Fleury's Algorithm in Action

Fleury's algorithm introduced a solution to finding an Euler tour or trail in a graph, calling for a sequential traversal and deletion of each edge in the graph within two possible cases. The first would be whether the graph has every vertex of even degree, and if such was the case, then the algorithm can start at any vertex in the graph. Since the first case contains every vertex of even degree, the chosen vertex is both the origin and terminus. The second would be when the graph has at most two vertices of odd degree, in which the algorithm must start at one of the odd degree vertices and creates an Euler trail from one odd vertex to the other.

The steps of the first case start by defining a connected graph *G = (V, E).* Since any vertex can be a starting point, we would denote the first vertex as $v_0 \in V$ with a trail $C_0 = v_0$. The trail C_n can be defined as the nth case for our format of the trail denoted as $C_n = v_0, e_1, v_1, \dots, e_n, v_n$. When the starting point has been established, the algorithm picks an arbitrary edge from the starting vertex denoted as v_n and traverses to another vertex v_{n+1} through e_{n+1} outside of the trail C_n. However, before deletion, the algorithm must avoid disconnecting the graph by checking whether e_{n+1} is a bridge within a few extra steps. The first is to have a trail C_n be initialized as C_0 where $n = 0$ and n increments to add edges and vertices onto the trail while the algorithm progresses to a bridge check, denoted as $G_n = G - E(C_n)$. The bridge check shows which edges can be deleted within the nth term of the trail until the graph has one final edge back to the starting vertex. Following the bridge check is accounting the next edge and vertex denoted as $C_{n+1} = C_n, e_{n+1}, v_{n+1}$. The algorithm continues this process of incrementing n to the successive iterations by $n = |E|$ since the algorithm checks whether n is incremented to the final stage. If the condition of $n = |E|$ is satisfied, the algorithm terminates with an Euler tour of $C = C_n$.