CALC Notes Day 27

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May 7, 2019

Written Homework Notes

$y_n = \frac{2^n}{n!}$ Show $y_n \to 0$ with Squeeze Theorm:
Trap y_n between 2 seq x_n, z_n
$x_n \leq y_n \leq z_n$ such that $x_n \to 0$, and $z_n \to$

$\boxed{\text{IDEA: Write out } \frac{2^n}{n!} = \frac{2*2*2*2(ntimes)}{n*(n-1)*(n-2)*...*2,1}}$ What is that less than? [

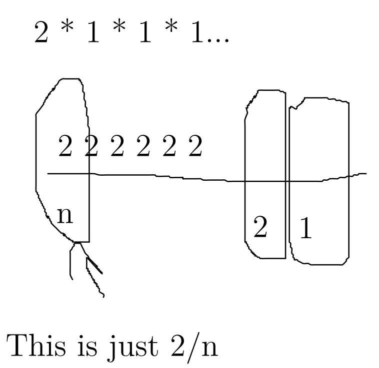

Figure 1: fig_1

For the first one, it equals 2. However for every other term, they're less than or equal to 1, thus the entire thing is less than or equal to 2
EXCEPT for the last one, which is $\frac{2}{n}$, so the entire thing is less than $\frac{4}{n}$. Since $\frac{4}{n}$

Integral Test!

If you have inf series $\sum_{n=1}^{\infty} x_n$ and

1. x_n can be "turned into function"
2. x_n is decreasing
3. $x_n \geq 0$

Then the convergent status of $\sum_{n=1}^{\infty} x_n$ is the same as $\int_1^{\infty} f(x_)dx$ "Can use integral to decide if series converges or diverges"

However, this only actually helps if you can **ACTUALLY DO THE INTEGRAL**

EX

Does

$$\sum_{n=3}^{\infty} \frac{lnn}{n}.$$

Converge or diverge?

1. f(x) = $\frac{lnx}{x}$
2. is above 0, since x $\geq$ 3
3. Decreasing? $(\frac{lnx}{x})' = \frac{\frac{1}{x}*x - lnx*1}{x^2} = \frac{1-lnx}{x^2}$
   $lnx \geq 3$
   ln 3 = 1.1
   so, ENTIRE derivative is less than or equal to 0, so function is decreasing.

These were the steps to say that we're ALLOWED to use the integral test – not the actual test itself

$$\int_3^\infty \frac{lnx}{x}dx$$

Use U sub, u = ln(x) , du = $\frac{1}{x}dx$

$$= \int udu$$

$$= \frac{u^2}{2}$$

$$= \frac{(lnx)^2}{2}\text{from } \infty \text{ to } 3$$

$$= \lim_{t\to\infty} \frac{(lnt)^2}{2} - \frac{(ln3)^2}{2}$$

$$= \frac{\infty}{2} = \infty$$

EX 2

$$\text{p} > 0 and$$

$$\sum_{n=1}^{\infty} \frac{1}{n^P}$$

Integral Test:

$$f(x) = \frac{1}{x^p}$$

since p is always greater than 0, this is always a real number greater than 0

$$\text{because n is inc, so } n^p \text{ is inc, so } \frac{1}{n^p} \text{ is dec}$$

Now we use the intergral test:

$$\int_1^\infty \frac{1}{x^p}dx = \begin{cases} convg & p > 1 \\ divg & p \leq 1 \end{cases}$$

one last thing...

$$\sum_{n=1}^{\infty} \frac{1}{n^3}.$$

Does this converge or diverge?

Converge, p is greater than 1, see above

$\boxed{\text{Could we estimate this to within 0.01?}}$

Try adding up the first 10 or terms of the series until we're within 0.01 of the infinite series

> Question: if I do
>
> $$\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + ... + \frac{1}{N^3}.$$
>
> How close am I do the infinite series?

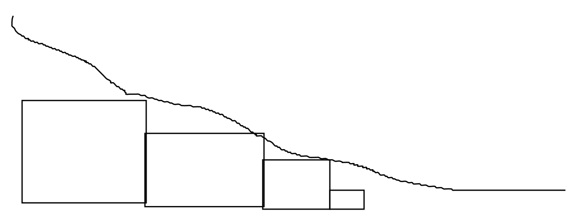

Figure 2: Adding up partial sums