less than $\frac{1}{16^2}$

Remember from yesterday, when we pair things together

less than $\frac{1}{16^2}$, and 0.05 is much larger than $\frac{1}{16^2}$ so we good

what we wanted: a number that has distance of less than 0.05 from value of infinite series

What did we learn from all of this?

If we have a series

$$\sum_{n=1}^{\infty}(-1)^n x_n \text{and 1 and 2 from AST are true}$$

then using 1st N terms gives estimate within distance x_{N+1} of from value of infinite series

So if you're trying to estimate the infinite series to within some number:

1. Figure out how much N must be to make $X_{N+!} <$ that number
2. then our guess is the sum of the first N terms

EX

$$\text{Approx} \sum_{n=2}^{\infty} \frac{(-1)^n}{lnn} \text{ to within } 0.01.$$