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In[*]:= (*See supporting information
        sec. "2. Supporting Information Theory S2: Derivation of aliasing
        correction formula using bSSFP modes" *)

In[*]:= (*Use mode description [A9,A10] and put into the Fourier series expansion (below)*)

In[*]:= mplus[φ_] := Sum[c0 * z^p * Exp[-i * p * φ], {p, 0, ∞}] + Sum[cm1 * zc^{p-1} * Exp[i * p * φ], {p, 1, ∞}]

In[*]:= (*Perform a discrete Fourier transformation [A11] (below)
        (N is not a legal parameter so I replaced it with "M" in this mathematica script)*)

In[*]:= cDFT[n_, M_] := 1/M * Sum[mplus[2 * π / M * k] * Exp[i * n * 2 * π / M * k], {k, 0, M - 1}]

In[*]:= (* DERIVATION of ALIASING CORRECTION*)

In[*]:= (*1.0) only zeroth DFT mode*)

In[*]:= (*1.1) use N=4*)

In[*]:= cDFT[0, 4] // FullSimplify
Out[*]=

$$\frac{c_0 - cm_1 (-1 + z^4) zc^3 - c_0 zc^4}{(-1 + z^4) (-1 + zc^4)}$$


In[*]:= (*1.1.1) separete c0 and cm1*)

In[*]:= 
$$\frac{c_0 - c_0 zc^4}{(-1 + z^4) (-1 + zc^4)}$$
 // FullSimplify
Out[*]=

$$\frac{c_0}{1 - z^4}$$


In[*]:= 
$$\frac{-cm_1 (-1 + z^4) zc^3}{(-1 + z^4) (-1 + zc^4)}$$
 // FullSimplify
Out[*]=

$$-\frac{cm_1 zc^3}{-1 + zc^4}$$


In[*]:= (*1.2) use N=6*)

In[*]:= cDFT[0, 6] // FullSimplify
Out[*]=

$$\frac{c_0 - cm_1 (-1 + z^6) zc^5 - c_0 zc^6}{(-1 + z^6) (-1 + zc^6)}$$


In[*]:= (*1.2.1) separete c0 and cm1*)

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In[\*]:= 
$$\frac{c0 - c0 zc^6}{(-1 + z^6) (-1 + zc^6)} // \text{FullSimplify}$$

Out[\*]= 
$$\frac{c0}{1 - z^6}$$

In[\*]:= 
$$\frac{-cm1 (-1 + z^6) zc^5}{(-1 + z^6) (-1 + zc^6)} // \text{FullSimplify}$$

Out[\*]= 
$$-\frac{cm1 zc^5}{-1 + zc^6}$$

In[\*]:= (\*1.3) use N=8\*)

In[\*]:= CDFT[0, 8] // FullSimplify

Out[\*]= 
$$\frac{c0 - cm1 (-1 + z^8) zc^7 - c0 zc^8}{(-1 + z^8) (-1 + zc^8)}$$

In[\*]:= (\*1.3.1) seperate c0 and cm1\*)

In[\*]:= 
$$\frac{c0 - c0 zc^8}{(-1 + z^8) (-1 + zc^8)} // \text{FullSimplify}$$

Out[\*]= 
$$\frac{c0}{1 - z^8}$$

In[\*]:= 
$$\frac{-cm1 (-1 + z^8) zc^7}{(-1 + z^8) (-1 + zc^8)} // \text{FullSimplify}$$

Out[\*]= 
$$-\frac{cm1 zc^7}{-1 + zc^8}$$

In[\*]:= (\*1.4) One can assume based on the former results that the equation for the DFT modes is:

$$c0DFT = \frac{c0}{1-z^N} + \frac{cm1 zc^{N-1}}{1-zc^N} = \frac{c0}{1-z^N} + \frac{cm1 zc^N}{zc(1-zc^N)} *$$

In[\*]:= (\*2.0) only first DFT mode\*)

In[\*]:= (\*2.1) use N=4\*)

In[\*]:= CDFT[1, 4] // FullSimplify

Out[\*]= 
$$\frac{-cm1 (-1 + z^4) zc^2 + c0 (z - z zc^4)}{(-1 + z^4) (-1 + zc^4)}$$

In[\*]:= (\*2.1.1) seperate c0 and cm1\*)

$$\text{In[*]} := \frac{c0 (z - z zc^4)}{(-1 + z^4) (-1 + zc^4)} // \text{FullSimplify}$$

Out[\*]=

$$\frac{c0 z}{1 - z^4}$$

$$\text{In[*]} := \frac{-cm1 (-1 + z^4) zc^2}{(-1 + z^4) (-1 + zc^4)} // \text{FullSimplify}$$

Out[\*]=

$$-\frac{cm1 zc^2}{-1 + zc^4}$$

In[\*]:= (\*2.2) use N=6\*)

In[\*]:= CDFT[1, 6] // FullSimplify

Out[\*]=

$$-\frac{c0 z}{-1 + z^6} - \frac{cm1 zc^4}{-1 + zc^6}$$

In[\*]:= (\*2.2.1) seperate c0 and cm1\*)

$$\text{In[*]} := -\frac{c0 z}{-1 + z^6} // \text{FullSimplify}$$

Out[\*]=

$$-\frac{c0 z}{-1 + z^6}$$

$$\text{In[*]} := -\frac{cm1 zc^4}{-1 + zc^6} // \text{FullSimplify}$$

Out[\*]=

$$-\frac{cm1 zc^4}{-1 + zc^6}$$

In[\*]:= (\*2.3) use N=8\*)

In[\*]:= CDFT[1, 8] // FullSimplify

Out[\*]=

$$\frac{-cm1 (-1 + z^8) zc^6 + c0 (z - z zc^8)}{(-1 + z^8) (-1 + zc^8)}$$

In[\*]:= (\*2.3.1) seperate c0 and cm1\*)

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In[*]:= 
$$\frac{c0 (z - z zc^8)}{(-1 + z^8) (-1 + zc^8)} // FullSimplify$$

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Out[*]= 
$$\frac{c0 z}{1 - z^8}$$

```

```
In[*]:= 
$$\frac{-cm1 (-1 + z^8) zc^6}{(-1 + z^8) (-1 + zc^8)} // FullSimplify$$

```

```
Out[*]= 
$$-\frac{cm1 zc^6}{-1 + zc^8}$$

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In[*]:= (*2.4) One can assume based on the  
former results that the equation for the DFT modes is:
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$$c1DFT = \frac{c0 * z}{1 - z^N} + \frac{cm1 zc^{N-2}}{1 - zc^N} = \frac{c0 z}{1 - z^N} + \frac{cm1 zc^{N-1}}{zc(1 - zc^N)} *$$

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In[*]:= (*3.0) only minus first DFT mode*)
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In[*]:= (*3.1) use N=4*)
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In[*]:= cDFT[-1, 4] // FullSimplify
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Out[*]= 
$$\frac{cm1 - cm1 z^4 - c0 z^3 (-1 + zc^4)}{(-1 + z^4) (-1 + zc^4)}$$

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```
In[*]:= (*3.1.1) separete c0 and cm1*)
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```
In[*]:= 
$$\frac{-c0 z^3 (-1 + zc^4)}{(-1 + z^4) (-1 + zc^4)} // FullSimplify$$

```

```
Out[*]= 
$$-\frac{c0 z^3}{-1 + z^4}$$

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In[*]:= 
$$\frac{cm1 - cm1 z^4}{(-1 + z^4) (-1 + zc^4)} // FullSimplify$$

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Out[*]= 
$$\frac{cm1}{1 - zc^4}$$

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```
In[*]:= (*3.2) use N=6*)
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In[*]:= cDFT[-1, 6] // FullSimplify
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Out[*]= 
$$\frac{cm1 - cm1 z^6 - c0 z^5 (-1 + zc^6)}{(-1 + z^6) (-1 + zc^6)}$$

```

In[\*]:= (\*3.2.1) sepearate c0 and cm1\*)

In[\*]:= 
$$\frac{-c0 z^5 (-1 + z c^6)}{(-1 + z^6) (-1 + z c^6)} // FullSimplify$$

Out[\*]= 
$$-\frac{c0 z^5}{-1 + z^6}$$

In[\*]:= 
$$\frac{cm1 - cm1 z^6}{(-1 + z^6) (-1 + z c^6)} // FullSimplify$$

Out[\*]= 
$$\frac{cm1}{1 - z c^6}$$

In[\*]:= (\*3.3) use N=8\*)

In[\*]:= CDFT[-1, 8] // FullSimplify

Out[\*]= 
$$\frac{cm1 - cm1 z^8 - c0 z^7 (-1 + z c^8)}{(-1 + z^8) (-1 + z c^8)}$$

In[\*]:= (\*3.3.1) sepearate c0 and cm1\*)

In[\*]:= 
$$\frac{-c0 z^7 (-1 + z c^8)}{(-1 + z^8) (-1 + z c^8)} // FullSimplify$$

Out[\*]= 
$$-\frac{c0 z^7}{-1 + z^8}$$

In[\*]:= 
$$\frac{cm1 - cm1 z^8}{(-1 + z^8) (-1 + z c^8)} // FullSimplify$$

Out[\*]= 
$$\frac{cm1}{1 - z c^8}$$

In[\*]:= (\*3.4) One can assume based on the former results that the equation for the DFT modes is:

$$cm1DFT = \frac{c0 * z^{N-1}}{1 - z^N} + \frac{cm1}{1 - z c^N} = \frac{c0 * z^{N-1}}{1 - z^N} + \frac{cm1 z c}{z c (1 - z c^N)} *$$

In[\*]:= (\*4.0) only minus second DFT mode\*)

In[\*]:= (\*4.1) use N=4\*)

In[\*]:= CDFT[-2, 4] // FullSimplify

Out[\*]= 
$$-\frac{c0 z^2}{-1 + z^4} - \frac{cm1 z c}{-1 + z c^4}$$

`In[*]:= (*4.1.1) sepearate c0 and cm1*)`

`In[*]:= - $\frac{c0 z^2}{-1 + z^4}$  // FullSimplify`

`Out[*]=`

$$-\frac{c0 z^2}{-1 + z^4}$$

`In[*]:= - $\frac{cm1 zc}{-1 + zc^4}$  // FullSimplify`

`Out[*]=`

$$-\frac{cm1 zc}{-1 + zc^4}$$

`In[*]:= (*4.2) use N=6*)`

`In[*]:= CDFT[-2, 6] // FullSimplify`

`Out[*]=`

$$-\frac{c0 z^4}{-1 + z^6} - \frac{cm1 zc}{-1 + zc^6}$$

`In[*]:= (*4.2.1) sepearate c0 and cm1*)`

`In[*]:= - $\frac{c0 z^4}{-1 + z^6}$  // FullSimplify`

`Out[*]=`

$$-\frac{c0 z^4}{-1 + z^6}$$

`In[*]:= - $\frac{cm1 zc}{-1 + zc^6}$  // FullSimplify`

`Out[*]=`

$$-\frac{cm1 zc}{-1 + zc^6}$$

`In[*]:= (*4.3) use N=8*)`

`In[*]:= CDFT[-2, 8] // FullSimplify`

`Out[*]=`

$$-\frac{c0 z^6}{-1 + z^8} - \frac{cm1 zc}{-1 + zc^8}$$

`In[*]:= (*4.3.1) sepearate c0 and cm1*)`

`In[*]:= - $\frac{c0 z^6}{-1 + z^8}$  // FullSimplify`

`Out[*]=`

$$-\frac{c0 z^6}{-1 + z^8}$$

```
In[*]:= -  $\frac{cm1\ zc}{-1 + zc^8}$  // FullSimplify
Out[*]= -  $\frac{cm1\ zc}{-1 + zc^8}$ 
```

(\*4.4) One can assume based on the former results that the equation for the DFT modes is:

$$cm2DFT = \frac{c\theta * z^{N-2}}{1-z^N} + \frac{cm1\ zc}{1-zc^N} = \frac{c\theta * z^{N-2}}{1-z^N} + \frac{cm1\ zc^2}{zc(1-zc^N)} *)$$

In[\*]:= (\*5.1) Summary:

$$\text{we have: } c\theta DFT = \frac{c\theta}{1-z^N} + \frac{cm1\ zc^N}{zc(1-zc^N)} \quad \text{and} \quad c1DFT = \frac{c\theta\ z^n}{1-z^N} + \frac{cm1\ zc^{N-1}}{zc(1-zc^N)}$$

One can assume the general equation is:

$$c_{n,DFT}(N) = \frac{c\theta\ z^n}{1-z^N} + \frac{cm1\ zc^{N-n}}{zc(1-zc^N)} *)$$

In[\*]:= (\*5.2) Summary:

$$\text{we have: } cm1DFT = \frac{c\theta * z^{N-1}}{1-z^N} + \frac{cm1\ zc}{zc(1-zc^N)} \quad \text{and} \quad cm2DFT = \frac{c\theta * z^{N-2}}{1-z^N} + \frac{cm1\ zc^2}{zc(1-zc^N)}$$

One can assume the general equation is:

$$c_{-n,DFT}(N) = \frac{c\theta * z^{N-n}}{1-z^N} + \frac{cm1\ zc^n}{zc(1-zc^N)} *)$$

In[\*]:= (\*These are the equations shown in the paper (aliasing correction) and which are validated in numerical simulations within computer precision\*)

In[\*]:= (\*Mathematica has somehow problems with odd N (N=5,7 etc) so the numerical simulation was necessary to prove the exactness of the above conjecture also for odd numbers\*)