In[ ]:= (*See supporting information
       sec. "2. Supporting Information Theory S2: Derivation of aliasing
          correction formula using bSSFP modes" *)

In[ ]:= (*Use mode description [A9,A10] and put into the Fourier series expansion (below)*)

In[ ]:= mplus[φ_] := Sum[c0 * z^p * Exp[-𝕚 * p * φ], {p, 0, ∞}] + Sum[cm1 * zc^{p-1} * Exp[𝕚 * p * φ], {p, 1, ∞}]

In[ ]:= (*Perform a discrete Fourier transformation [A11] (below)
        (N is not a legal parameter so I replaced it with "M" in this mathematica script)*)

In[ ]:= cDFT[n_, M_] := $\frac{1}{M}$ * Sum[mplus[$\frac{2*π}{M}$ * k] * Exp[𝕚 * n * $\frac{2*π}{M}$ * k], {k, 0, M - 1}]

In[ ]:= (* DERIVATION of ALIASING CORRECTION*)

In[ ]:= (*1.0) only zeroth DFT mode*)

In[ ]:= (*1.1) use N=4*)

In[ ]:= cDFT[0, 4] // FullSimplify

Out[ ]=
$$\frac{c0 - cm1 \left(-1 + z^4\right) zc^3 - c0 \, zc^4}{\left(-1 + z^4\right) \left(-1 + zc^4\right)}$$

In[ ]:= (*1.1.1) seperate c0 and cm1*)

In[ ]:= $\frac{c0 - c0 \, zc^4}{\left(-1 + z^4\right) \left(-1 + zc^4\right)}$ // FullSimplify

Out[ ]=
$$\frac{c0}{1 - z^4}$$

In[ ]:= $\frac{-cm1 \left(-1 + z^4\right) zc^3}{\left(-1 + z^4\right) \left(-1 + zc^4\right)}$ // FullSimplify

Out[ ]=
$$-\frac{cm1 \, zc^3}{-1 + zc^4}$$

(*1.2) use N=6*)

In[ ]:= cDFT[0, 6] // FullSimplify

Out[ ]=
$$\frac{c0 - cm1 \left(-1 + z^6\right) zc^5 - c0 \, zc^6}{\left(-1 + z^6\right) \left(-1 + zc^6\right)}$$

In[ ]:= (*1.2.1) seperate c0 and cm1*)

In[ ]:= $\dfrac{c0 - c0\, zc^6}{(-1 + z^6)\,(-1 + zc^6)}$ // FullSimplify

Out[ ]=

$\dfrac{c0}{1 - z^6}$

In[ ]:= $\dfrac{-cm1\,(-1 + z^6)\,zc^5}{(-1 + z^6)\,(-1 + zc^6)}$ // FullSimplify

Out[ ]=

$-\dfrac{cm1\, zc^5}{-1 + zc^6}$

In[ ]:= (*1.3) use N=8*)

In[ ]:= **cDFT[0, 8] // FullSimplify**

Out[ ]=

$\dfrac{c0 - cm1\,(-1 + z^8)\,zc^7 - c0\, zc^8}{(-1 + z^8)\,(-1 + zc^8)}$

In[ ]:= (*1.3.1) seperate c0 and cm1*)

In[ ]:= $\dfrac{c0 - c0\, zc^8}{(-1 + z^8)\,(-1 + zc^8)}$ // FullSimplify

Out[ ]=

$\dfrac{c0}{1 - z^8}$

In[ ]:= $\dfrac{-cm1\,(-1 + z^8)\,zc^7}{(-1 + z^8)\,(-1 + zc^8)}$ // FullSimplify

Out[ ]=

$-\dfrac{cm1\, zc^7}{-1 + zc^8}$

In[ ]:= (*1.4) One can assume based on the
     former results that the equation for the DFT modes is:

$c0DFT = \dfrac{c0}{1 - z^N} + \dfrac{cm1\, zc^{N-1}}{1 - zc^N} = \dfrac{c0}{1 - z^N} + \dfrac{cm1\, zc^N}{zc\,(1 - zc^N)}$ *)

In[ ]:= (*2.0) only first DFT mode*)

In[ ]:= (*2.1) use N=4*)

In[ ]:= **cDFT[1, 4] // FullSimplify**

Out[ ]=

$\dfrac{-cm1\,(-1 + z^4)\,zc^2 + c0\,(z - z\, zc^4)}{(-1 + z^4)\,(-1 + zc^4)}$

In[ ]:= **(*2.1.1) seperate c0 and cm1*)**

In[ ]:= $\dfrac{c0\ (z - z\ zc^4)}{(-1 + z^4)\ (-1 + zc^4)}$ **// FullSimplify**

Out[ ]=

$\dfrac{c0\ z}{1 - z^4}$

In[ ]:= $\dfrac{-\,cm1\ (-1 + z^4)\ zc^2}{(-1 + z^4)\ (-1 + zc^4)}$ **// FullSimplify**

Out[ ]=

$-\dfrac{cm1\ zc^2}{-1 + zc^4}$

In[ ]:= **(*2.2) use N=6*)**

In[ ]:= **cDFT[1, 6] // FullSimplify**

Out[ ]=

$-\dfrac{c0\ z}{-1 + z^6} - \dfrac{cm1\ zc^4}{-1 + zc^6}$

In[ ]:= **(*2.2.1) seperate c0 and cm1*)**

In[ ]:= $-\dfrac{c0\ z}{-1 + z^6}$ **// FullSimplify**

Out[ ]=

$-\dfrac{c0\ z}{-1 + z^6}$

In[ ]:= $-\dfrac{cm1\ zc^4}{-1 + zc^6}$ **// FullSimplify**

Out[ ]=

$-\dfrac{cm1\ zc^4}{-1 + zc^6}$

In[ ]:= **(*2.3) use N=8*)**

In[ ]:= **cDFT[1, 8] // FullSimplify**

Out[ ]=

$\dfrac{-\,cm1\ (-1 + z^8)\ zc^6 + c0\ (z - z\ zc^8)}{(-1 + z^8)\ (-1 + zc^8)}$

In[ ]:= **(*2.3.1) seperate c0 and cm1*)**

In[ ]:= $\dfrac{c0\ (z - z\ zc^8)}{(-1 + z^8)\ (-1 + zc^8)}$ // FullSimplify

Out[ ]=

$\dfrac{c0\ z}{1 - z^8}$

In[ ]:= $\dfrac{-\,cm1\ (-1 + z^8)\ zc^6}{(-1 + z^8)\ (-1 + zc^8)}$ // FullSimplify

Out[ ]=

$-\,\dfrac{cm1\ zc^6}{-1 + zc^8}$

In[ ]:= (*2.4) One can assume based on the
    former results that the equation for the DFT modes is:
    c1DFT = $\dfrac{c0*z}{1-z^N} + \dfrac{cm1\ zc^{N-2}}{1-zc^N} = \dfrac{c0\ z}{1-z^N} + \dfrac{cm1\ zc^{N-1}}{zc\ (1-zc^N)}$ *)

In[ ]:= (*3.0) only minus first DFT mode*)

In[ ]:= (*3.1) use N=4*)

In[ ]:= cDFT[-1, 4] // FullSimplify

Out[ ]=

$\dfrac{cm1 - cm1\ z^4 - c0\ z^3\ (-1 + zc^4)}{(-1 + z^4)\ (-1 + zc^4)}$

In[ ]:= (*3.1.1) seperate c0 and cm1*)

In[ ]:= $\dfrac{-\,c0\ z^3\ (-1 + zc^4)}{(-1 + z^4)\ (-1 + zc^4)}$ // FullSimplify

Out[ ]=

$-\,\dfrac{c0\ z^3}{-1 + z^4}$

In[ ]:= $\dfrac{cm1 - cm1\ z^4}{(-1 + z^4)\ (-1 + zc^4)}$ // FullSimplify

Out[ ]=

$\dfrac{cm1}{1 - zc^4}$

In[ ]:= (*3.2) use N=6*)

In[ ]:= cDFT[-1, 6] // FullSimplify

Out[ ]=

$\dfrac{cm1 - cm1\ z^6 - c0\ z^5\ (-1 + zc^6)}{(-1 + z^6)\ (-1 + zc^6)}$

In[ ]:= **(*3.2.1) seperate c0 and cm1*)**

In[ ]:= $\dfrac{-c0\ z^5\ \left(-1+zc^6\right)}{\left(-1+z^6\right)\ \left(-1+zc^6\right)}$ **// FullSimplify**

Out[ ]=

$$-\frac{c0\ z^5}{-1+z^6}$$

In[ ]:= $\dfrac{cm1-cm1\ z^6}{\left(-1+z^6\right)\ \left(-1+zc^6\right)}$ **// FullSimplify**

Out[ ]=

$$\frac{cm1}{1-zc^6}$$

In[ ]:= **(*3.3) use N=8*)**

In[ ]:= **cDFT[-1, 8] // FullSimplify**

Out[ ]=

$$\frac{cm1-cm1\ z^8-c0\ z^7\ \left(-1+zc^8\right)}{\left(-1+z^8\right)\ \left(-1+zc^8\right)}$$

In[ ]:= **(*3.3.1) seperate c0 and cm1*)**

In[ ]:= $\dfrac{-c0\ z^7\ \left(-1+zc^8\right)}{\left(-1+z^8\right)\ \left(-1+zc^8\right)}$ **// FullSimplify**

Out[ ]=

$$-\frac{c0\ z^7}{-1+z^8}$$

In[ ]:= $\dfrac{cm1-cm1\ z^8}{\left(-1+z^8\right)\ \left(-1+zc^8\right)}$ **// FullSimplify**

Out[ ]=

$$\frac{cm1}{1-zc^8}$$

In[ ]:= **(*3.4) One can assume based on the**
    former results that the equation for the DFT modes is:
    $cm1DFT = \dfrac{c0*z^{N-1}}{1-z^N}+\dfrac{cm1}{1-zc^N}=\dfrac{c0*z^{N-1}}{1-z^N}+\dfrac{cm1\ zc}{zc\left(1-zc^N\right)}$ ***)**

In[ ]:= **(*4.0) only minus second DFT mode*)**

In[ ]:= **(*4.1) use N=4*)**

In[ ]:= **cDFT[-2, 4] // FullSimplify**

Out[ ]=

$$-\frac{c0\ z^2}{-1+z^4}-\frac{cm1\ zc}{-1+zc^4}$$

In[ ]:= **(*4.1.1) seperate c0 and cm1*)**

In[ ]:= $-\dfrac{c0\ z^2}{-1+z^4}$ **// FullSimplify**

Out[ ]=

$$-\dfrac{c0\ z^2}{-1+z^4}$$

In[ ]:= $-\dfrac{cm1\ zc}{-1+zc^4}$ **// FullSimplify**

Out[ ]=

$$-\dfrac{cm1\ zc}{-1+zc^4}$$

In[ ]:= **(*4.2) use N=6*)**

In[ ]:= **cDFT[-2, 6] // FullSimplify**

Out[ ]=

$$-\dfrac{c0\ z^4}{-1+z^6}-\dfrac{cm1\ zc}{-1+zc^6}$$

In[ ]:= **(*4.2.1) seperate c0 and cm1*)**

In[ ]:= $-\dfrac{c0\ z^4}{-1+z^6}$ **// FullSimplify**

Out[ ]=

$$-\dfrac{c0\ z^4}{-1+z^6}$$

In[ ]:= $-\dfrac{cm1\ zc}{-1+zc^6}$ **// FullSimplify**

Out[ ]=

$$-\dfrac{cm1\ zc}{-1+zc^6}$$

In[ ]:= **(*4.3) use N=8*)**

In[ ]:= **cDFT[-2, 8] // FullSimplify**

Out[ ]=

$$-\dfrac{c0\ z^6}{-1+z^8}-\dfrac{cm1\ zc}{-1+zc^8}$$

In[ ]:= **(*4.3.1) seperate c0 and cm1*)**

In[ ]:= $-\dfrac{c0\ z^6}{-1+z^8}$ **// FullSimplify**

Out[ ]=

$$-\dfrac{c0\ z^6}{-1+z^8}$$

$In[\bullet]:=$ $-\dfrac{cm1\ zc}{-1+zc^8}$ // FullSimplify

$Out[\bullet]=$

$-\dfrac{cm1\ zc}{-1+zc^8}$

(*4.4) One can assume based on the
  former results that the equation for the DFT modes is:
  $cm2DFT = \dfrac{c0*z^{N-2}}{1-z^N} + \dfrac{cm1\ zc}{1-zc^N} = \dfrac{c0*z^{N-2}}{1-z^N} + \dfrac{cm1\ zc^2}{zc(1-zc^N)}$ ***)**

$In[\bullet]:=$ **(*5.1) SUmmary:**
  we have: $c0DFT = \dfrac{c0}{1-z^N} + \dfrac{cm1\ zc^N}{zc(1-zc^N)}$ **and** $c1DFT = \dfrac{c0\ z^n}{1-z^N} + \dfrac{cm1\ zc^{N-1}}{zc(1-zc^N)}$

  One can assume the general equation is:
  $c_{n,DFT}(N) = \dfrac{c0\ z^n}{1-z^N} + \dfrac{cm1\ zc^{N-n}}{zc(1-zc^N)}$ ***)**

$In[\bullet]:=$ **(*5.2) SUmmary:**
  we have: $cm1DFT = \dfrac{c0*z^{N-1}}{1-z^N} + \dfrac{cm1\ zc}{zc(1-zc^N)}$ **and** $cm2DFT = \dfrac{c0*z^{N-2}}{1-z^N} + \dfrac{cm1\ zc^2}{zc(1-zc^N)}$

  One can assume the general equation is:
  $c_{-n,DFT}(N) = \dfrac{c0*z^{N-n}}{1-z^N} + \dfrac{cm1\ zc^n}{zc(1-zc^N)}$ ***)**

$In[\bullet]:=$ **(*These are the equations shown in the paper (aliasing correction) and**
  which are validated in numerical simulations within computer precision*)

  (*Mathematica has somehow problems to _simplify_ the
  expressions (this does not mean that they do not exists)with
  odd N (N=5,7 etc) so the numerical simulation was necessary to
  prove the exactness of the above conjecture also for odd numbers*)