Calc notes Day 19

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11.1 Sequences

Written as (X_N)

Convergence

EX:

$\sqrt[2]{n+2} \begin{cases} x_1 & \sqrt[2]{3} \\ x_2 & \sqrt[2]{4} \\ x_3 & \sqrt[2]{5} \end{cases}$ *etc*... Another way to describe sequence: recursive def:

$$x_{n=1} = \frac{1}{1+x_n}, x_1 = 1.$$

EX:

$$x_2 = \frac{1}{1+x_1} = \frac{1}{1+1} = \frac{1}{2}.$$

what does it mean to say seq x_n approaches / converges to a limit ?

IDEA $(X_n) \to L$ means:

No matter how close you want to get to x_n to get to L, it will happen if n is large enough
EX : $(\frac{1}{n})$
$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \cdot \frac{1}{5}$
This converges to 0.
Reason: for any tiny number you give me, if I go far enough in (x_n), then x_n is that close to 0.

Illustration

Can we make $(\frac{1}{n})$ be within 0.1 of 0?
if $n > 10 \to 0 < \frac{1}{n} < \frac{1}{10}$
this means that dist from $\frac{1}{n}$ to 0 is $< \frac{1}{10}$ after n = 10.
This tells us that we can get beneath 0.1, but so what?
EX part 2:
What about under 0.0005?

Convergence in "math speak"

For every $\epsilon > 0$, then there is N such that if $n > N$, then the distance between x_n and L is $< \epsilon$ if $n > 2000$, then $0 < \frac{1}{n} < 0.0005$

- If you go FAR ENOUGH, (past N), then n will be less than any given bounds

This is a lot to write, how can we show that $(x_n) \to L$ without doing that? 2 Techniques to show $(x_n) \to L$:

- Algebra + simplification + $\frac{1}{n} \to 0$

$$\frac{2^2+n}{2n^2-1} = \frac{(n^2+n) * \frac{1}{n^2}}{(2n^2-1) * \frac{1}{n^2}} = \frac{1+\frac{1}{n}}{2-\frac{1}{n} * \frac{1}{2}} = \frac{1+0}{2-0*0} = \frac{1}{2}.$$

- Calc / L'H

EX: What's the limit of $\frac{ln(n)}{n} = x_n$?

ONLY POSITIVE INTEGERS ARE ALLOWED FOR n in SEQUENCES

Moving back to the problem above:
If we graph $\frac{ln(n)}{n}$: We can only plug in positive **INTEGERS**. See notebook CALC III day 18 and 19 for fig 1.
The graph of $\frac{ln(n)}{n}$ is dots, which are a small part of graph $\frac{ln(x)}{x}$.
Can treat this like a **FUNCTION** anyway, and can use calculus on it:

$$\lim_{x\to\infty} \frac{ln(x)}{x} \to \frac{\infty}{\infty} \to \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0.$$

Therefore:

$$\lim_{n\to\infty} \frac{ln(n)}{n} \to 0.$$

too!

RULE: If you can write the terms of x_n as values of a function $f(n), \lim_{x\to\infty} f(x) = L$ then $(x_n) \to L$

What can't be turned into a function?

n! is an example of a sequence with CANNOT be turned into a function f(x) (in a simple way)

Increasing / Decreasing

What does it mean to say (x_n) is increasing?

For every n, $x_{n+1} > x_n$
Decreasing?
For every n, $x_{n+1} < x_n$

How to decide if seq is increasing or decreasing?

$x_n = \frac{n}{n+1}$
rewrite as a function:
$\frac{x}{x+1}$ is a function, take derivative, derivative is inc