In[2]:= (*Derivation of Eq.[9]*)

In[1]:= $$\text{c0t[P0\_] := P0} * \frac{1 + r^2}{1 - r^2} * (1 - \text{E2} * r)$$

In[3]:= (*Use absolute values of P0 in Eq.[1]*)

In[4]:= $$\text{P0t[E1\_, E2\_, M0\_] := } k * \frac{\text{M0} * (1 - \text{E1}) * \text{Sin}[\alpha]}{1 - \text{E1} * \text{E2}^2 + \left(\text{E2}^2 - \text{E1}\right) * \text{Cos}[\alpha]} * \text{Exp}[-\text{TE} / \text{T2}]$$

In[5]:= (*Use E1 solution from other mathematica file (I just copy-pasted below)*)

In[6]:= $$\text{E1t[E2\_, r\_] := } \frac{\text{E2} - 2\,r + \text{E2}\,r^2 + \text{E2}\left(1 - 2\,\text{E2}\,r + r^2\right)\text{Cos}[\alpha]}{\text{E2}\left(1 - 2\,\text{E2}\,r + r^2\right) + \left(\text{E2} - 2\,r + \text{E2}\,r^2\right)\text{Cos}[\alpha]}$$

In[7]:= (*Use E2 solution from Eq.[6] and insert x=
cm1/c0 and assume c0 and cm1 are absolute values in the following*)

In[8]:= $$\text{E2t[r\_] := } \frac{r + \frac{\text{cm1}}{\text{c0}}}{1 + \frac{\text{cm1}}{\text{c0}} * r}$$

In[9]:= (*Perform self consistency approach according
to the description before Eq.[9] in the manuscript*)

In[21]:= (*1) Insert the P0 defintion into the mode expression c0*)

In[13]:= c0t[P0t[E1, E2, M0]] // FullSimplify

Out[13]=
$$\frac{e^{-\frac{\text{TE}}{\text{T2}}}\,(-1 + \text{E1})\,k\,\text{M0}\,(-1 + \text{E2}\,r)\left(1 + r^2\right)\text{Sin}[\alpha]}{\left(-1 + r^2\right)\left(-1 + \text{E1}\,\text{E2}^2 + \left(\text{E1} - \text{E2}^2\right)\text{Cos}[\alpha]\right)}$$

In[23]:= (* 2) Define c0 in dependence of E1 and E2*)

In[14]:= $$\text{c0t2[E1\_, E2\_] := } \frac{e^{-\frac{\text{TE}}{\text{T2}}}\,(-1 + \text{E1})\,k\,\text{M0}\,(-1 + \text{E2}\,r)\left(1 + r^2\right)\text{Sin}[\alpha]}{\left(-1 + r^2\right)\left(-1 + \text{E1}\,\text{E2}^2 + \left(\text{E1} - \text{E2}^2\right)\text{Cos}[\alpha]\right)}$$

In[25]:= (*3) Insert the E1 defintion into the mode expression c0*)

In[15]:= c0t2[E1t[E2, r], E2] // FullSimplify

Out[15]=
$$\frac{2\,e^{-\frac{\text{TE}}{\text{T2}}}\,k\,\text{M0}\,r\,(-1 + \text{E2}\,r)\,\text{Tan}\left[\frac{\alpha}{2}\right]}{\text{E2}\left(-1 + r^2\right)}$$

In[27]:= (* 4) Define c0 in dependence of E2 and r*)

In[16]:= $$\text{c0t3[E2\_, r\_] := } \frac{2\,e^{-\frac{\text{TE}}{\text{T2}}}\,k\,\text{M0}\,r\,(-1 + \text{E2}\,r)\,\text{Tan}\left[\frac{\alpha}{2}\right]}{\text{E2}\left(-1 + r^2\right)}$$

In[29]:= (*5) Insert the E2 defintion into the mode expression c0*)

In[17]:= `c0t3[E2t[r], r] // FullSimplify`

Out[17]=

$$\frac{2\, c0\, e^{-\frac{TE}{T2}}\, k\, M0\, r\, \mathrm{Tan}\left[\frac{\alpha}{2}\right]}{cm1 + c0\, r}$$

In[31]:= `(*6) Solve the expression of c0 in dependence of {M0,cm1,c0,r} for M0*)`

In[18]:= $$\mathrm{Solve}\left[c0 == \frac{2\, c0\, e^{-\frac{TE}{T2}}\, k\, M0\, r\, \mathrm{Tan}\left[\frac{\alpha}{2}\right]}{cm1 + c0\, r}, M0\right] \text{ // FullSimplify}$$

Out[18]=

$$\left\{\left\{M0 \to \frac{e^{\frac{TE}{T2}}\, (cm1 + c0\, r)\, \mathrm{Cot}\left[\frac{\alpha}{2}\right]}{2\, k\, r}\right\}\right\}$$

In[19]:= `(*Check if equation is equal to Eq.[9]*)`

In[20]:= $$\frac{e^{\frac{TE}{T2}}\, (cm1 + c0\, r)\, \mathrm{Cot}\left[\frac{\alpha}{2}\right]}{2\, k\, r} - \left(\frac{c0 + cm1 / r}{2 * k * \mathrm{Tan}\left[\frac{\alpha}{2}\right]} * e^{\frac{TE}{T2}}\right) \text{ // FullSimplify}$$

Out[20]=

0

In[32]:= `(*Since it is zero Eq.[9] is correct*)`