Polynomial regression

The example belows uses a temperature-energy dataset in order to illustrate how to perform a non linear regression.

Workflow:

1. Preparation
    - Extract the dataset from the _power_demand_vs*temperature.csv*
    - Explore the dataset and check for missing values
    - Plot the distribution
    - Divide the dataset into train and test
    - Create an evaluation function
2. First experiment
    - Create a linear model
    - Train the model on X_train and y_train
    - Evaluate the model on X_test and y_test
    - Visualize the prediction of the model
3. Second experiment
    - Create a polynomial regression model with degree 2
    - Train the model on X_train and y_train
    - Evaluate the model on X_test and y_test
    - Visualize the prediction of the model
4. Third experiment
    - repeat the steps done in the second experiment but with degree 3
5. Third experiment
    - repeat the steps done in the second experiment but with degree 4
6. Compare the evaluation of each model

In [ ]:

```
# Code source: Filippo Orazi
# License: BSD 3 clause

import matplotlib.pyplot as plt
import numpy as np
import scipy.stats
from sklearn import datasets, linear_model
from sklearn.metrics import mean_squared_error, r2_score
import pandas as pd
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
from sklearn.pipeline import Pipeline

random_state = 42 # this will be used to guarantee the repeatability of the experime
```

Dataset preparation

Load the dataset from a `.csv` file

This cell allows full compatibility between execution in Google Colab and in local

In [ ]:

```
try:
  import google.colab.files
  IN_COLAB = True
except:
  IN_COLAB = False
# from google.colab import files
if IN_COLAB:
    uploaded = files.upload()
```

The file must be available in the same directory, or uploaded in the Colab environment in the execution of the previous cell

Set the date column as index

In [ ]:

```
df = pd.read_csv('./power_demand_vs_temperature.csv')
df.index = df["date"]
df = df.drop("date", axis=1)
df.head()
```

Out[ ]:

	demand	temp
date		
2015-01-01	1.736065	1.7
2015-01-02	1.831672	2.2
2015-01-03	1.714934	14.4
2015-01-04	1.628577	15.6
2015-01-05	2.045394	0.0

Explore the dataset and check for missing values

In [ ]:

```
df.describe()
```

Out[ ]:

	demand	temp
count	1096.000000	1096.000000
mean	1.831796	16.927737
std	0.329434	10.791581
min	1.316033	-15.000000
25%	1.581654	8.900000
50%	1.731479	18.900000
75%	2.024869	26.100000
max	2.804025	33.900000

In [ ]:

```
print("The dataframe has " + str(df.isnull().sum().sum()) + " invalid rows")
```

```
The dataframe has 0 invalid rows
```

Create X and y

In [ ]:

```
X = df.drop("demand", axis=1)
Y = df["demand"]
print("X has shape" + str(X.shape))
print("Y has shape" + str(Y.shape))
```

```
X has shape(1096, 1)
Y has shape(1096,)
```

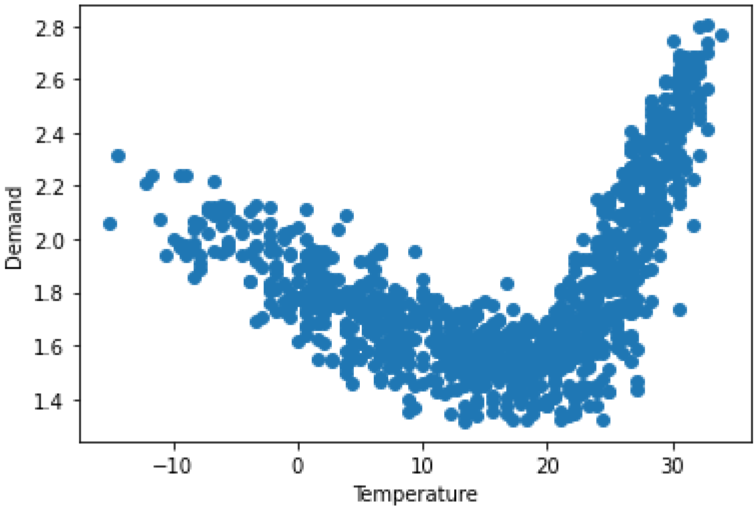
Plot the distribution

In [ ]:

```
plt.xlabel("Temperature")
plt.ylabel("Demand")
plt.plot(X, Y, 'o')
```

Out[ ]:

```
[<matplotlib.lines.Line2D at 0x23858aaa460>]
```



Divide the dataset in train and test splits

In [ ]:

```
Xtrain, Xtest, Ytrain, Ytest = train_test_split(X, Y, random_state=random_state)
print(f"Training set and test set have {Xtrain.shape[0]} and {Xtest.shape[0]} elemen
```

```
Training set and test set have 822 and 274 elements respectively
```

Create an evaluation function to compute, print and return the metrics: rmse r2 f-statistic and p-value

In [ ]:

```
# Computation of F-statistic and p-value for the regression
# http://facweb.cs.depaul.edu/sjost/csc423/documents/f-test-reg.htm

# n is number of observations
# p is number of regression parameters
def SSM(y_true, y_pred):
    to_ret = 0
    y_mean = np.mean(y_true)
    for i in range(len(y_true)):
        to_ret+=(y_mean - y_pred[i])**2
    return to_ret
def SST(y_true, y_pred):
    to_ret = 0
    y_mean = np.mean(y_true)
```

```
    for i in range(len(y_true)):
        to_ret+=(y_pred[i] - y_mean)**2
    return to_ret

def SSE(y_true, y_pred):
    to_ret = 0
    for i in range(len(y_true)):
        to_ret+=(y_true[i] - y_pred[i])**2
    return to_ret

def DFM(p):
    return p-1

def DFE(n, p):
    return n-p

def DFT(n):
    return n-1

def MSM(SSM, DFM):
    return SSM / DFM

def MSE(SSE, DFE):
    return SSE / DFE

def MST (SST, DFT):
    return SST / DFT

f_table = [{"Confidence Interval": [0, 0.900], "F-value": 1.89},
           {"Confidence Interval": [0, 0.950], "F-value": 2.28},
           {"Confidence Interval": [0, 0.975], "F-value": 2.68},
           {"Confidence Interval": [0, 0.990], "F-value": 2.22},
           {"Confidence Interval": [0, 0.999], "F-value": 4.71}]

levels = [0.100, 0.050, 0.025, 0.010, 0.001]

def my_evaluate(n, p, y_pred, y_true):
    F = MSM(SSM(y_true, y_pred), DFM(p)) / MSE(SSE(y_true, y_pred), DFE(n, p))
    level = DFE(n,p)/DFM(p)
    distances = [abs(level - l) for l in levels]
    f_table_i = np.argmin(distances)
    my_values = f_table[f_table_i]
    # interval = my_values["F-value"]
    r2 = SSM(y_true, y_pred) / SST(y_true, y_pred)
    print("Mean squared error: \t" + str(MSE(SSE(y_true, y_pred), DFE(n, p))))
    print("r2 score: \t\t" + str(r2_score(y_true, y_pred)))
    print("f-statistic: \t\t" + str(F))
    return MSE(SSE(y_true, y_pred), DFE(n, p)), r2_score(y_true, y_pred), F
```

First experiment

Create a linear model

Train the model on X_train and y_train

Evaluate the model on X_test and y_test

In [ ]:

```
model = LinearRegression()
model.fit(Xtrain, Ytrain)
y_pred = model.predict(Xtest)
vals00, vals01, vals02 = my_evaluate(Xtest.shape[0], Xtest.shape[1]+1, y_pred, Ytest
```

```
Mean squared error:      0.10047643370714782
r2 score:                0.16497160632995378
f-statistic:             42.936354203555396
```

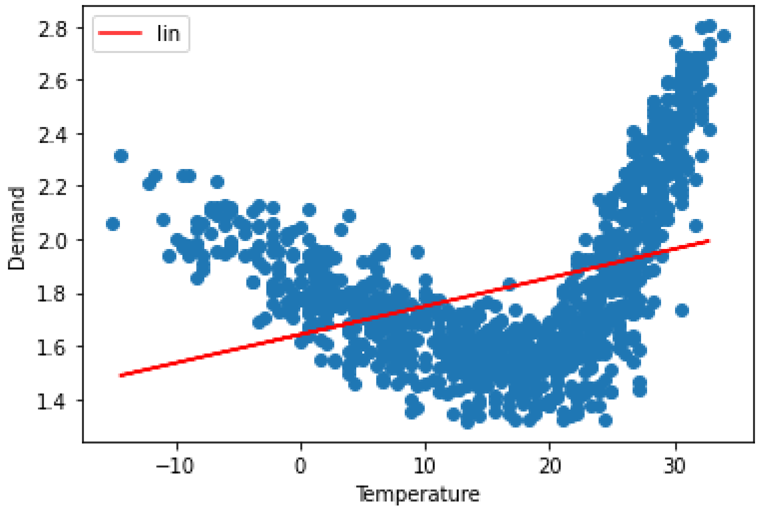
Visualize the prediction of the model

In [ ]:

```
plt.xlabel("Temperature")
plt.ylabel("Demand")
plt.plot(X, Y, 'o')
plt.plot(Xtest, y_pred, label="lin", color="red")
plt.legend(loc="upper left")
```

Out[ ]:

```
<matplotlib.legend.Legend at 0x238589ae970>
```



Second experiment - Polynomial regression

We can clearly see that the linear regression model cannot really approximate the data distribution.

We can now try with a non linear regression model:

1. Use the sklearn fucntion *PolynomialFeature* to create a new array of features. Set *degree=2* and _include*bias=False*
2. Train a Linear regression model with the new features
3. Evaluate the model
4. Visualize the predicted values of the model

Polynomial degree = 2

In [ ]:

```
features = PolynomialFeatures(degree = 2, include_bias = False)
```

In [ ]:

```
model_2 = LinearRegression()
pipeline = Pipeline(steps=[('t', features), ('m', model_2)])
pipeline.fit(Xtrain, Ytrain)
y_pred_2 = pipeline.predict(Xtest)
vals10, vals11, vals12 = my_evaluate(Xtrain.shape[0], Xtrain.shape[1] + 1, y_pred_2,
```

```
Mean squared error:      0.0105525523550657
r2 score:                0.7356133663568162
```

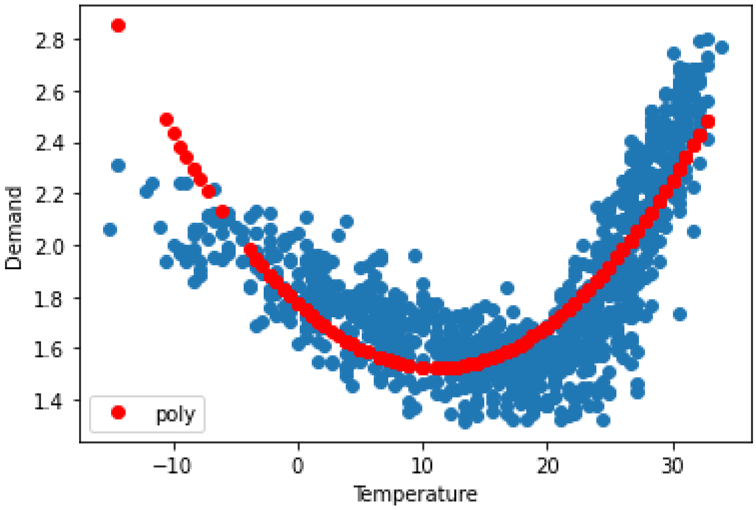
```
f-statistic:            2187.822635223873
```

In [ ]:

```
plt.xlabel("Temperature")
plt.ylabel("Demand")
plt.plot(X, Y, 'o')
plt.plot(Xtest, y_pred_2, 'o', label="poly", color="red")
plt.legend(loc="lower left")
```

Out[ ]:

```
<matplotlib.legend.Legend at 0x238516cd280>
```



Third experiment

Polynomial degree = 3

In [ ]:

```
features = PolynomialFeatures(degree = 3, include_bias = False)
```

In [ ]:

```
model_3 = LinearRegression()
pipeline = Pipeline(steps=[('t', features), ('m', model_3)])
pipeline.fit(Xtrain, Ytrain)
y_pred_3 = pipeline.predict(Xtest)
vals20, vals21, vals22 = my_evaluate(Xtrain.shape[0], Xtrain.shape[1] + 1, y_pred_3,
```

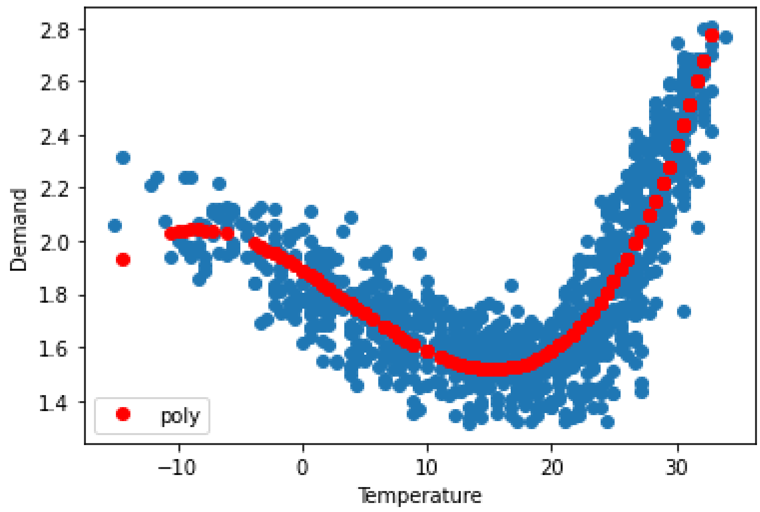
```
Mean squared error:     0.006682531835192724
r2 score:               0.83257395588548
f-statistic:            4255.404031253318
```

In [ ]:

```
plt.xlabel("Temperature")
plt.ylabel("Demand")
plt.plot(X, Y, 'o')
plt.plot(Xtest, y_pred_3, 'o', label="poly", color="red")
plt.legend(loc="lower left")
```

Out[ ]:

```
<matplotlib.legend.Legend at 0x23858bfd0d0>
```



Fourth experiment

Polynomial degree = 4

In [ ]:

```
features = PolynomialFeatures(degree = 4, include_bias = False)
```

In [ ]:

```
model_4 = LinearRegression()
pipeline = Pipeline(steps=[('t', features), ('m', model_4)])
pipeline.fit(Xtrain, Ytrain)
y_pred_4 = pipeline.predict(Xtest)
vals30, vals31, vals32 = my_evaluate(Xtrain.shape[0], Xtrain.shape[1] + 1, y_pred_4,
```

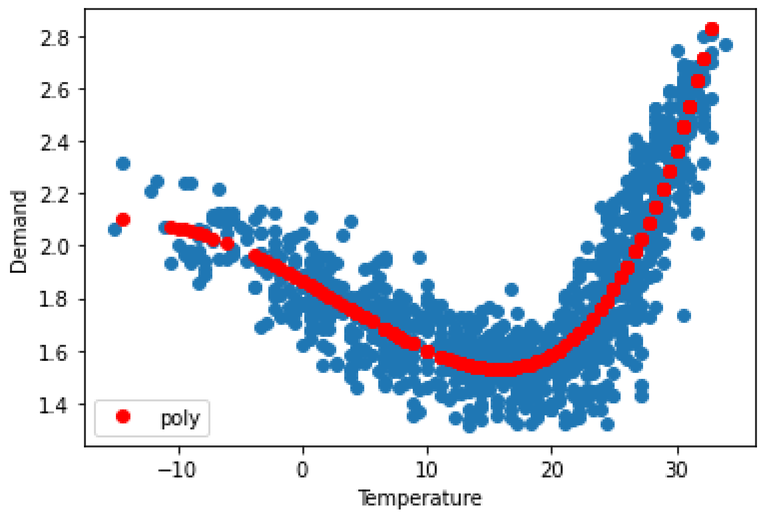
```
Mean squared error:      0.006581262402840308
r2 score:                0.8351111889083287
f-statistic:             4423.244586521021
```

In [ ]:

```
plt.xlabel("Temperature")
plt.ylabel("Demand")
plt.plot(X, Y, 'o')
plt.plot(Xtest, y_pred_4, 'o', label="poly", color="red")
plt.legend(loc="lower left")
```

Out[ ]:

```
<matplotlib.legend.Legend at 0x238571a7100>
```



Compare the performance of the four models

In [ ]:

```
vals_0 = np.array([[vals00, vals01, vals02]]).T
vals_1 = np.array([[vals10, vals11, vals12]]).T
vals_2 = np.array([[vals20, vals21, vals22]]).T
vals_3 = np.array([[vals30, vals31, vals32]]).T

compars = pd.DataFrame(np.concatenate((vals_0, vals_1, vals_2, vals_3), axis=1))
compars.columns = ["linear", "polynomial d = 2", "polynomial d = 3", "polynomial d =
compars.index = ["rmse", "r2", "f-statistic"]
compars.head()
```

Out[ ]:

	linear	polynomial d = 2	polynomial d = 3	polynomial d = 4
rmse	0.100476	0.010553	0.006683	0.006581
r2	0.164972	0.735613	0.832574	0.835111
f-statistic	42.936354	2187.822635	4255.404031	4423.244587

In [ ]: