Calc III Notes Day 26

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Notes: Web Assign 5 involves section 11.3, postponed until Friday.
Questions 6-8 not doable yet!
Inf Series Questions:

Does it converge (add up to some finite number?)

or Diverge (doesn't add to some finite number)?

So far, we know

1. How to check if geom. series converges / diverges
2. $(x_n) \to$ 0, then $\sum_{i=1}^{n} x_n$ conv?

   no, if $\sum_{i=1}^{n} x_n$ conv, then terms $x_n \to$ 0

   I.E, if x_n does not $\to$ 0, then $\sum_{i=1}^{n} x_n$ diverges

   The alternative is not neccesarily true

   EX: $1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + ...$

   The terms go to zero, but the sum still diverges (1 / 2 + 1 /2 = 1, etc)

If the terms go to 0, the serives might converge OR it might diverge, rest of quarter is how to check this

Big picture: finding out if the denominator or numerator goes to infinity faster

Stepping back for a bit..

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$$

This sum is the area of all rectangles in this infinite sum
Area of each rectangle is greater than the area under the curve y = 1 / x
Doing the integral of y = 1 / x gives a diverging improper integral, which by the comparison test means that the thing larger than it also diverges (goes to infinity)
EX 2:

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{1}{1^3} + \frac{1}{2^3} + 1^{3^3}.$$

If we set the rectangles to start area from the left,
These rectangles all fit under $y = \frac{1}{x^3}$
IE purple area $>$ sum of series
However if area starts to the left, the integral is $\int_0^\infty \frac{1}{x^3} dx$ Which fails
Instead, we can "chop off" the first term which equals 1
and do $\int_1^\infty \frac{1}{x^3} dx$ which equals $\frac{1}{2}$.
Since all the boxes are less than the integral, the sum from 2 to ∞ is less than 1 / 2.
adding back the 1 from the first term, then the entire thing is $< \frac{3}{2}$.
Thus, the original series converges.

Putting all of this together: Integral Test

How do decide whether something converges or diverges with integrals?

Say $\sum_{n=1}^{\infty} x_n$ satisfies:

1. We can turn func into f(x).
2. x_n decreasing
3. (x_n) is positive

Then convergence status of $\sum_{n=1}^{\infty} x_n$ is SAME as the convergence status of $\int_1^\infty f(x) dx$.
aka

$$\begin{cases} \text{integral converges} & \text{series converges} \\ \text{integral diverges} & \text{series diverges} \end{cases}$$

Example:

does series

$$\sum_{n=3}^{\infty} \frac{ln(n)}{n}.$$

Converge or diverge?

1. turn x_n into function:

   $f(x) = \frac{lnx}{x}$

2. $\frac{lnn}{n}$ decreasing?

   Usually take first derivative of f(x)

3. are terms $\frac{lnn}{n} > 0$?

   all n are greater than 3, which implies that ln n > 0, and n > 0, so everything is greater than 0