homework 3

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1 Problem 1

Q: 设有如下三类模式样本集 w_1，w_2 和 w_3，其先验概率相等，求 S_w 和 S_b

$$
\begin{aligned}
w_1 &:(1,0)^T,(2,0)^T,(1,1)^T \\
w_2 &:(-1,0)^T,(0,1)^T,(-1,1)^T \\
w_3 &:(-1,-1)^T,(0,-1)^T,(0,-2)^T
\end{aligned}
$$

A: 先验概率相等, $P(w)=1/3$, 均值和协方差矩阵为

$$
\begin{aligned}
m_1 &= (\frac{4}{3},\frac{1}{3})^T, m_2 = (-\frac{2}{3},\frac{2}{3})^T, m_3 = (-\frac{1}{3},-\frac{4}{3})^T \\
C_1 &= E[(x-m_1)(x-m_1)^T] = \frac{1}{3}\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \\
C_2 &= E[(x-m_2)(x-m_2)^T] = \frac{1}{3}\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \\
C_3 &= E[(x-m_3)(x-m_3)^T] = \frac{1}{3}\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix}
\end{aligned}
$$

因此

$$
\begin{aligned}
S_w &= \sum_{i=1}^{3} P(w_i)C_i \\
&= \frac{1}{9}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
m_0 &= \sum_{i=1}^{3} P(w_i)m_i = (\frac{1}{9},-\frac{1}{9})^T \\
S_b &= \sum_{i=1}^{3} P(w_i)(m_i-m_0)(m_i-m_0)^T \\
&= \frac{1}{3}[(\frac{11}{9},\frac{4}{9})^T(\frac{11}{9},\frac{4}{9}) + (\frac{-7}{9},\frac{7}{9})^T(\frac{-7}{9},\frac{7}{9}) + (\frac{-4}{9},\frac{-11}{9})^T(\frac{-4}{9},\frac{-11}{9})] \\
&= \frac{1}{81}\begin{pmatrix} 62 & 13 \\ 13 & 62 \end{pmatrix}
\end{aligned}
$$

2 Problem 2

Q: 设有如下两类样本集，其出现的概率相等：

$$w_1 :\{(0,0,0)^T,(2,0,0)^T,(2,0,1)^T,(1,2,0)^T\}$$
$$w_2 :\{(0,0,1)^T,(0,1,0)^T,(0,-2,1)^T,(1,1,-2)^T\}$$

用 K-L 变换，分别把特征空间维数降到二维和一维，并画出样本在该空间中的位置 (可用 matlab 计算).

A: 样本均值为 $m=(0.75,0.25,0.125)^T$, 将所有样本减去均值得到新的样本集为

$w_1 :(-0.75,-0.25,-0.125)^T,(1.25,-0.25,-0.125)^T,(1.25,-0.25,0.875)^T,(0.25,1.75,-0.125)^T$

$w_2 :(-0.75,-0.25,0.875)^T,(-0.75,0.75,-0.125)^T,(-0.75,-2.25,0.875)^T,(0.25,0.75,-2.125)^T$

$$R=\sum_{i=1}^{2}P(w_i)E(xx^T)=\begin{pmatrix}0.6875 & 0.1875 & -0.09375\\ 0.1875 & 1.1875 & -0.53125\\ -0.09375 & -0.53125 & 0.859375\end{pmatrix}$$

求解 R 的特征值和特征向量,$\lambda_1=1.625,\lambda_2=0.64876246,\lambda_3=0.46061254$, 对应的一个特征向量组

$$\phi_1=(0.21538745,0.78975397,-0.57436653)^T,$$
$$\phi_2=(0.95853318,-0.05858624,0.27889386)^T,$$
$$\phi_3=(-0.18660756,0.61061961,0.76962413)^T$$

选取 λ_1,λ_2 对应的特征向量作为变换矩阵, 由 $y=(\phi_1,\phi_2)^Tx$ 得到变换后的二维模式特征为

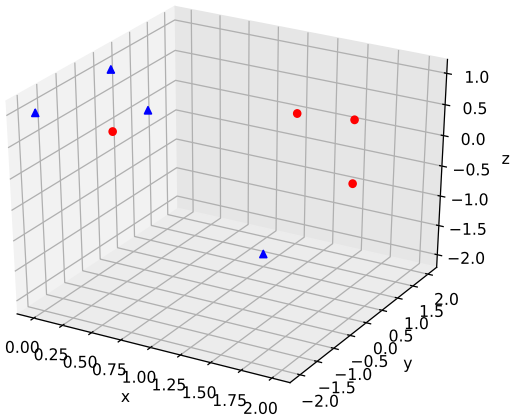
$w_1 :(0,0)^T,(0.4307749,1.91706637)^T,(-0.14359163,2.19596023)^T,(1.7948954,0.8413607)^T$

$w_2 :(-0.57436653,0.27889386)^T,(0.78975397,-0.05858624)^T,$
$(-2.15387448,0.39606634)^T,(2.15387448,0.34215922)^T$

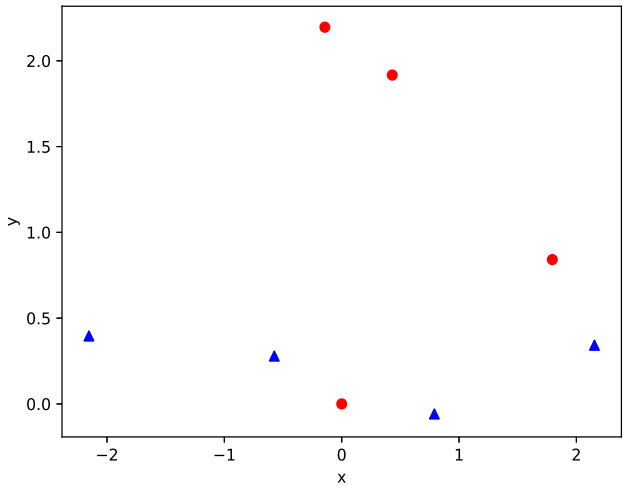
选择 λ_1 对应的特征向量作为变换矩阵, 由 $y=\phi_1^Tx$ 得到一维的模式特征为

$$w_1 :0.,0.4307749,-0.14359163,1.7948954$$
$$w_2 :-0.57436653,0.78975397,-2.15387448,2.15387448$$

原始数据的空间分布图如下



降到二维的空间分布



降到一维的空间分布