Question 1 **20 Marks**

(a) Place the following numbers in order, starting with the smallest:

$$\frac{3}{2} \qquad 1\cdot 4 \qquad \sqrt{2}$$

$1\cdot 4,\ \sqrt{2},\ \frac{3}{2}.$ $\quad \left(\sqrt{2} = 1\cdot 414...,\quad \frac{3}{2} = 1\cdot 5.\right)$

(b) Which one of the following is **not** a rational number? Explain your answer.

$$3\frac{1}{7} \qquad 3\cdot 142 \qquad \frac{22}{7} \qquad \pi$$

Answer: π Reason: It cannot be written as a fraction.

(c) **(i)** Find the values of $\frac{4n^2+1}{13}$, where $n \in \{17, 19, 21\}$.

n	$\frac{4n^2+1}{13}$
17	$\frac{4\times(17)^2+1}{13} = \frac{1157}{13} = 89$
19	$\frac{4\times(19)^2+1}{13} = \frac{1445}{13}$ or $111^{2}/_{13}$
21	$\frac{4\times(21)^2+1}{13} = \frac{1765}{13}$ or $135^{10}/_{13}$

(ii) State which **one** of your answers is a natural number, and explain your choice.

Answer: 89. Reason: It is a positive whole number.

Question 2 **15 Marks**

(a) John thinks that he has a method for finding **all** prime numbers.
He says that if he uses the formulas in the table below, he will generate the prime numbers.
He also says that these formulas will generate **only** the prime numbers.

(i) Complete the table.

p	$6p + 1$	$6p + 5$
0	**1**	**5**
1	7	11
2	13	17
3	19	23
4	25	29
5	31	35

(ii) Give two reasons why his method is not fully correct.

*There are a number of different reasons – **any** two will suffice.*
*Reasons related to "**all** prime numbers":*

The formulas do not generate 2, which is prime.

The formulas do not generate 3, which is prime.

*Reasons related to "**only** prime numbers":*

The formulas generate 1, which is not prime.

The formulas generate 25, which is not prime.

The formulas generate 35, which is not prime.

(b) The Swiss mathematician and physicist, Euler, first noticed (in 1772) that the expression $n^2 - n + 41$ gives a prime number for all positive integer values of n less than 41.

Explain why it does not give a prime number for $n = 41$.

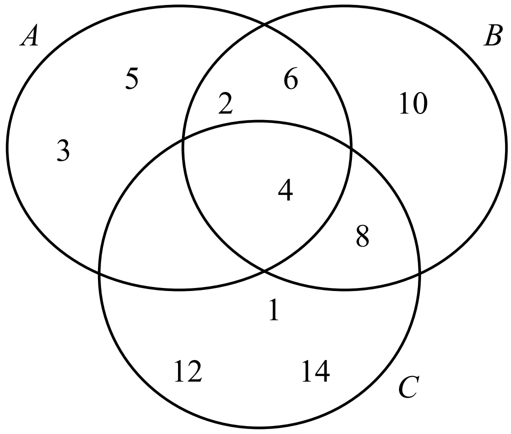
$41^2 - 41 + 41 = 41^2$, which has 41 as a factor.

Question 3 **25 Marks**

(a) The sets A, B, and C are as follows:

$A = \{2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$, and $C = \{1, 4, 8, 12, 14\}$.

(i) Complete the Venn diagram.



(ii) List the elements of each of the following sets:

$A \cap B =$	$\{2, 4, 6\}$
$B \setminus (A \cap C) =$	$\{2, 6, 8, 10\}$
$(B \setminus A) \cup (B \setminus C) =$	$\{2, 6, 8, 10\}$

(iii) Write down a null set, in terms of A, B, and C.

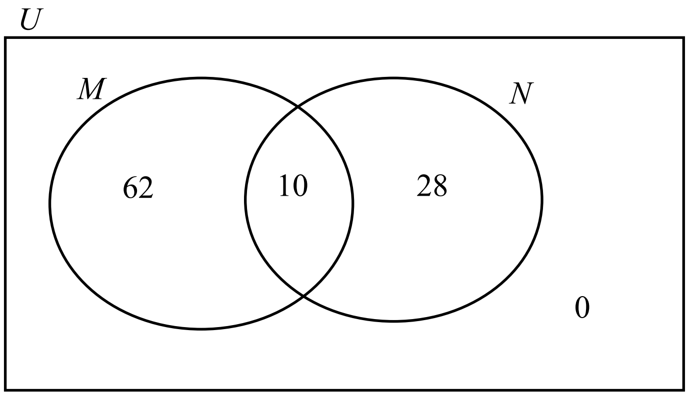
$(A \cap C) \setminus B$ *or equivalent.*

(b) In a table quiz, 100 questions were asked. Team M answered 72 questions correctly. Team N answered 38 questions correctly.

(i) Find, with the aid of the Venn diagram, the minimum number of questions answered correctly by both teams.

$72 + 38 = 110.$
$110 - 100 = 10.$
Minimum $= 10$

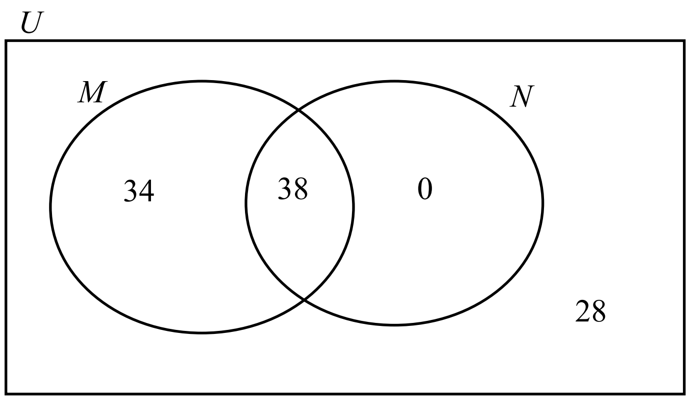
To make $\#(M \cap N)$ *as small as possible, make* $\#(M \cup N)' = 0$.



(ii) Find, with the aid of the Venn diagram, the maximum number of questions answered correctly by both teams.

Maximum $= 38$

To make $M \cap N$ *as big as possible, make the smaller set a subset of the larger set.*



Question 4 **25 Marks**

(a) Factorise fully $9a^2 - 6ab + 12ac - 8bc$.

$$\begin{aligned} 9a^2 - 6ab + 12ac - 8bc &= 3a(3a-2b) + 4c(3a-2b) \\ &= (3a-2b)(3a+4c). \end{aligned}$$

(b) Factorise $9x^2 - 16y^2$.

$$9x^2 - 16y^2 = (3x-4y)(3x+4y).$$

(c) Use factors to simplify the following: $\dfrac{2x^2+4x}{2x^2+x-6}$.

$$\begin{aligned} \frac{2x^2+4x}{2x^2+x-6} &= \frac{2x(x+2)}{(x+2)(2x-3)} \\ &= \frac{2x}{2x-3}. \end{aligned}$$

Question 5 **10 Marks**

Solve the following inequality and show the solution on the number line.

$$-17 \le 1 - 3x < 13, \quad x \in \mathbb{Z}$$

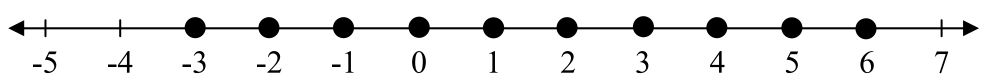
One method:

$$\begin{array}{lrcrcr} & -17 & \le & 1-3x & < & 13 \\ -1: & -18 & \le & -3x & < & 12 \\ \div(-3): & 6 & \ge & x & > & -4 \\ \text{i.e.} & -4 & < & x & \le & 6. \end{array}$$

Or:

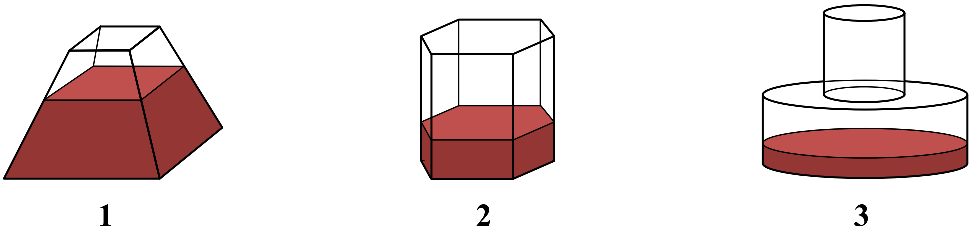
$$\begin{array}{rcl c rcl} -17 & \le & 1-3x & \text{and} & 1-3x & < & 13 \\ 3x & \le & 18 & \text{and} & -3x & < & 12 \\ x & \le & 6 & \text{and} & x & > & -4 \end{array}$$

i.e. $-4 < x \le 6$.

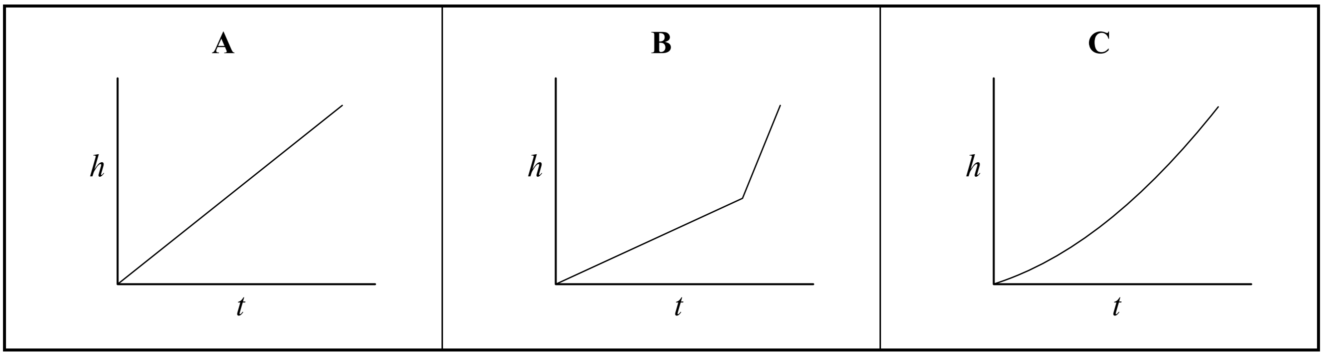


Question 6 **10 Marks**

Below are three containers, labelled **1**, **2**, and **3**.
Water is poured into each container at a constant rate, until it is full.



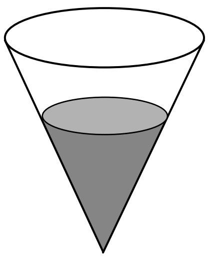
The three graphs, **A**, **B**, and **C**, show the height of the water, *h*, in the containers after time *t*.

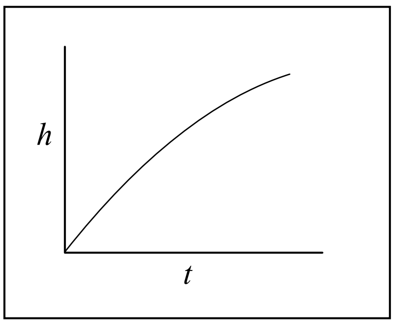


(a) Write **A**, **B**, and **C** in the table below to match each container to its corresponding graph.

Container	**1**	**2**	**3**
Graph	C	A	B

(b) Another container is shown below. Water is also poured into this container at a constant rate until it is full. Sketch the graph you would expect to get when plotting height (h) against time (t) for this container.





Question 7 **25 Marks**

Last year Elena had a gross income of €36 960.
She had to pay Universal Social Charge (USC) and income tax on her gross income.
The rates and bands of USC are as follows.

Income band	Rate of USC
Up to €10 036	2%
Between €10 036 and €16 016	4%
Above €16 016	7%

(i) Find the amount of USC that was deducted from Elena's gross income last year.

USC @ 2%: $0\cdot 02\times 10036 = €200\cdot 72$.

USC @ 4%: $16016 - 10036 = €5980$, and $0\cdot 04\times 5980 = €239\cdot 20$.

USC @ 7%: $36960 - 16016 = €20944$, and $0\cdot 07\times 20944 = €1466\cdot 08$.

Total USC $= €1906$.

(ii) The standard rate of income tax was 20% and the higher rate was 41%.
The standard rate cut-off point was €32 800.
Elena paid a total of €4965·60 income tax last year.

Find Elena's tax credits for the year.

Tax @ 20%: $0\cdot 20\times 32800 = €6560\cdot 00$.

Tax @ 41%: $36960 - 32800 = €4160$, and $0\cdot 41\times 4160 = €1705\cdot 60$.

Gross Tax: $€8265\cdot 60$.

Tax Credits: $8265\cdot 60 - 4965\cdot 60 = €3300$.

(iii) Find Elena's total deduction (USC and income tax) as a percentage of her gross income.
Give your answer correct to the nearest percent.

Total Deductions: $1906 + 4965\cdot 60 = €6871\cdot 60$.

Total Deductions as % of Gross Income:

$$\frac{6871\cdot 60}{36960}\times 100 = 18\cdot 59... = 19\%$$, correct to the nearest percent.

Question 8 **15 Marks**

The table shows the height, in metres, of a ball at various times after being kicked into the air.

(i) Is the pattern of heights in the table linear, quadratic, or exponential? Explain your answer.

Time (seconds)	0	0·5	1	1·5	2	2·5	3
Height (metres)	0·3	3·4	5·7	7·2	7·9	7·8	6·9

First difference: 3·1 2·3 1·5 0·7 –0·1 –0·9

Second difference: –0·8 –0·8 –0·8 –0·8 –0·8

Answer: Quadratic.

Reason: The first differences are not all the same, but the second differences are.

(ii) Estimate the height of the ball after 3·5 seconds.

5·2 metres.

Second difference:		–0·8	–0·8	
First difference:	–0·1	–0·9	–1·7	
Height (m):	7·9	7·8	6·9	5·2
Time (s):	2	2·5	3	3·5

(iii) Estimate the total time the ball spends in the air. Justify your answer.

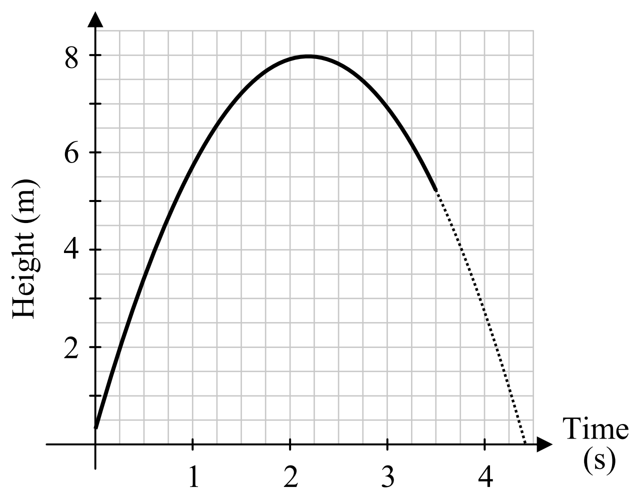
Continuing the method for **(ii)**:

Second difference:		–0·8	–0·8	–0·8	–0·8	
First difference:	–0·1	–0·9	–1·7	–2·5	–3·3	
Height (m):	7·9	7·8	6·9	5·2	2·7	–0·6
Time (s):	2	2·5	3	3·5	4	4·5

Answer: The ball spends roughly 4·4 seconds in the air. Its height is 0 just before 4·5 seconds.

Or, graphically:

From the graph, the ball spends roughly 4·4 seconds in the air



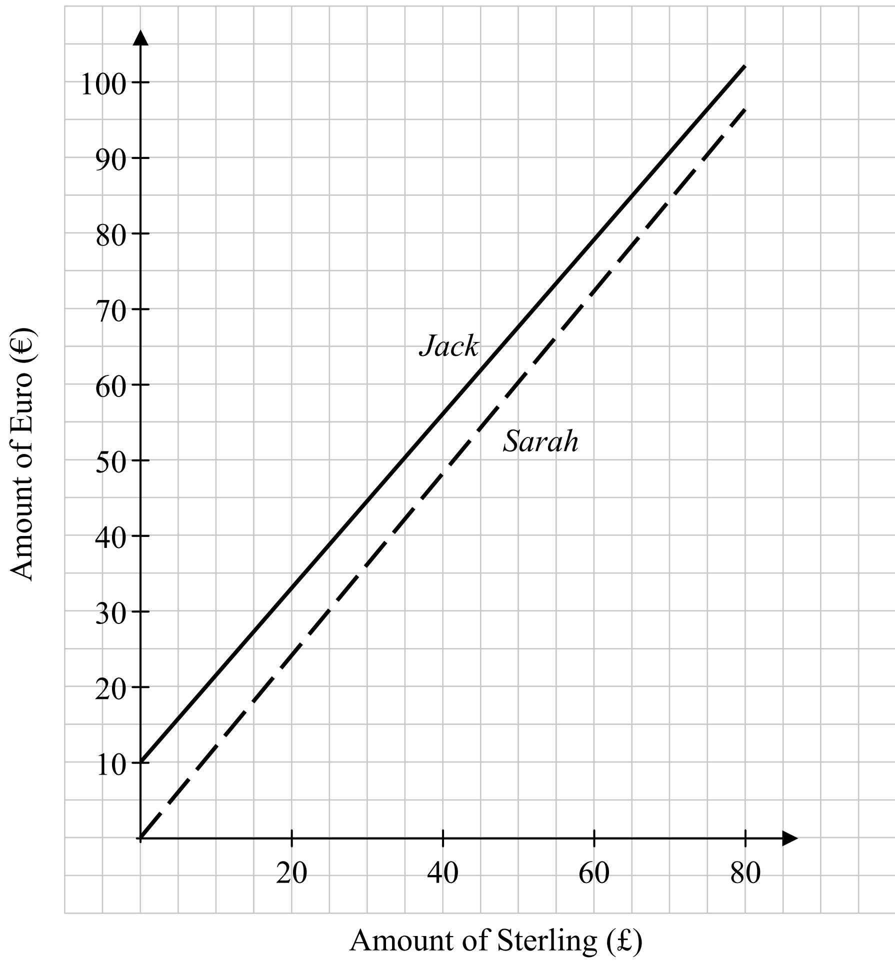
Question 9 **30 Marks**

Jack and Sarah are going on a school tour to England. They investigate how much different amounts of sterling (£) will cost them in euro (€). They each go to a different bank.

Their results are shown in the table below.

Amount of sterling (£)	Cost in euro (€) for Jack	Cost in euro (€) for Sarah
20	33	24
40	56	48
60	79	72
80	102	96

(i) On the grid below, draw graphs to show how much the sterling will cost Jack and Sarah, for up to £80.



(ii) Using the table, or your graph, find the slope (rate of change) of Jack's graph. Explain what this value means. Refer to both euro and sterling in your explanation.

$$\text{Slope} = \frac{56-33}{40-20} = \frac{23}{20}, \textit{ or } 1\cdot15.$$

Explanation: Each extra £1 costs Jack an extra €1·15.

Or:

Explanation: Each £1 costs Jack €1·15, after an initial fee of €10.

(iii) Write down a formula to represent what Jack must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

$e = 1\cdot15s + 10$, where s is the amount, in sterling, and e is the amount, in euro.

(iv) Write down a formula to represent what Sarah must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

$$\text{Slope} = \frac{48-24}{40-20} = \frac{6}{5}, \textit{ or } 1\cdot2. \qquad y\text{-intercept} = 0$$

$e = 1\cdot2s$, where s is the amount, in sterling, and e is the amount, in euro.

(v) Using your formulas from **(iii)** and **(iv)**, or otherwise, find the amount of sterling Jack and Sarah could buy that would cost them the same amount each in euro.

Using formulas:

$e = 1\cdot15s + 10$ and $e = 1\cdot2s$, so $1\cdot15s + 10 = 1\cdot2s$,

i.e. $s = 200$ and $e = 240$.

Amount of sterling: £200.

From table:

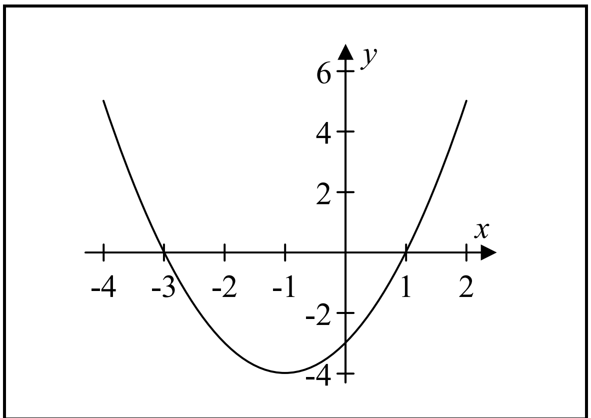
Each time the amount of sterling goes up by 20, the difference between the costs decreases by €1.

This difference is €9 for £20.

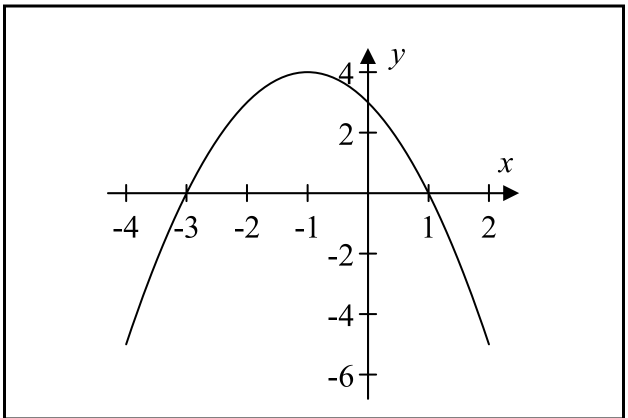
So after 9 increases, i.e. increase of $9 \times 20 = £180$, the costs are the same, i.e. for £200.

Question 10 **15 Marks**

(a) The graphs of the functions $f(x)=x^2+2x-3$ and $g(x)=-x^2-2x+3$ are shown below.
Identify each graph by writing $f(x)$ or $g(x)$ in the space provided below the graph.

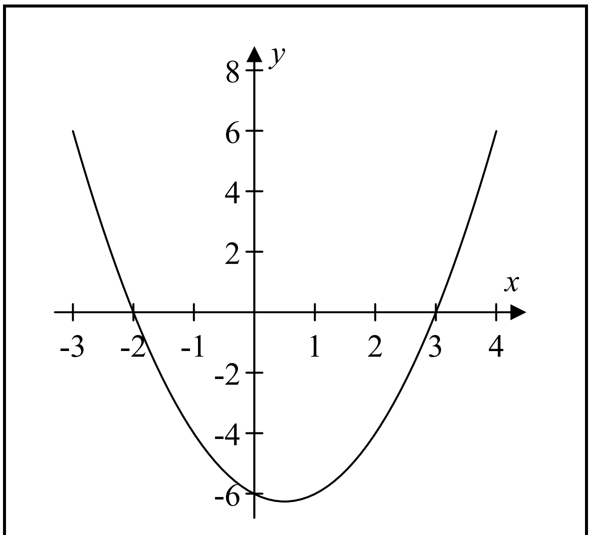


Answer: $f(x)$

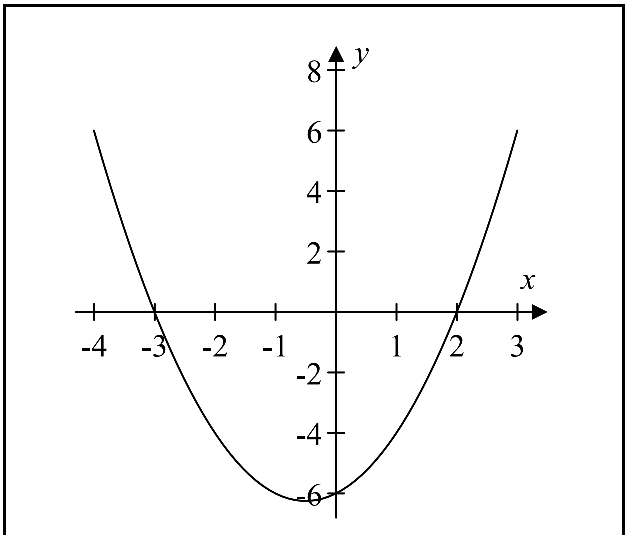


Answer: $g(x)$

(b) The graphs of the functions $y=h(x)$ and $y=k(x)$ are shown below.



$h(x)$



$k(x)$

Write down the roots of each function.
Hence, or otherwise, write down an equation for each function.

Roots of $h(x)$:	$x=-2$ and $x=3$.
Equation:	$h(x)=(x+2)(x-3)$, or $h(x)=x^2-x-6$.

[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $h(0)=-6$, which corresponds to the graph.]

Roots of $k(x)$:	$x=-3$ and $x=2$.
Equation:	$k(x)=(x+3)(x-2)$, or $k(x)=x^2+x-6$.

[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $k(0)=-6$, which corresponds to the graph.].

Question 11 **20 Marks**

x is a real number.
One new number is formed by increasing x by 1.
A second new number is formed by decreasing x by 2.

(i) Write down each of these new numbers, in terms of x.

Increase x by 1:	$x+1$
Decrease x by 2:	$x-2$

(ii) The product of these two new numbers is 1.
Use this information to write an equation in x.

$(x+1)(x-2)=1$ *or equivalent.*

(iii) Solve this equation to find the two possible values of x.
Give each of your answers correct to 3 decimal places.

$$(x+1)(x-2)=1$$
$$\Rightarrow \quad x^2-x-3=0$$
$$\Rightarrow \quad x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}$$
$\Rightarrow \quad x=2\cdot 3028\ldots$ and $x=-1\cdot 3028\ldots$

$\Rightarrow \quad x=2\cdot 303$ and $x=-1\cdot 303$, correct to three decimal places.

Question 12 **25 Marks**

(a) Simplify $(6x-3)(2x-1)$.

$(6x-3)(2x-1) = 12x^2 - 12x + 3$

(b) Simplify $\left(3x^3-2x^2-3x+2\right)\div\left(x-1\right)$.

$$
\begin{array}{r}
3x^2+x-2 \\
x-1\overline{)3x^3-2x^2-3x+2} \\
\underline{3x^3-3x^2} \qquad\qquad \\
x^2-3x+2 \\
\underline{x^2-x} \qquad \\
-2x+2 \\
\underline{-2x+2} \\
0
\end{array}
$$

Answer $= 3x^2 + x - 2$.

Or:

	$3x^2$	x	-2
x	$3x^3$	x^2	$-2x$
-1	$-3x^2$	$-x$	2

Answer $= 3x^2 + x - 2$.

(c) **(i)** Solve the simultaneous equations:

$2x - 3y = 18$ ①

$5x + 9y = -10$ ②

① × 3: $6x - 9y = 54$

②: $\underline{5x + 9y = -10}$

$11x = 44$

÷11: $x = 4$

Sub in x = 4 in ①:

$2(4) - 3y = 18$

$8 - 3y = 18$

$-3y = 18 - 8$

$-3y = 10$

×(−1): $3y = -10$

÷3: $y = -10 \div 3 = -10/3$ *or equivalent*

Answer: $x = 4$ and $y = -10/3$.

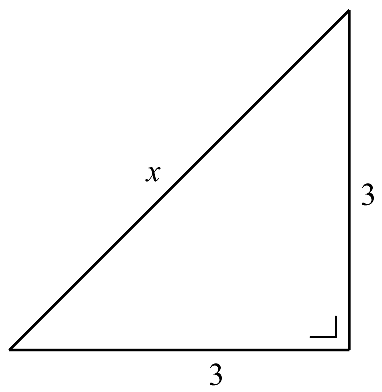
(ii) Verify your answer to **(c)(i)**.

Note: *Only need to check the equation that* ***wasn't*** *used to find the second variable. In this case, we only need use* ②.

$$5(4)+9\left(-\frac{10}{3}\right) \quad = \quad 20-30 \quad = \quad -10.$$

Question 13 **15 Marks**

(i) Use the diagram on the right to calculate the value of x.
Give your answer in surd form.



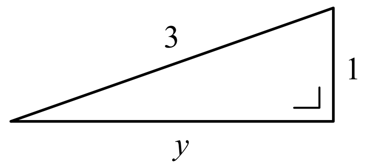
$$x = \sqrt{3^2 + 3^2}$$
$$= \sqrt{18} \quad \text{or} \quad 3\sqrt{2}$$

Or:

$$\sin 45° = \frac{3}{x}$$
$$\frac{1}{\sqrt{2}} = \frac{3}{x}$$
$$x = 3\sqrt{2}$$

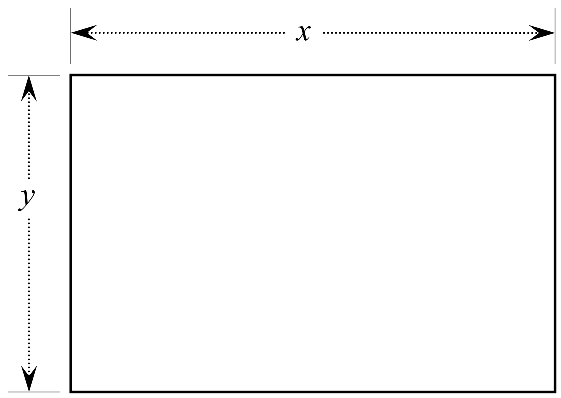
(ii) Use the diagram below to calculate the value of y. Give your answer in surd form.

$$y = \sqrt{3^2 - 1^2} = \sqrt{8} \quad \text{or} \quad 2\sqrt{2}.$$



(iii) A rectangle with sides of length x and y is drawn using the values of x and y from parts **(i)** and **(ii)**, as shown below.

Write the **perimeter** of this rectangle in the form $a\sqrt{2}$, where $a \in \mathbb{N}$.



$$\text{Perimeter} = 2x + 2y$$
$$= 2\sqrt{18} + 2\sqrt{8}$$
$$= 2\left(3\sqrt{2}\right) + 2\left(2\sqrt{2}\right)$$
$$= 10\sqrt{2}.$$

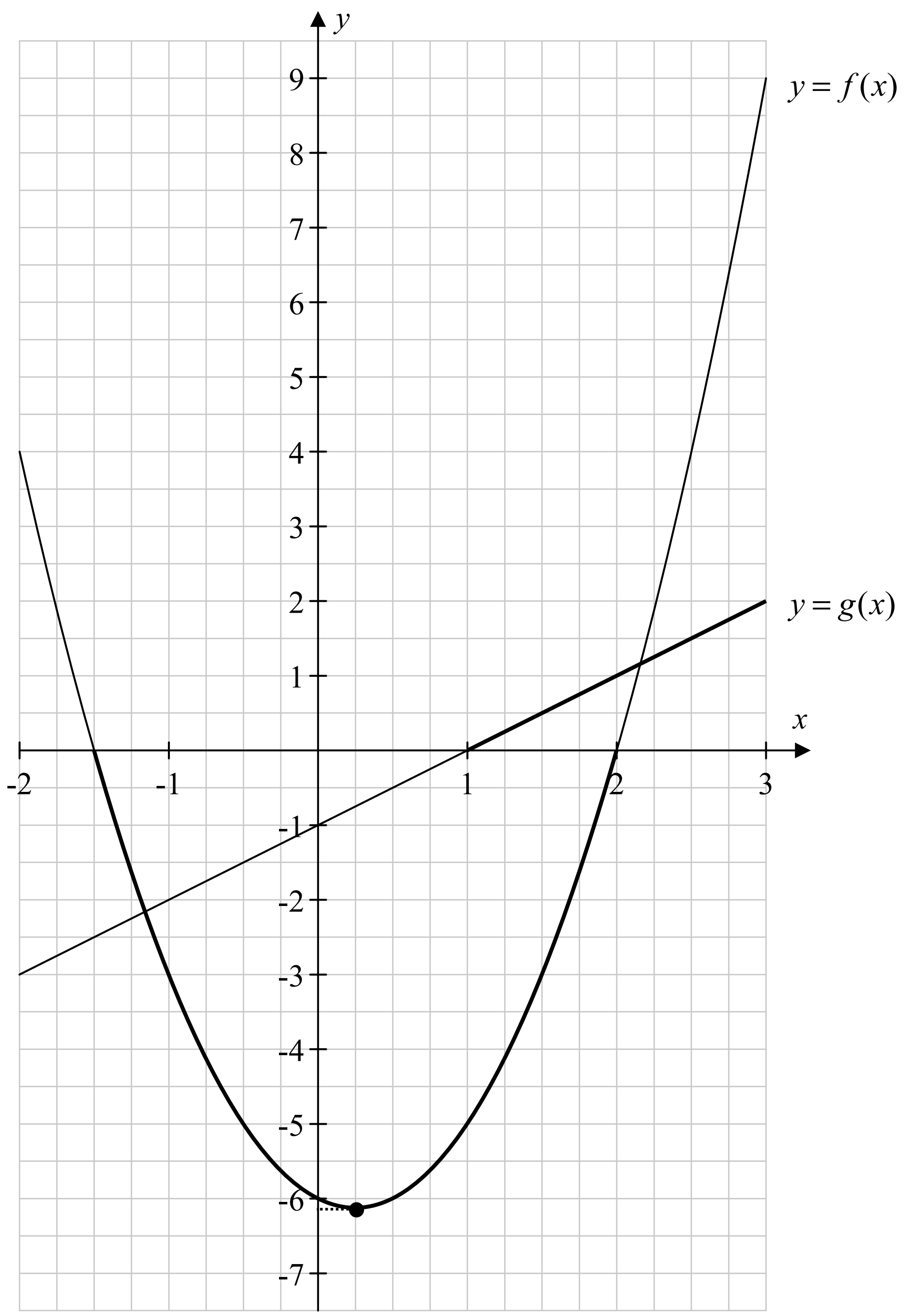
Question 14 **50 Marks**

(i) g is the function $g : x \mapsto x - 1$, where $x \in \mathbb{R}$. Find the value of each of the following.

> $g(3) = 3 - 1 = 2.$
>
> $g(-2) = -2 - 1 = -3.$

(ii) f is the function $f : x \mapsto 2x^2 - x - 6$, where $x \in \mathbb{R}$.

Using the same axes and scales, draw the graphs of the functions $y = f(x)$ and $y = g(x)$ in the domain $-2 \le x \le 3$.



Graphing g:	*Graphing f*:
Straight line, so only need the two points from **(i)**: $(3,2)$ and $(-2,-3)$.	$f(-2)=4$ $f(-1)=-3$ $f(0)=-6$ $f(1)=-5$ $f(2)=0$ $f(3)=9$
Or:	*Or*:
$g(x) = x - 1$	$f(x) = 2x^2 - x - 6$

x	x	- 1	y
-2	-2	- 1	-3
-1	-1	- 1	-2
0	0	- 1	-1
1	1	- 1	0
2	2	- 1	1
3	3	- 1	2

x	$2x^2$	$- x$	- 6	y
-2	8	+2	- 6	4
-1	2	+ 1	- 6	- 3
0	0	0	- 6	- 6
1	2	- 1	- 6	- 5
2	8	- 2	- 6	0
3	18	- 3	- 6	9

Use your graphs from **(ii)** to estimate:

(iii) the minimum value of $f(x)$

$f_{\min}(x) = -6 \cdot 1$ *... see graph*

(iv) the range of values of x for which $f(x) < 0$

$-1 \cdot 5 < x < 2$ *... see graph*

(v) the range of values of x for which $g(x) \geq 0$.

$x \geq 1$ *... see graph*