```
In[1]:= (*See supporting information
 sec. "2. Supporting Information Theory S2: Derivation of aliasing
    correction formula using bSSFP modes" *)

In[2]:= (*Use mode description [A9,A10] and put into the Fourier series expansion (below)*)
```

In[3]:= $\texttt{mplus}[\varphi\_] := \texttt{Sum}\left[c0 * z^p * \texttt{Exp}[-i * p * \varphi], \{p, 0, \infty\}\right] + \texttt{Sum}\left[cm1 * zc^{p-1} * \texttt{Exp}[i * p * \varphi], \{p, 1, \infty\}\right]$

```
In[4]:= (*Perform a discrete Fourier transformation [A11] (below)
 (N is not a legal parameter so I replaced it with "M" in this mathematica script)*)
```

In[5]:= $\texttt{cDFT}[n\_, M\_] := \frac{1}{M} * \texttt{Sum}\left[\texttt{mplus}\left[\frac{2*\pi}{M} * k\right] * \texttt{Exp}\left[i * n * \frac{2*\pi}{M} * k\right], \{k, 0, M-1\}\right]$

```
In[6]:= (* DERIVATION of ALIASING CORRECTION*)

In[7]:= (*1.0) only zeroth DFT mode*)

In[8]:= (*1.1) use N=4*)

In[9]:= cDFT[0, 4] // FullSimplify
```

Out[9]= $$\frac{c0 - cm1\left(-1 + z^4\right) zc^3 - c0\, zc^4}{\left(-1 + z^4\right)\left(-1 + zc^4\right)}$$

```
In[10]:= (*1.1.1) seperate c0 and cm1*)
```

In[11]:= $\frac{c0 - c0\, zc^4}{\left(-1 + z^4\right)\left(-1 + zc^4\right)}$ // FullSimplify

Out[11]= $$\frac{c0}{1 - z^4}$$

In[12]:= $\frac{-cm1\left(-1 + z^4\right) zc^3}{\left(-1 + z^4\right)\left(-1 + zc^4\right)}$ // FullSimplify

Out[12]= $$-\frac{cm1\, zc^3}{-1 + zc^4}$$

```
In[13]:= (*1.2) use N=6*)

In[14]:= cDFT[0, 6] // FullSimplify
```

Out[14]= $$\frac{c0 - cm1\left(-1 + z^6\right) zc^5 - c0\, zc^6}{\left(-1 + z^6\right)\left(-1 + zc^6\right)}$$

```
In[15]:= (*1.2.1) seperate c0 and cm1*)
```

In[16]:= $\frac{c0 - c0\,zc^6}{\left(-1+z^6\right)\left(-1+zc^6\right)}$ // FullSimplify

Out[16]=

$$\frac{c0}{1-z^6}$$

In[17]:= $\frac{-cm1\left(-1+z^6\right)zc^5}{\left(-1+z^6\right)\left(-1+zc^6\right)}$ // FullSimplify

Out[17]=

$$-\frac{cm1\,zc^5}{-1+zc^6}$$

In[18]:= (*1.3) use N=8*)

In[19]:= cDFT[0, 8] // FullSimplify

Out[19]=

$$\frac{c0 - cm1\left(-1+z^8\right)zc^7 - c0\,zc^8}{\left(-1+z^8\right)\left(-1+zc^8\right)}$$

In[20]:= (*1.3.1) seperate c0 and cm1*)

In[21]:= $\frac{c0 - c0\,zc^8}{\left(-1+z^8\right)\left(-1+zc^8\right)}$ // FullSimplify

Out[21]=

$$\frac{c0}{1-z^8}$$

In[22]:= $\frac{-cm1\left(-1+z^8\right)zc^7}{\left(-1+z^8\right)\left(-1+zc^8\right)}$ // FullSimplify

Out[22]=

$$-\frac{cm1\,zc^7}{-1+zc^8}$$

In[23]:= (*1.4) One can assume based on the former results that the equation for the DFT modes is: c0DFT= $\frac{c0}{1-z^N}+\frac{cm1\ zc^{N-1}}{1-zc^N}=\frac{c0}{1-z^N}+\frac{cm1\ zc^N}{zc\left(1-zc^N\right)}$ *)

In[24]:= (*2.0) only first DFT mode*)

In[25]:= (*2.1) use N=4*)

In[26]:= cDFT[1, 4] // FullSimplify

Out[26]=

$$\frac{-cm1\left(-1+z^4\right)zc^2 + c0\left(z - z\,zc^4\right)}{\left(-1+z^4\right)\left(-1+zc^4\right)}$$

In[27]:= (*2.1.1) seperate c0 and cm1*)

In[28]:= $$\frac{c0\left(z - z\,zc^4\right)}{\left(-1+z^4\right)\left(-1+zc^4\right)}$$ // FullSimplify

Out[28]=
$$\frac{c0\,z}{1-z^4}$$

In[29]:= $$\frac{-cm1\left(-1+z^4\right)zc^2}{\left(-1+z^4\right)\left(-1+zc^4\right)}$$ // FullSimplify

Out[29]=
$$-\frac{cm1\,zc^2}{-1+zc^4}$$

In[30]:= (*2.2) use N=6*)

In[31]:= cDFT[1, 6] // FullSimplify

Out[31]=
$$-\frac{c0\,z}{-1+z^6}-\frac{cm1\,zc^4}{-1+zc^6}$$

In[32]:= (*2.2.1) seperate c0 and cm1*)

In[33]:= $$-\frac{c0\,z}{-1+z^6}$$ // FullSimplify

Out[33]=
$$-\frac{c0\,z}{-1+z^6}$$

In[34]:= $$-\frac{cm1\,zc^4}{-1+zc^6}$$ // FullSimplify

Out[34]=
$$-\frac{cm1\,zc^4}{-1+zc^6}$$

In[35]:= (*2.3) use N=8*)

In[36]:= cDFT[1, 8] // FullSimplify

Out[36]=
$$\frac{-cm1\left(-1+z^8\right)zc^6+c0\left(z-z\,zc^8\right)}{\left(-1+z^8\right)\left(-1+zc^8\right)}$$

In[37]:= (*2.3.1) seperate c0 and cm1*)

In[38]:= $\frac{c0\left(z - z\,zc^{8}\right)}{\left(-1 + z^{8}\right)\left(-1 + zc^{8}\right)}$ // FullSimplify

Out[38]=

$$\frac{c0\,z}{1 - z^{8}}$$

In[39]:= $\frac{-cm1\left(-1 + z^{8}\right)zc^{6}}{\left(-1 + z^{8}\right)\left(-1 + zc^{8}\right)}$ // FullSimplify

Out[39]=

$$-\frac{cm1\,zc^{6}}{-1 + zc^{8}}$$

In[40]:= (*2.4) One can assume based on the former results that the equation for the DFT modes is: $c1DFT = \frac{c0*z}{1-z^{N}} + \frac{cm1\ zc^{N-2}}{1-zc^{N}} = \frac{c0\ z}{1-z^{N}} + \frac{cm1\ zc^{N-1}}{zc\left(1-zc^{N}\right)}$ *)

In[41]:= (*3.0) only minus first DFT mode*)

In[42]:= (*3.1) use N=4*)

In[43]:= cDFT[-1, 4] // FullSimplify

Out[43]=

$$\frac{cm1 - cm1\,z^{4} - c0\,z^{3}\left(-1 + zc^{4}\right)}{\left(-1 + z^{4}\right)\left(-1 + zc^{4}\right)}$$

In[44]:= (*3.1.1) seperate c0 and cm1*)

In[45]:= $\frac{-c0\,z^{3}\left(-1 + zc^{4}\right)}{\left(-1 + z^{4}\right)\left(-1 + zc^{4}\right)}$ // FullSimplify

Out[45]=

$$-\frac{c0\,z^{3}}{-1 + z^{4}}$$

In[46]:= $\frac{cm1 - cm1\,z^{4}}{\left(-1 + z^{4}\right)\left(-1 + zc^{4}\right)}$ // FullSimplify

Out[46]=

$$\frac{cm1}{1 - zc^{4}}$$

In[47]:= (*3.2) use N=6*)

In[48]:= cDFT[-1, 6] // FullSimplify

Out[48]=

$$\frac{cm1 - cm1\,z^{6} - c0\,z^{5}\left(-1 + zc^{6}\right)}{\left(-1 + z^{6}\right)\left(-1 + zc^{6}\right)}$$

In[49]:= (*3.2.1) seperate c0 and cm1*)

In[50]:= $\frac{-c0\, z^5 \left(-1 + zc^6\right)}{\left(-1 + z^6\right) \left(-1 + zc^6\right)}$ // FullSimplify

Out[50]=

$$-\frac{c0\, z^5}{-1 + z^6}$$

In[51]:= $\frac{cm1 - cm1\, z^6}{\left(-1 + z^6\right) \left(-1 + zc^6\right)}$ // FullSimplify

Out[51]=

$$\frac{cm1}{1 - zc^6}$$

In[52]:= (*3.3) use N=8*)

In[53]:= cDFT[-1, 8] // FullSimplify

Out[53]=

$$\frac{cm1 - cm1\, z^8 - c0\, z^7 \left(-1 + zc^8\right)}{\left(-1 + z^8\right) \left(-1 + zc^8\right)}$$

In[54]:= (*3.3.1) seperate c0 and cm1*)

In[55]:= $\frac{-c0\, z^7 \left(-1 + zc^8\right)}{\left(-1 + z^8\right) \left(-1 + zc^8\right)}$ // FullSimplify

Out[55]=

$$-\frac{c0\, z^7}{-1 + z^8}$$

In[56]:= $\frac{cm1 - cm1\, z^8}{\left(-1 + z^8\right) \left(-1 + zc^8\right)}$ // FullSimplify

Out[56]=

$$\frac{cm1}{1 - zc^8}$$

In[57]:= (*3.4) One can assume based on the former results that the equation for the DFT modes is: cm1DFT= $\frac{c0*z^{N-1}}{1-z^N} + \frac{cm1}{1-zc^N} = \frac{c0*z^{N-1}}{1-z^N} + \frac{cm1\ zc}{zc\left(1-zc^N\right)}$ *)

In[58]:= (*4.0) only minus second DFT mode*)

In[59]:= (*4.1) use N=4*)

In[60]:= cDFT[-2, 4] // FullSimplify

Out[60]=

$$-\frac{c0\, z^2}{-1 + z^4} - \frac{cm1\, zc}{-1 + zc^4}$$

In[61]:= (*4.1.1) seperate c0 and cm1*)

In[62]:= $-\frac{c0\,z^2}{-1+z^4}$ // FullSimplify

Out[62]=

$$-\frac{c0\,z^2}{-1+z^4}$$

In[63]:= $-\frac{cm1\,zc}{-1+zc^4}$ // FullSimplify

Out[63]=

$$-\frac{cm1\,zc}{-1+zc^4}$$

In[64]:= (*4.2) use N=6*)

In[65]:= cDFT[-2, 6] // FullSimplify

Out[65]=

$$-\frac{c0\,z^4}{-1+z^6}-\frac{cm1\,zc}{-1+zc^6}$$

In[66]:= (*4.2.1) seperate c0 and cm1*)

In[67]:= $-\frac{c0\,z^4}{-1+z^6}$ // FullSimplify

Out[67]=

$$-\frac{c0\,z^4}{-1+z^6}$$

In[68]:= $-\frac{cm1\,zc}{-1+zc^6}$ // FullSimplify

Out[68]=

$$-\frac{cm1\,zc}{-1+zc^6}$$

In[69]:= (*4.3) use N=8*)

In[70]:= cDFT[-2, 8] // FullSimplify

Out[70]=

$$-\frac{c0\,z^6}{-1+z^8}-\frac{cm1\,zc}{-1+zc^8}$$

In[71]:= (*4.3.1) seperate c0 and cm1*)

In[72]:= $-\frac{c0\,z^6}{-1+z^8}$ // FullSimplify

Out[72]=

$$-\frac{c0\,z^6}{-1+z^8}$$

In[73]:= $-\frac{\text{cm1 zc}}{-1+\text{zc}^8}$ // FullSimplify

Out[73]=

$$-\frac{\text{cm1 zc}}{-1+\text{zc}^8}$$

In[74]:= (*4.4) One can assume based on the former results that the equation for the DFT modes is: $\text{cm2DFT}=\frac{\text{c0}*\text{z}^{N-2}}{1-\text{z}^N}+\frac{\text{cm1 zc}}{1-\text{zc}^N}=\frac{\text{c0}*\text{z}^{N-2}}{1-\text{z}^N}+\frac{\text{cm1 zc}^2}{\text{zc}\left(1-\text{zc}^N\right)}$ *)

In[75]:= (*5.1) Summary:
we have: $\text{c0DFT}=\frac{\text{c0}}{1-\text{z}^N}+\frac{\text{cm1 zc}^N}{\text{zc}\left(1-\text{zc}^N\right)}$ and $\text{c1DFT}=\frac{\text{c0 z}}{1-\text{z}^N}+\frac{\text{cm1 zc}^{N-1}}{\text{zc}\left(1-\text{zc}^N\right)}$
One can assume the general equation is:
$c_{n,DFT}(N)=\frac{\text{c0 z}^n}{1-\text{z}^N}+\frac{\text{cm1 zc}^{N-n}}{\text{zc}\left(1-\text{zc}^N\right)}$ *)

In[76]:= (*5.2) Summary:
we have: $\text{cm1DFT}=\frac{\text{c0}*\text{z}^{N-1}}{1-\text{z}^N}+\frac{\text{cm1 zc}}{\text{zc}\left(1-\text{zc}^N\right)}$ and $\text{cm2DFT}=\frac{\text{c0}*\text{z}^{N-2}}{1-\text{z}^N}+\frac{\text{cm1 zc}^2}{\text{zc}\left(1-\text{zc}^N\right)}$
One can assume the general equation is:
$c_{-n,DFT}(N)=\frac{\text{c0}*\text{z}^{N-n}}{1-\text{z}^N}+\frac{\text{cm1 zc}^n}{\text{zc}\left(1-\text{zc}^N\right)}$ *)

In[77]:= (*These are the equations shown in the paper (aliasing correction) and which are validated in numerical simulations within computer precision*)

In[78]:= (*Mathematica has somehow problems to ***simplify*** ***the expressions*** (this does not mean that they do not exists)with odd N (N=5,7 etc) so the numerical simulation was necessary to prove the exactness of the above conjecture also for odd numbers*)

In[79]:= (*6.0) try out other values of N yourself*)

In[80]:= cDFT[0, 12] // FullSimplify

Out[80]=

$$-\frac{\text{c0}}{-1+\text{z}^{12}}-\frac{\text{cm1 zc}^{11}}{-1+\text{zc}^{12}}$$

In[81]:= cDFT[0, 16] // FullSimplify

Out[81]=

$$\frac{\text{c0}-\text{cm1}\left(-1+\text{z}^{16}\right)\text{zc}^{15}-\text{c0 zc}^{16}}{\left(-1+\text{z}^{16}\right)\left(-1+\text{zc}^{16}\right)}$$

In[82]:= cDFT[0, 20] // FullSimplify

Out[82]=

$$-\frac{\text{c0}}{-1+\text{z}^{20}}-\frac{\text{cm1 zc}^{19}}{-1+\text{zc}^{20}}$$

In[83]:= (*7.0) try out other values of n yourself*)

In[84]:=
```
cDFT[-3, 8] // FullSimplify
```

Out[84]=

$$-\frac{c0\, z^5}{-1 + z^8} - \frac{cm1\, zc^2}{-1 + zc^8}$$

In[85]:=
```
cDFT[2, 8] // FullSimplify
```

Out[85]=

$$-\frac{c0\, z^2}{-1 + z^8} - \frac{cm1\, zc^5}{-1 + zc^8}$$