1. $\int \frac{dx}{2+\cos x} = \int \frac{2\left(\frac{dt}{1+t^2}\right)}{\frac{2}{1}+\left(\frac{1-t^2}{1+t^2}\right)}$

$= 2\int \frac{\frac{dt}{1+t^2}}{\frac{2(1+t^2)+1-t^2}{1+t^2}}$ $\quad 2\int \frac{dt}{2(1+t^2)+1-t^2}$

$2\int \frac{dt}{2+2t^2+1-t^2}$

$2\int \frac{dt}{t^2+3}$ $= 2\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)+C$

$u^2=t^2 \quad a^2=3$
$u=t \quad a=\sqrt{3}$

$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right)+C$ //

2. $\int \frac{dx}{5+4\operatorname{sen}x} = \int \frac{2\left(\frac{dt}{1+t^2}\right)}{5+4\left(\frac{2t}{1+t^2}\right)} = \int \frac{\frac{2dt}{1+t^2}}{\frac{5(1+t^2)+8t}{1+t^2}}$

$2\int \frac{dt}{5t^2+8t+5} = \frac{2}{5}\int \frac{dt}{t^2+8/5\,t+\frac{16}{25}+1-\frac{16}{25}}$

$t \quad 4/5$

$t \quad 4/5$

$\frac{9}{25}$

$\frac{2}{5}\int \frac{dt}{\left(t+\frac{4}{5}\right)^2+\frac{9}{25}}$

$u^2 = \left(t+\frac{4}{5}\right)^2 \quad a^2 = \frac{9}{25}$

$u = \left(t+\frac{4}{5}\right) \quad a = \frac{3}{5}$

$\frac{2}{5}\cdot\frac{5}{3}\tan^{-1}\left(\frac{t+\frac{4}{5}}{\frac{3}{5}}\right)+C$

$\frac{2}{3}\tan^{-1}\left(\frac{t+\frac{4}{5}}{\frac{3}{5}}\right)+C$

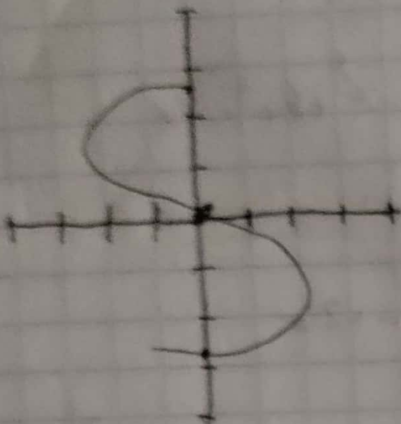
$\frac{\frac{5t+4}{5}}{\frac{3}{5}} = \frac{5t+4}{3}$

$\frac{2}{3}\tan^{-1}\left(\frac{5\tan\left(\frac{x}{2}\right)+4}{3}\right)+C$

3)

$x = -2y + y^3$

$(y)(-2 + y^2)$



| $x$ | $y$ |
|---|---|
| | $-\sqrt{2}$ |
| | $0$ |
| | $\sqrt{2}$ |

$0 \quad \pm\sqrt{2}$

$$\int_{-\sqrt{2}}^{0} -2y + y^3 = -\frac{2y^2}{2} - \frac{y^4}{4}\Big|_{-\sqrt{2}}^{0} = 1$$

$1 \times 2 = 2\ u^2$

4) $y = x^3 + 3x^2 + 2x$ $\quad$ $y = 2x^2 + 4x$

$x^3 - x^2 - 2x$

$(x)(x^2 - x - 2)$

$x \quad -2$

$x \quad 1$

$x = 0 \quad x = 1$

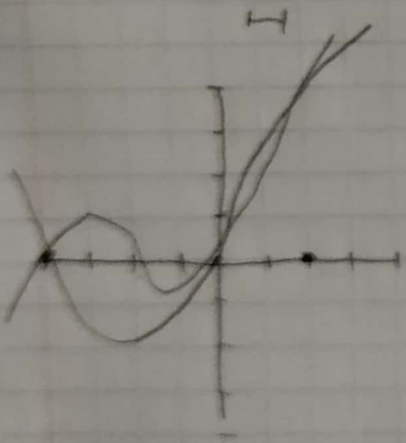
$x = -2$

$$\int_{-2}^{0} x^3 + 3x^2 + 2x - 2x^2 - 4x$$

$$\int_{-2}^{0} x^3 + x^2 - 2x = \frac{x^4}{4} + \frac{x^3}{3} - \frac{2x^2}{2} = -\frac{8}{3}$$

$$\int_{0}^{1} 2x^2 + 4x - x^3 - 3x^2 - 2x$$

$$\int_{0}^{1} -x^3 - x^2 + 2x \to -\frac{x^4}{4} - \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} = \frac{5}{12}$$



$$\frac{5}{12} + \frac{8}{3} = \frac{37}{12}\, u^2$$