homework3

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1. Problem 1 of Page 195

$\Delta_1 = 5, \Delta_2 = 1, \Delta_3 = 1, \Delta_4 = -7,$

$$L_1 = \begin{pmatrix} 1 & & & \\ \frac{2}{5} & 1 & & \\ \frac{-4}{5} & & 1 & \\ 0 & & & 1 \end{pmatrix}, L_1^{-1} = \begin{pmatrix} 1 & & & \\ \frac{-2}{5} & 1 & & \\ \frac{4}{5} & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

then

$$A^{(1)} = L_1^{-1} A^{(0)} = \begin{pmatrix} 5 & 2 & -4 & 0 \\ 0 & 1/5 & -2/5 & 1 \\ 0 & -2/5 & 9/5 & 0 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & -2 & 1 & \\ & 5 & & 1 \end{pmatrix}, L_2^{-1} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & 2 & 1 & \\ & -5 & & 1 \end{pmatrix}$$

$$A^{(2)} = L_2^{-1} A^{(1)} = \begin{pmatrix} 5 & 2 & -4 & 0 \\ 0 & 1/5 & -2/5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & -3 \end{pmatrix}$$

$$L_3 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 2 & 1 \end{pmatrix}, L_3^{-1} = \begin{pmatrix} 1 & & & \\ & & & \\ & & 1 & \\ & & -2 & 1 \end{pmatrix}$$

$$\begin{aligned} A^{(3)} = L_3^{-1} A^{(2)} &= \begin{pmatrix} 5 & 2 & -4 & 0 \\ 0 & 1/5 & -2/5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -7 \end{pmatrix} \\ &= \text{diag}(5, 1/5, 1, -7) \begin{pmatrix} 1 & 2/5 & -4/5 & 0 \\ 0 & 1/5 & -2 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = DU \end{aligned}$$

thus

$$L = L_1 L_2 L_3 = \begin{pmatrix} 1 & & & \\ 2/5 & 1 & & \\ -4/5 & -2 & 1 & \\ 0 & 5 & 2 & 1 \end{pmatrix}$$

Doolittle decomposition:

$$A = L(DU) = L\hat{U}$$

here

$$\hat{U} = DU = \begin{pmatrix} 5 & 2 & -4 & 0 \\ 0 & 1/5 & -2/5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$

2. Problem 3 of Page 195

Proof:
since

$$L_1 = \begin{pmatrix} 1 & & & \\ c_{21} & 1 & & \\ \vdots & & \ddots & \\ c_{n1} & & & 1 \end{pmatrix}, L_1^{-1} = \begin{pmatrix} 1 & & & \\ -c_{21} & 1 & & \\ \vdots & & \ddots & \\ -c_{n1} & & & 1 \end{pmatrix}$$

here $c_{i1} = a_{i1}/a_{11}$ $(i = 2, 3, ..., n)$,

$$A^{(1)} = L_1^{-1} A^{(0)} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ 0 & \vdots & & \vdots \\ 0 & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

thus

$$B = \begin{pmatrix} a_{22} & a_{23} & \cdots & a_{2n} \\ a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & \vdots & \\ a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix}$$

since A is a real symmetric positive-definite matrix, B is a real symmetric positive-definite matrix and its diagonal elements are unchanging.

3. Problem 4 of Page 195

$\Delta_1 = 5, \Delta_2 = 1, \Delta_3 = 1,$

$$L_1 = \begin{pmatrix} 1 & & \\ 2/5 & 1 & \\ -4/5 & & 1 \end{pmatrix}, A^{(1)} = L_1^{-1} A^{(0)} = \begin{pmatrix} 5 & 2 & -4 \\ 0 & 1/5 & -2/5 \\ 0 & -2/5 & 9/5 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} 1 & & \\ & 1 & \\ & -2 & 1 \end{pmatrix}, A^{(2)} = L_2^{-1} A^{(1)} = \begin{pmatrix} 5 & 2 & -4 \\ 0 & 1/5 & -2/5 \\ 0 & 0 & 1 \end{pmatrix} = \mathrm{diag}(5, 1/5, 1) \begin{pmatrix} 1 & 2/5 & -4/5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

thus

$$
\begin{aligned}
G = L\tilde{D} = L_1 L_2 \mathrm{diag}(\sqrt{5}, \sqrt{1/5}, 1) \\
= \begin{pmatrix} \sqrt{5} & & \\ 2/\sqrt{5} & 1/\sqrt{5} & \\ -4/\sqrt{5} & -2/\sqrt{5} & 1 \end{pmatrix}
\end{aligned}
$$

$$
A = GG^T
$$

4. Problem 2 of Page 219

$T_{12}(c, s) : c = 2/\sqrt(13), s = 3/\sqrt(13),$

$$
T_{12} = \begin{pmatrix} 2/\sqrt{13} & 3/\sqrt{13} & & \\ -3/\sqrt{13} & 2/\sqrt{13} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}, T_{12}\boldsymbol{x} = (\sqrt{13}, 0, 0, 5)^T
$$

$T_14(c, s) : c = \sqrt{13}/\sqrt{38}, s = 5/\sqrt{38},$

$$
T_{14} = \begin{pmatrix} \sqrt{13}/\sqrt{38} & & & 5/\sqrt{38} \\ & 1 & & \\ & & 1 & \\ -5/\sqrt{38} & & & \sqrt{13}/\sqrt{38} \end{pmatrix}, T_{12}\boldsymbol{x} = (\sqrt{38}, 0, 0, 0)^T
$$

thus

$$
T = T_{14}T_{12} = \begin{pmatrix} 2/\sqrt{38} & 3/\sqrt{38} & 0 & 5/\sqrt{38} \\ -3/\sqrt{13} & 2/\sqrt{13} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -10/\sqrt{13*38} & -15/\sqrt{13*38} & 0 & \sqrt{13/38} \end{pmatrix}, T\boldsymbol{x} = \sqrt{38}\boldsymbol{e}_1
$$

5. Problem 4 of Page 219

6. Problem 7 of Page 219

7. Problem 8 of Page 219