Importing Numpy, Matplotlib and Scipy, also math

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.linalg
import math
```

Exercise 1

Creation of A in R^n*n random matrix and x_true column vector of n ones

In [ ]:
```
n=2
A = np.array([[1, 2], [3, 4]])
x_true = np.ones(n).reshape(-1,1)
```

*Computes the right-hand side of the linear system b = Ax_true**

In [ ]:
```
b = np.dot(A, x_true)
```

Computes the condition number in 2-norm of the matrix A.

In [ ]:
```
condition_number_2 = np.linalg.cond(A, 2)
print(condition_number_2)
```

```
14.933034373659268
```

It is ill-conditioned? **No, condition number is low**

What if we use the ∞-norm instead of the 2-norm?

In [ ]:
```
condition_number_inf = np.linalg.cond(A, np.inf)
print(condition_number_inf)
```

```
20.999999999999993
```

Still low condition number, so it is not conditioned

Solves the linear system Ax = b with the function np.linalg.solve().

In [ ]:
```
x = np.linalg.solve(A, b)
```

Computes the relative error between the solution computed before and the true solution xtrue

In [ ]:
```
rel_err = (np.linalg.norm(np.subtract(x, x_true), 2))/(np.linalg.norm(x_t
print(rel_err)
```

```
2.830524433501838e-16
```

Plot a graph (using matplotlib.pyplot) with the relative errors as a function of n and (in a new window) the condition number in 2-norm K2(A) and in ∞-norm, as a function of n

In [ ]:
```
def err_and_cond(n, vander = False, hilbert = False):
  # A = []
  # for i in range(0, n):
  #   A.append(np.arange((i*n)+1, n+(i*n)+1))
  #A= np.matrix(A)
  A = np.random.rand(n, n)
  if vander:
    A= np.vander(np.arange(1, n+1))
  if hilbert:
    A=scipy.linalg.hilbert(n)
  x_true = np.ones(n).reshape(-1,1)
  b = np.dot(A, x_true)
  condition_number_2 = np.linalg.cond(A, 2)
  condition_number_inf = np.linalg.cond(A, np.inf)
  x = np.linalg.solve(A, b)
  rel_err = (np.linalg.norm(np.subtract(x, x_true), 2))/(np.linalg.norm(x
  return rel_err, condition_number_2, condition_number_inf
```

Getting values

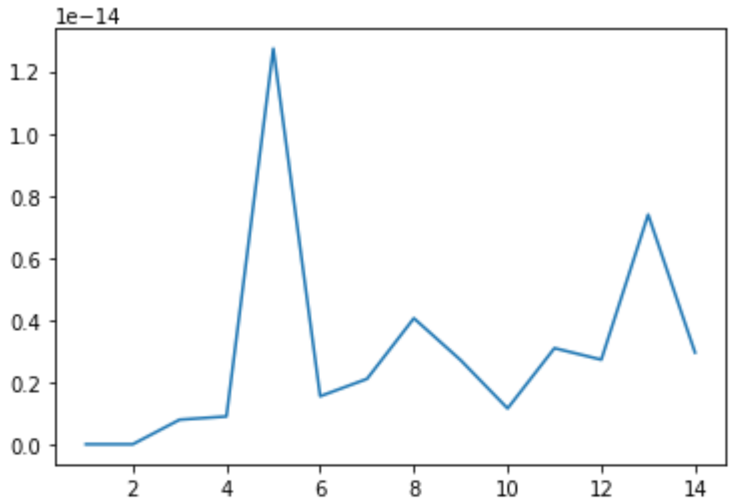
In [ ]:
```
def get_values(start, end, step, vander= False, hilbert= False):
  indeces = np.arange(start, end, step)
  rel_errs, conds_2, conds_inf = [], [], []
  for n in indeces:
    rel_err, cond_2, cond_inf = err_and_cond(n, vander, hilbert)
    rel_errs.append(rel_err)
    conds_2.append(cond_2)
    conds_inf.append(cond_inf)
  return indeces, rel_errs, conds_2, conds_inf
```

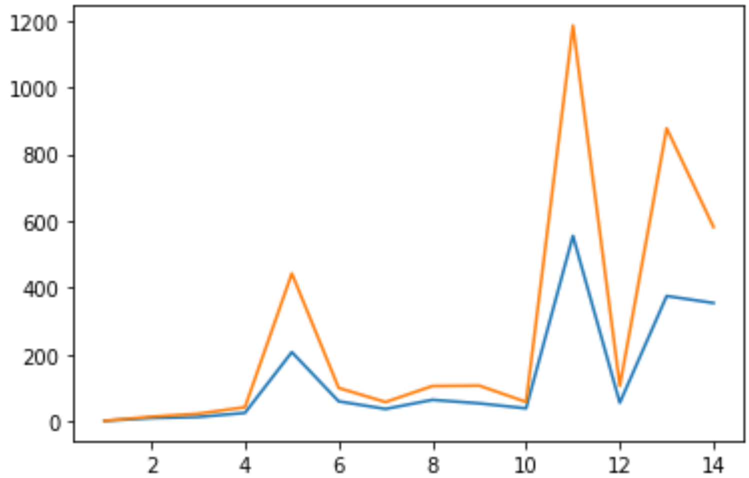
In [ ]:
```
indeces, rel_errs, conds_2, conds_inf = get_values(1, 15, 1)
```

Plotting

In [ ]:
```
def my_plot(indeces, rel_errs, conds_2, conds_inf):
  fig1 = plt.figure()
  plt.plot(indeces, rel_errs)
  plt.show()
  fig2 = plt.figure()
  plt.plot(indeces, conds_2)
  plt.plot(indeces, conds_inf)
  plt.show()
```

In [ ]:
```
my_plot(indeces, rel_errs, conds_2, conds_inf)
```

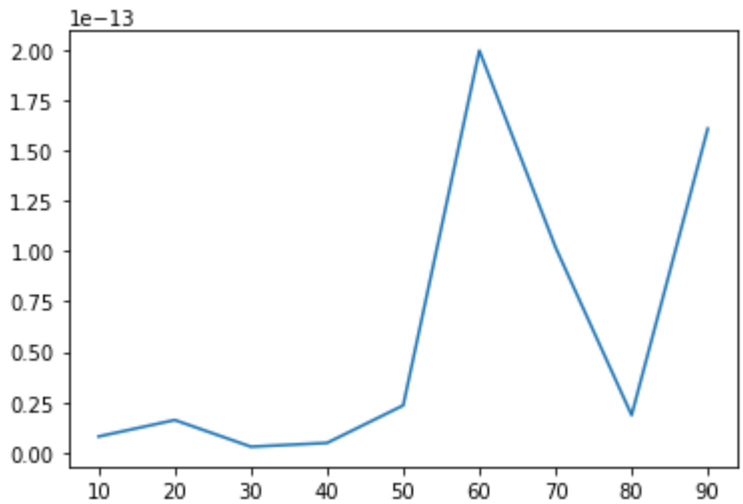


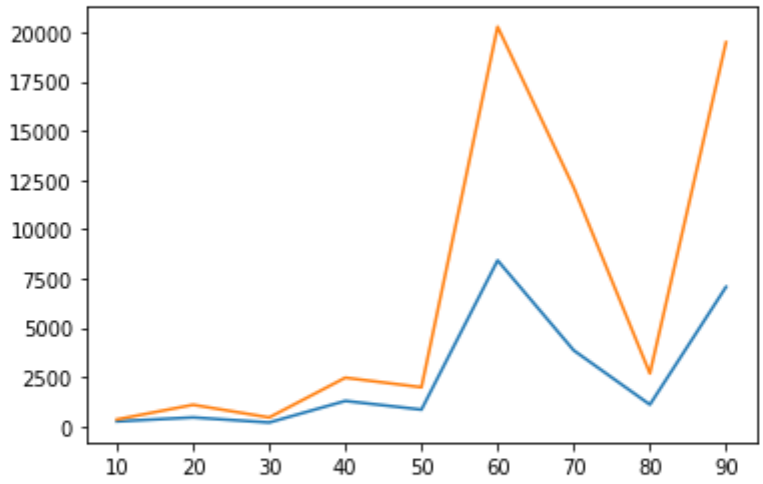


Exercise 2

Random Matrix

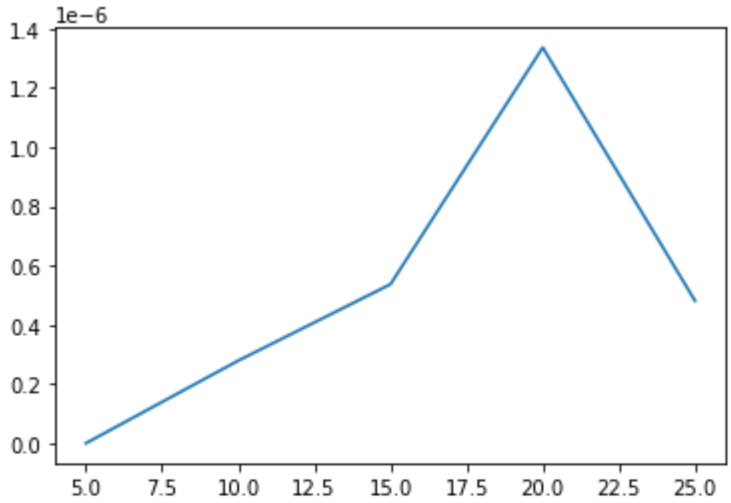
In [ ]:
```
i, err, cond_2, cond_inf = get_values(10, 100, 10)
my_plot(i, err, cond_2, cond_inf)
```

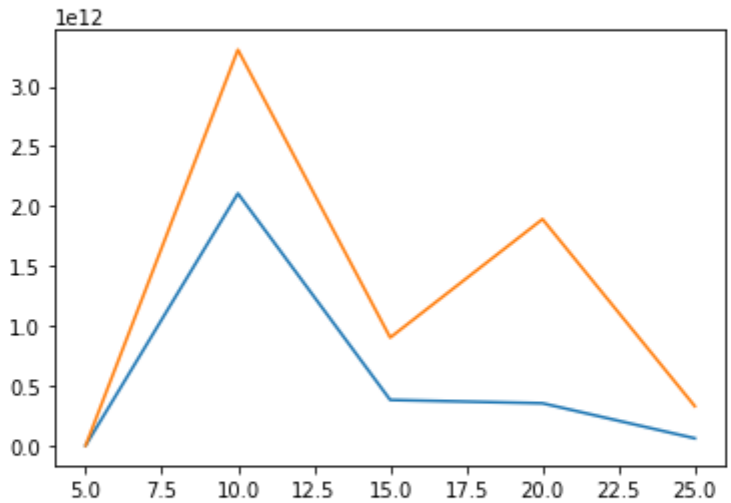




Vandermonde Matrix

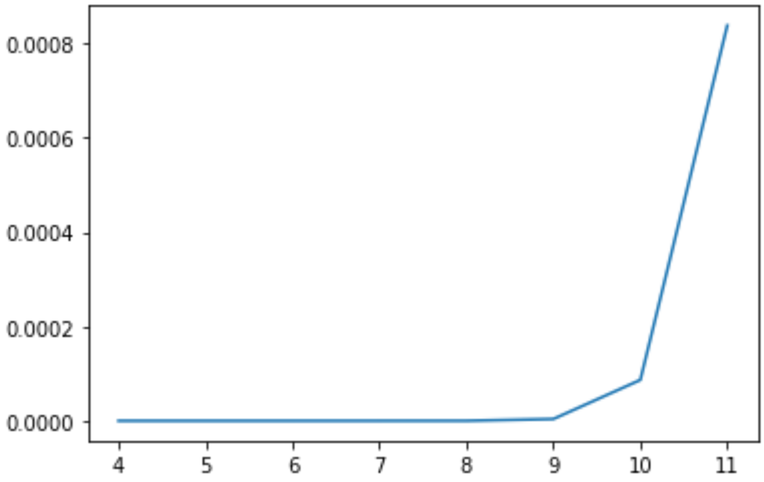
In [ ]:
```
i, err, cond_2, cond_inf = get_values(5, 30, 5, vander=True)
my_plot(i, err, cond_2, cond_inf)
```

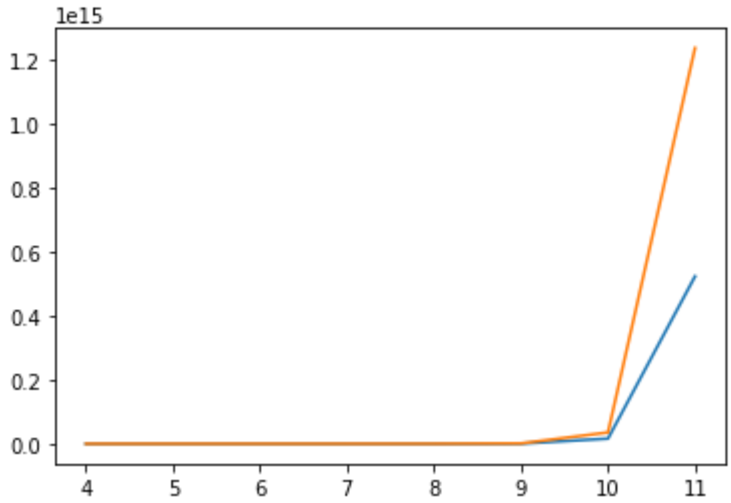




Hilbert Matrix

In [ ]:
```
i, err, cond_2, cond_inf = get_values(4, 12, 1, hilbert=True)
my_plot(i, err, cond_2, cond_inf)
```





Exercise 3

Compute machine_epsilon, which is defined as the smallest floating point number such that it holds: fl(1 + machine_epsilon) > 1

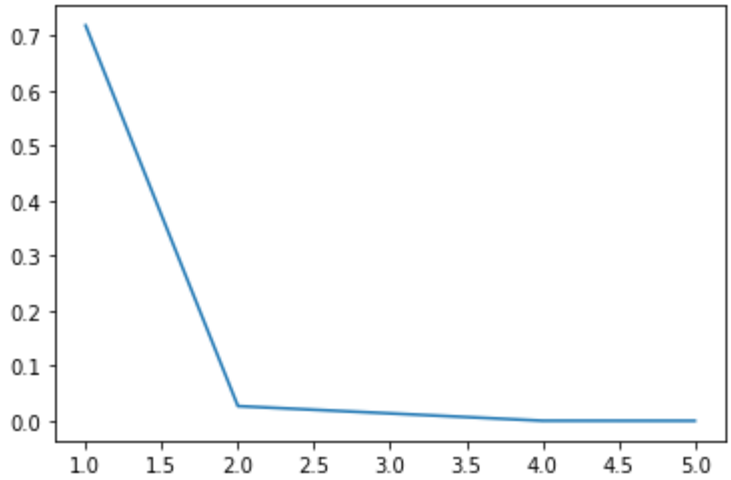
In [ ]:
```
machine_epsilon = 1
while float(1+machine_epsilon) > 1:
  machine_epsilon = machine_epsilon/2
print(machine_epsilon)
```

```
1.1102230246251565e-16
```

Let's consider the sequence an = (1 + 1 n) n. It is well known that: limn→∞ an = e where e is the Euler costant. Choose different values for n, compute an and compare it to the real value of the Euler costant. What happens if you choose a large value of n? Guess the reason

In [ ]:
```
def a_n(n):
  return (1+(1/n))**n
```

In [ ]:
```
ns = [1, 50, 100, 10000, 999999999]
ecs = []
for n in ns:
  ecs.append(a_n(n))
abs_errs = []
for ec in ecs:
  abs_errs.append(math.e - ec)
plt.figure()
plt.plot(np.arange(1,len(ns)+1), abs_errs)
plt.show()
```



Let's consider the matrices: A = 4 2 1 3 , B = 4 2 2 1 Compute the rank of A and B and their eigenvalues. Are A and B full-rank matrices? Can you infer some relationship between the values of the eigenvalues and the full-rank condition? Please, corroborate your deduction with other examples.

In [ ]:
```
A = np.matrix([[4, 2], [1, 3]])
B = np.matrix([[4, 2], [2 ,1]])
A_rank = np.linalg.matrix_rank(A)
B_rank = np.linalg.matrix_rank(B)
A_eig = np.linalg.eig(A)
B_eig = np.linalg.eig(B)
print(A_rank, B_rank)
print(A_eig[0])
print(B_eig[0])
```

```
2 1
[5. 2.]
[5. 0.]
```

A is a full rank matrix, while B is not (is rank 1). If there is eigval 0, the det = 0, so the matrix is not full ranked.

Example

In [ ]:
```
#Full rank and not a eigval = 0
C = np.matrix([[1, 2, 3], [6, 12, 9], [9, 2, 1]])
print(np.linalg.matrix_rank(C))
print(np.linalg.eig(C)[0])

#Not full rank and not a eigval = 0
D = np.matrix([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
print(np.linalg.matrix_rank(D))
print(np.linalg.eig(D)[0])

#Not full rank because a eigval = 0
E = np.matrix([[1, 0, 3], [2, 0, 4], [5, 0, 9]])
print(np.linalg.matrix_rank(E))
print(np.linalg.eig(E)[0])
```

```
3
[15.46698572 -3.8716714   2.40468568]
2
[ 1.61168440e+01 -1.11684397e+00 -1.30367773e-15]
2
[ 0.         -0.56776436 10.56776436]
```