Calc III Notes Day 28

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May 8, 2019

Continued From last Class

Estimating Inf series / integrals

suppose you have some inf series $\sum_{n=1}^{\infty} x_n$ and it converges... but you don't know the value.

If you want to approximate $\sum_{n=1}^{\infty} x_n$ The best way to do it is just to add up a "bunch" of terms

Aka use the sum of the first N terms.

Question: How far off is $\sum_{n=1}^{N} x_n$ from $\sum_{n=1}^{\infty} x_n$?

Type of question: Approximate $\sum_{n=1}^{\infty} \frac{1}{n^3}$ to within 0.01:

$$\sum_{n=1}^{N} x_n = x_1 + x_1 + x_3 ... + x_N$$
$$\sum_{n=1}^{\infty} x_n = x_1 + x_2 + x_3 + ... + x_N + x_{N+1} ...$$

This the sum to N is off by $x_{N+1} + x_{N+2} ...$

KEY:: $\sum_{n=1}^{\infty} x_n < \int_N^{\infty} f(x)dx$ Basically, if we can get the integral to go to converge than we can squeeze the series.

Set up

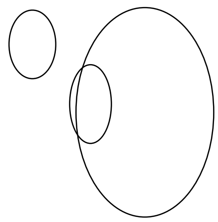

Figure 1: newfig

$$\begin{aligned}
\int_N^\infty \frac{1}{x^3}dx &= \lim_{t\to\infty}\int_N^t x^{-3}dx \\
&= \lim_{t\to\infty}\frac{x^{-2}}{-2}\text{from N to t} \\
&= \lim_{t\to\infty}\frac{t^{-2}}{-2}-\frac{N^{-2}}{-2} \\
&= \lim_{t\to\infty}-\frac{1}{2t^2}+\frac{1}{2N^2} \\
&= \frac{1}{2N^2}
\end{aligned}$$

Now set this to be less than our goal

$$\begin{aligned}
\frac{1}{2N^2} &< 0.01 \\
1 &< 2N^2 * 0.01 \\
100 &< 2N^2 \\
50 &< N^2 \\
N^2 &> 50
\end{aligned}$$

if N is greater than $\sqrt{50}$, then the sum is within 0.01 of the infinite sum $\boxed{\begin{array}{l}\text{THIS ONLY WORKS IF IT SATISFIES}\\ \text{THE INTEGRAL HYPOTHESIS!!!!}\end{array}}$

Telescoping Series

$$\sum_{n=4}^{\infty}(\frac{1}{n} - \frac{1}{n+2}) = \frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \frac{1}{7} + \frac{1}{9}.$$

Everything cancels except for the first two first parts ($\frac{1}{4}$ and $\frac{1}{5}$)

Web Assign-esque problem

find value of

$$\sum_{n=4}^{\infty}\frac{1}{n^2+n}$$

How to guess: Partial Fraction Decomposition

$$\frac{1}{n^2+n} = \frac{1}{n(n+1)}$$

$$\frac{1}{n(n+1} = \frac{A}{n} + \frac{B}{n+1}$$

reminder: if we multiply both sides by denominator $n(n+1)$ then things cancel such that

$$1 = A(n+1) + B(n)$$

Easiest way: plug in "easy" values of n

$$n = 0$$
$$1 = A * 1 + B * 0$$
$$A = 1$$
$$n = -1$$
$$1 = B(-1)$$
$$B = -1$$
$$= \sum_{n=4}^{\infty}(\frac{A}{n} + \frac{B}{n+1})$$
$$= \sum_{n=4}^{\infty}(\frac{1}{n} - \frac{1}{n+1})$$

11.4 Final thing: Comparison Test

If $0 \leq x_n \leq y_n$, then
if $\sum_{n=1}^{\infty} y_n$ converges, then $\sum_{n=1}^{\infty} x_n$ converges
if $\sum_{n=1}^{\infty} x_n$ diverges, then $\sum_{n=1}^{\infty} y_n$ diverges

Key things we want to compare these two: (Because they're what we know)

1. $\frac{1}{n^p}$
$$\begin{cases} conv & p > 1 div \\ p \leq 1 \end{cases}$$
2. r^n

EX

$$\sum_{n=4}^{\infty} \frac{lnn}{n}$$
$$= \frac{lnn}{n} > \frac{1}{n} \qquad = n \geq 4, lnn \geq ln4 > 1$$

so the original sum is also divergent by comparison test