homework1

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yes. Call the set of all real symmetric matrix "$\mathcal{S}$".

- obviously zero matrix $O \in \mathcal{S}$;
- $\forall A \in \mathcal{S}, \forall \alpha \in \mathcal{R}$, $(\alpha A)^T = \alpha A$, αA is still symmetric, so $\alpha A \in \mathcal{S}$;
- $\forall A, B \in \mathcal{S}$, $(A+B)^T = (A+B)$, $(A+B)$ is symmetric, $(A+B) \in \mathcal{S}$

in summary, the set of all real symmetric matrix is closed under addition and scalar multiplication, so it's a linear space.

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Proof:

Let $a_1 \cdot 1 + a_2 \cdot \cos^2 t + a_3 \cdot \cos 2t = 0, a_1, a_2, a_3 \in \mathcal{R}$. Suppose $1, \cos^2 t, \cos 2t$ is linear independent,then it must has $a_1 = a_2 = a_3 = 0$. But as we all know,$\cos 2t = 2\cos^2 t - 1$, if $a_1 = 1, a_2 = -2, a_3 = 1$, it also has $a_1 \cdot 1 + a_2 \cdot \cos^2 t + a_3 \cdot \cos 2t = 0$,which is contrary to the hypothesis. So $1, \cos^2 t, \cos 2t$ is linear dependent.

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let (η_1, η_2, η_3) is the new coordinate of vector $\mathbf{x}$, then $\mathbf{x} = \eta_1\mathbf{x}_1 + \eta_2\mathbf{x}_2 + \eta_3\mathbf{x}_3,$,which equals to

$$(\mathbf{x}_1^T, \mathbf{x}_2^T, \mathbf{x}_3^T)(\eta_1, \eta_2, \eta_3)^T = \mathbf{x}^T$$

so $(\eta_1, \eta_2, \eta_3)^T = (\mathbf{x}_1^T, \mathbf{x}_2^T, \mathbf{x}_3^T)^{-1}\mathbf{x}^T = (33, -82, 154)^T$.
new coordinate of $\mathbf{x}$: $(33, -82, 154)$

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let $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$,$\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{y}_4)$, then the original equation equals to

$$\begin{cases} \mathbf{x}(1,2,0,0)^T = \mathbf{y}(0,0,1,0)^T & (1) \\ \mathbf{x}(0,1,2,0)^T = \mathbf{y}(0,0,0,1)^T & (2) \\ \mathbf{y}(1,2,0,0)^T = \mathbf{x}(0,0,1,0)^T & (3) \\ \mathbf{y}(0,1,2,0)^T = \mathbf{x}(0,0,0,1)^T & (4) \end{cases}$$

so