```
In[•]:= (*See supporting information
         sec. "2. Supporting Information Theory S2: Derivation of aliasing
            correction formula using bSSFP modes" *)

In[•]:= (*Use mode description [A9,A10] and put into the Fourier series expansion (below)*)
```

In[•]:= mplus[φ_] := Sum[c0 * z^p * Exp[-i * p * φ], {p, 0, ∞}] + Sum[cm1 * zc^(p-1) * Exp[i * p * φ], {p, 1, ∞}]

$$\text{mplus}[\varphi\_] := \text{Sum}\left[c0 * z^{p} * \text{Exp}[-i * p * \varphi], \{p, 0, \infty\}\right] + \text{Sum}\left[cm1 * zc^{p-1} * \text{Exp}[i * p * \varphi], \{p, 1, \infty\}\right]$$

```
In[•]:= (*Perform a discrete Fourier transformation [A11] (below)
         (N is not a legal parameter so I replaced it with "M" in this mathematica script)*)
```

In[•]:=

$$\text{cDFT}[n\_, M\_] := \frac{1}{M} * \text{Sum}\left[\text{mplus}\left[\frac{2*\pi}{M} * k\right] * \text{Exp}\left[i * n * \frac{2*\pi}{M} * k\right], \{k, 0, M-1\}\right]$$

```
In[•]:= (* DERIVATION of ALIASING CORRECTION*)

In[•]:= (*1.0) only zeroth DFT mode*)

In[•]:= (*1.1) use N=4*)

In[•]:= cDFT[0, 4] // FullSimplify
```

Out[•]=

$$\frac{c0 - cm1\left(-1 + z^{4}\right) zc^{3} - c0\, zc^{4}}{\left(-1 + z^{4}\right)\left(-1 + zc^{4}\right)}$$

```
In[•]:= (*1.1.1) seperate c0 and cm1*)
```

In[•]:=

$$\frac{c0 - c0\, zc^{4}}{\left(-1 + z^{4}\right)\left(-1 + zc^{4}\right)} \text{ // FullSimplify}$$

Out[•]=

$$\frac{c0}{1 - z^{4}}$$

In[•]:=

$$\frac{-cm1\left(-1 + z^{4}\right) zc^{3}}{\left(-1 + z^{4}\right)\left(-1 + zc^{4}\right)} \text{ // FullSimplify}$$

Out[•]=

$$-\frac{cm1\, zc^{3}}{-1 + zc^{4}}$$

```
(*1.2) use N=6*)

In[•]:= cDFT[0, 6] // FullSimplify
```

Out[•]=

$$\frac{c0 - cm1\left(-1 + z^{6}\right) zc^{5} - c0\, zc^{6}}{\left(-1 + z^{6}\right)\left(-1 + zc^{6}\right)}$$

```
In[•]:= (*1.2.1) seperate c0 and cm1*)
```

In[∘]:= $\frac{c0 - c0\,zc^6}{\left(-1 + z^6\right)\left(-1 + zc^6\right)}$ // FullSimplify

Out[∘]=
$$\frac{c0}{1 - z^6}$$

In[∘]:= $\frac{-cm1\left(-1 + z^6\right)zc^5}{\left(-1 + z^6\right)\left(-1 + zc^6\right)}$ // FullSimplify

Out[∘]=
$$-\frac{cm1\,zc^5}{-1 + zc^6}$$

In[∘]:= (*1.3) use N=8*)

In[∘]:= cDFT[0, 8] // FullSimplify

Out[∘]=
$$\frac{c0 - cm1\left(-1 + z^8\right)zc^7 - c0\,zc^8}{\left(-1 + z^8\right)\left(-1 + zc^8\right)}$$

In[∘]:= (*1.3.1) seperate c0 and cm1*)

In[∘]:= $\frac{c0 - c0\,zc^8}{\left(-1 + z^8\right)\left(-1 + zc^8\right)}$ // FullSimplify

Out[∘]=
$$\frac{c0}{1 - z^8}$$

In[∘]:= $\frac{-cm1\left(-1 + z^8\right)zc^7}{\left(-1 + z^8\right)\left(-1 + zc^8\right)}$ // FullSimplify

Out[∘]=
$$-\frac{cm1\,zc^7}{-1 + zc^8}$$

In[∘]:= (*1.4) One can assume based on the former results that the equation for the DFT modes is: $c0DFT = \frac{c0}{1-z^N} + \frac{cm1\ zc^{N-1}}{1-zc^N} = \frac{c0}{1-z^N} + \frac{cm1\ zc^N}{zc\left(1-zc^N\right)}$ *)

In[∘]:= (*2.0) only first DFT mode*)

In[∘]:= (*2.1) use N=4*)

In[∘]:= cDFT[1, 4] // FullSimplify

Out[∘]=
$$\frac{-cm1\left(-1 + z^4\right)zc^2 + c0\left(z - z\,zc^4\right)}{\left(-1 + z^4\right)\left(-1 + zc^4\right)}$$

In[ ]:= (*2.1.1) seperate c0 and cm1*)

In[ ]:= $\frac{c0\left(z - z\,zc^4\right)}{\left(-1 + z^4\right)\left(-1 + zc^4\right)}$ // FullSimplify

Out[ ]=

$$\frac{c0\,z}{1 - z^4}$$

In[ ]:= $\frac{-cm1\left(-1 + z^4\right)zc^2}{\left(-1 + z^4\right)\left(-1 + zc^4\right)}$ // FullSimplify

Out[ ]=

$$-\frac{cm1\,zc^2}{-1 + zc^4}$$

In[ ]:= (*2.2) use N=6*)

In[ ]:= cDFT[1, 6] // FullSimplify

Out[ ]=

$$-\frac{c0\,z}{-1 + z^6} - \frac{cm1\,zc^4}{-1 + zc^6}$$

In[ ]:= (*2.2.1) seperate c0 and cm1*)

In[ ]:= $-\frac{c0\,z}{-1 + z^6}$ // FullSimplify

Out[ ]=

$$-\frac{c0\,z}{-1 + z^6}$$

In[ ]:= $-\frac{cm1\,zc^4}{-1 + zc^6}$ // FullSimplify

Out[ ]=

$$-\frac{cm1\,zc^4}{-1 + zc^6}$$

In[ ]:= (*2.3) use N=8*)

In[ ]:= cDFT[1, 8] // FullSimplify

Out[ ]=

$$\frac{-cm1\left(-1 + z^8\right)zc^6 + c0\left(z - z\,zc^8\right)}{\left(-1 + z^8\right)\left(-1 + zc^8\right)}$$

In[ ]:= (*2.3.1) seperate c0 and cm1*)

In[∘]:= $\frac{\text{c0}\left(z - z\,\text{zc}^8\right)}{\left(-1+z^8\right)\left(-1+\text{zc}^8\right)}$ // FullSimplify

Out[∘]=

$$\frac{\text{c0}\,z}{1-z^8}$$

In[∘]:= $\frac{-\text{cm1}\left(-1+z^8\right)\text{zc}^6}{\left(-1+z^8\right)\left(-1+\text{zc}^8\right)}$ // FullSimplify

Out[∘]=

$$-\frac{\text{cm1}\,\text{zc}^6}{-1+\text{zc}^8}$$

In[∘]:= (*2.4) One can assume based on the former results that the equation for the DFT modes is: $\text{c1DFT}=\frac{\text{c0}*z}{1-z^N}+\frac{\text{cm1 zc}^{N-2}}{1-\text{zc}^N}=\frac{\text{c0 }z}{1-z^N}+\frac{\text{cm1 zc}^{N-1}}{\text{zc}\left(1-\text{zc}^N\right)}$ *)

In[∘]:= (*3.0) only minus first DFT mode*)

In[∘]:= (*3.1) use N=4*)

In[∘]:= cDFT[-1, 4] // FullSimplify

Out[∘]=

$$\frac{\text{cm1}-\text{cm1}\,z^4-\text{c0}\,z^3\left(-1+\text{zc}^4\right)}{\left(-1+z^4\right)\left(-1+\text{zc}^4\right)}$$

In[∘]:= (*3.1.1) seperate c0 and cm1*)

In[∘]:= $\frac{-\text{c0}\,z^3\left(-1+\text{zc}^4\right)}{\left(-1+z^4\right)\left(-1+\text{zc}^4\right)}$ // FullSimplify

Out[∘]=

$$-\frac{\text{c0}\,z^3}{-1+z^4}$$

In[∘]:= $\frac{\text{cm1}-\text{cm1}\,z^4}{\left(-1+z^4\right)\left(-1+\text{zc}^4\right)}$ // FullSimplify

Out[∘]=

$$\frac{\text{cm1}}{1-\text{zc}^4}$$

In[∘]:= (*3.2) use N=6*)

In[∘]:= cDFT[-1, 6] // FullSimplify

Out[∘]=

$$\frac{\text{cm1}-\text{cm1}\,z^6-\text{c0}\,z^5\left(-1+\text{zc}^6\right)}{\left(-1+z^6\right)\left(-1+\text{zc}^6\right)}$$

In[∘]:= (*3.2.1) seperate c0 and cm1*)

In[∘]:= $\frac{-c0\, z^5 \left(-1 + zc^6\right)}{\left(-1 + z^6\right)\left(-1 + zc^6\right)}$ // FullSimplify

Out[∘]=

$$-\frac{c0\, z^5}{-1 + z^6}$$

In[∘]:= $\frac{cm1 - cm1\, z^6}{\left(-1 + z^6\right)\left(-1 + zc^6\right)}$ // FullSimplify

Out[∘]=

$$\frac{cm1}{1 - zc^6}$$

In[∘]:= (*3.3) use N=8*)

In[∘]:= cDFT[-1, 8] // FullSimplify

Out[∘]=

$$\frac{cm1 - cm1\, z^8 - c0\, z^7 \left(-1 + zc^8\right)}{\left(-1 + z^8\right)\left(-1 + zc^8\right)}$$

In[∘]:= (*3.3.1) seperate c0 and cm1*)

In[∘]:= $\frac{-c0\, z^7 \left(-1 + zc^8\right)}{\left(-1 + z^8\right)\left(-1 + zc^8\right)}$ // FullSimplify

Out[∘]=

$$-\frac{c0\, z^7}{-1 + z^8}$$

In[∘]:= $\frac{cm1 - cm1\, z^8}{\left(-1 + z^8\right)\left(-1 + zc^8\right)}$ // FullSimplify

Out[∘]=

$$\frac{cm1}{1 - zc^8}$$

In[∘]:= (*3.4) One can assume based on the
former results that the equation for the DFT modes is:
cm1DFT= $\frac{c0 * z^{N-1}}{1-z^N} + \frac{cm1}{1-zc^N} = \frac{c0 * z^{N-1}}{1-z^N} + \frac{cm1\ zc}{zc\left(1-zc^N\right)}$ *)

In[∘]:= (*4.0) only minus second DFT mode*)

In[∘]:= (*4.1) use N=4*)

In[∘]:= cDFT[-2, 4] // FullSimplify

Out[∘]=

$$-\frac{c0\, z^2}{-1 + z^4} - \frac{cm1\, zc}{-1 + zc^4}$$

In[•]:= (*4.1.1) seperate c0 and cm1*)

In[•]:= $-\frac{c0\, z^2}{-1+z^4}$ // FullSimplify

Out[•]=

$$-\frac{c0\, z^2}{-1+z^4}$$

In[•]:= $-\frac{cm1\, zc}{-1+zc^4}$ // FullSimplify

Out[•]=

$$-\frac{cm1\, zc}{-1+zc^4}$$

In[•]:= (*4.2) use N=6*)

In[•]:= cDFT[-2, 6] // FullSimplify

Out[•]=

$$-\frac{c0\, z^4}{-1+z^6}-\frac{cm1\, zc}{-1+zc^6}$$

In[•]:= (*4.2.1) seperate c0 and cm1*)

In[•]:= $-\frac{c0\, z^4}{-1+z^6}$ // FullSimplify

Out[•]=

$$-\frac{c0\, z^4}{-1+z^6}$$

In[•]:= $-\frac{cm1\, zc}{-1+zc^6}$ // FullSimplify

Out[•]=

$$-\frac{cm1\, zc}{-1+zc^6}$$

In[•]:= (*4.3) use N=8*)

In[•]:= cDFT[-2, 8] // FullSimplify

Out[•]=

$$-\frac{c0\, z^6}{-1+z^8}-\frac{cm1\, zc}{-1+zc^8}$$

In[•]:= (*4.3.1) seperate c0 and cm1*)

In[•]:= $-\frac{c0\, z^6}{-1+z^8}$ // FullSimplify

Out[•]=

$$-\frac{c0\, z^6}{-1+z^8}$$

In[∘]:= $-\frac{\text{cm1 zc}}{-1+\text{zc}^8}$ // FullSimplify

Out[∘]=

$$-\frac{\text{cm1 zc}}{-1+\text{zc}^8}$$

(*4.4) One can assume based on the
former results that the equation for the DFT modes is:
$\text{cm2DFT}=\frac{\text{c0}*z^{N-2}}{1-z^N}+\frac{\text{cm1 zc}}{1-\text{zc}^N}=\frac{\text{c0}*z^{N-2}}{1-z^N}+\frac{\text{cm1 zc}^2}{\text{zc}\left(1-\text{zc}^N\right)}$ *)

(*5.1) Summary:
we have: $\text{c0DFT}=\frac{\text{c0}}{1-z^N}+\frac{\text{cm1 zc}^N}{\text{zc}\left(1-\text{zc}^N\right)}$ and $\text{c1DFT}=\frac{\text{c0 z}}{1-z^N}+\frac{\text{cm1 zc}^{N-1}}{\text{zc}\left(1-\text{zc}^N\right)}$
One can assume the general equation is:
$c_{n,\text{DFT}}(N)=\frac{\text{c0 z}^n}{1-z^N}+\frac{\text{cm1 zc}^{N-n}}{\text{zc}\left(1-\text{zc}^N\right)}$ *)

(*5.2) Summary:
we have: $\text{cm1DFT}=\frac{\text{c0}*z^{N-1}}{1-z^N}+\frac{\text{cm1 zc}}{\text{zc}\left(1-\text{zc}^N\right)}$ and $\text{cm2DFT}=\frac{\text{c0}*z^{N-2}}{1-z^N}+\frac{\text{cm1 zc}^2}{\text{zc}\left(1-\text{zc}^N\right)}$
One can assume the general equation is:
$c_{-n,\text{DFT}}(N)=\frac{\text{c0}*z^{N-n}}{1-z^N}+\frac{\text{cm1 zc}^n}{\text{zc}\left(1-\text{zc}^N\right)}$ *)

In[∘]:= (*These are the equations shown in the paper (aliasing correction) and
which are validated in numerical simulations within computer precision*)

(*Mathematica has somehow problems to *simplify the
expressions* (this does not mean that they do not exists)with
odd N (N=5,7 etc) so the numerical simulation was necessary to
prove the exactness of the above conjecture also for odd numbers*)