Question 1 **(15 marks)**

(a) **(i)** The columns in the table below represent the following sets of numbers:
Natural numbers ($\mathbb{N}$), Integers ($\mathbb{Z}$), Rational numbers ($\mathbb{Q}$),
Irrational numbers ($\mathbb{R}\backslash\mathbb{Q}$) and Real numbers ($\mathbb{R}$).

Complete the table by writing either '**Yes**' or '**No**' into each box indicating whether each of the numbers $\sqrt{5},\ 8,\ -4,\ 3\frac{1}{2},\ \frac{3\pi}{4}$ is or is not an element of each.
(One box has already been filled in. The '**Yes**' indicates that the number 8 is an element of the set of Real numbers, $\mathbb{R}$).

Number/Set	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$(\mathbb{R}\backslash\mathbb{Q})$	$\mathbb{R}$
$\sqrt{5}$	**No**	**No**	**No**	**Yes**	**Yes**
8	**Yes**	**Yes**	**Yes**	**No**	**Yes**
-4	**No**	**Yes**	**Yes**	**No**	**Yes**
3½	**No**	**No**	**Yes**	**No**	**Yes**
$\frac{3\pi}{4}$	**No**	**No**	**No**	**Yes**	**Yes**

(ii) In the case of $\sqrt{5}$ explain your choice in relation to the set of Irrational numbers ($\mathbb{R}\backslash\mathbb{Q}$) (i.e. give a reason for writing either 'Yes' or 'No').

$\sqrt{5}$ cannot be written as a fraction

(b) Use the properties of surds to show that $\sqrt{98}-\sqrt{18}+\sqrt{2}$ simplifies to $5\sqrt{2}$.

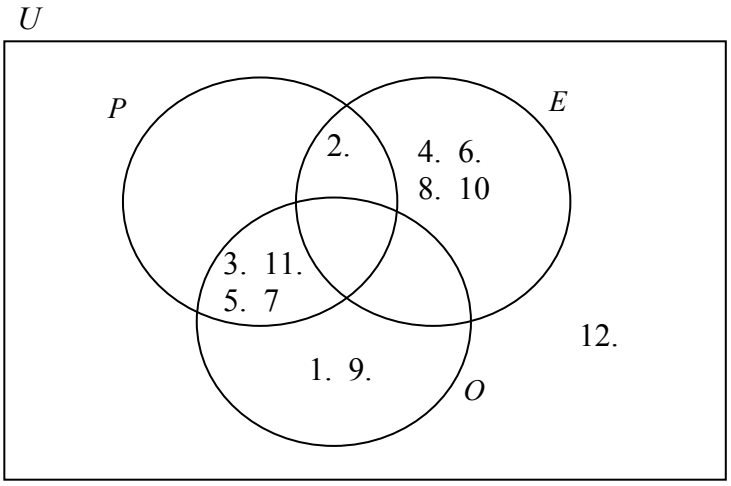
$$7\sqrt{2}-3\sqrt{2}+\sqrt{2}=5\sqrt{2}$$

[4]

Question 2 **(25 marks)**

$U = \{1, 2, 3, ..., 12\}$. P is the set of prime numbers less than 12. E is the set of even numbers less than 12. O is the set of odd numbers less than 12.

(a) Represent these sets on the Venn diagram.



(b) Name any set on this diagram (after part **(a)** has been completed) that is a null set.

P\(E∪O), P∩E∩O, E∩O, (E∩O)\P

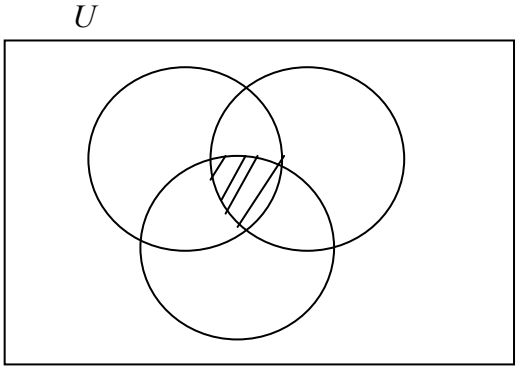
(c) If a number is drawn at random from **set *P***, what is the probability that it is even?

$\frac{1}{5}$

Question 3 **(40 marks)**

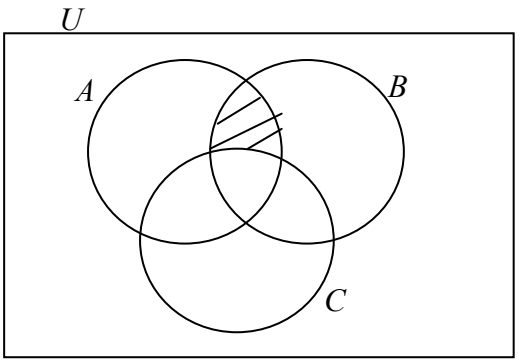
(a) For diagrams **(i)** and **(ii)** below, shade in the named region.

(i)



$A \cap B \cap C$

(ii)



$(A \cap B) \setminus C$

(b) The box on the right contains six statements,
(**note**: P', is the complement of a set P).

A number of the statements are incorrect.

Write down one incorrect statement.

(iv) A\B = B\A or (iii) (A\B)\C = A\(B\C)

Statements

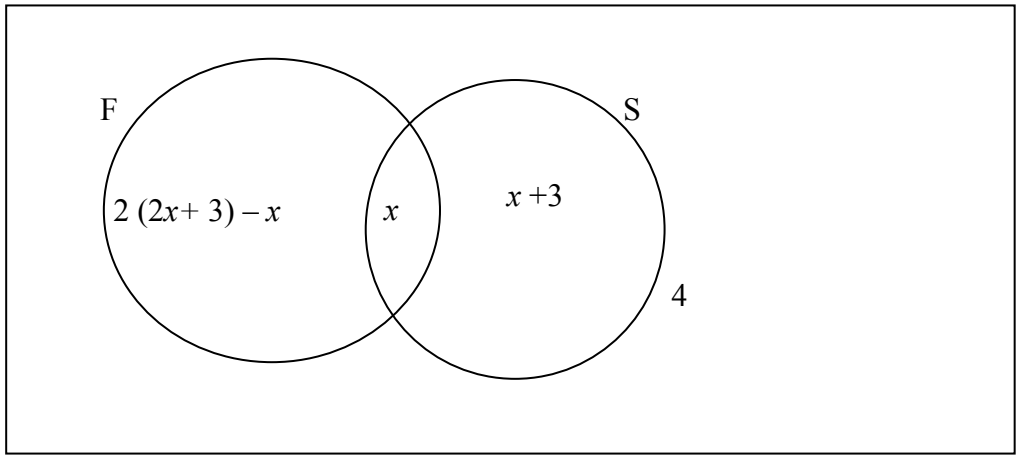
(i) $A \cup B = B \cup A$
(ii) $(A \cup B) \cup C = A \cup (B \cup C)$
(iii) $(A \setminus B) \setminus C = A \setminus (B \setminus C)$
(iv) $(A \cap B)' = U \setminus (A \cap B)$
(v) $A \setminus B = B \setminus A$
(vi) $B \setminus (A \cup C) = (B \cup C) \setminus A \setminus C$

Draw a diagram **or** give an example to explain your choice.

Diagram or explanation

(c) A group of 38 students were asked if they had ever been to France or Spain.
The number who had been to Spain only was 3 more than the number who had been to both countries.
Twice as many had been to France as Spain.
4 students had not been to either country.

Find how many had been to both countries.



$2\,(2x+3) - x + x + x + 3 = 34 \Rightarrow \boldsymbol{x = 5}$

[6]

Question 4 **(35 marks)**

The minimum wage per hour for different categories of workers is shown in the table.
By law the Under 18 minimum wage is set at 70% of the minimum wage for an experienced adult worker.

(a) Verify that this is true for the rates shown in the table on the right.

Category	Min. Wage per hour
Experienced adult worker	€8·65
Aged under 18	€6·06
Over 18 in first year from date of first employment	€6·92
Over 18 in second year of first employment	€7·79
Source: www.citizensinformation.ie	

8·65 × 0·7 = 6·055 which is 6·06 correct to two dps.
Or any other check.

(b) The government has decided that it is going to reduce all minimum wage rates by 6%. Calculate the new minimum wage for an experienced adult worker, correct to two decimal places, after this reduction.

8·65 × 0·94 = €8·13

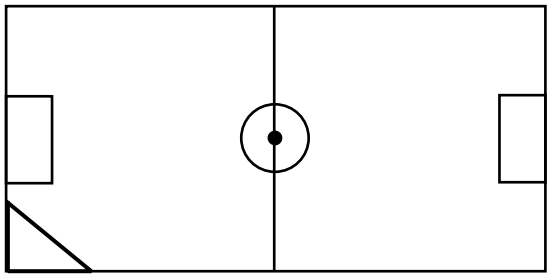
(c) John is an experienced adult worker. After the reduction he says "If the minimum wage were to be increased by 6% then I would be back earning €8·65 per hour."
Is John's statement correct? Explain your answer.

No.
8·13 × 1·06 = €8·62
This is not as high as the original starting point.

[7]

Question 5 **(10 marks)**

Two members of a soccer club want to find out if their football pitch has been lined out properly.



(a) They have a 10 metre tape measure, a calculator, pen and paper.
By using only these, explain how they could test if the angle at each corner is a right angle.

Start at the corner flag. Use the tape measure to measure a certain distance out along the side-line. e.g. 5 m.
Then measure a certain distance out along the goal-line. e.g. 4 m.
Then measure the distance between these two end points
Using Pythagoras Theorem see if the calculated distance is the same as the measured distance.

(b) By using only a trundle wheel, calculator, pen and paper, explain how the two members could test if the 'centre-circle' on the pitch is really a circle. (You may assume that the centre spot on the pitch is the centre of the circle).

Use the trundle wheel to measure the radius. i.e distance from the centre spot to anywhere on the circumference.
Use Circumference = $2\pi r$ to calculate the circumference.
Then use the trundle wheel to measure the circumference on the circle and see if the two match.

Question 6 **(15 marks)**

Car A and Car B set off from a starting point *S* at the same time. They travel the same route to destination *D*, which is 70 km away. Car A travels at an average speed of 50 km/h and car B travels at an average speed of 45 km/h.

How far will car B have travelled by the time car A arrives at destination *D*?

Car A: (Time to reach D) T = D/S= 70/ 50 = 1·4 h

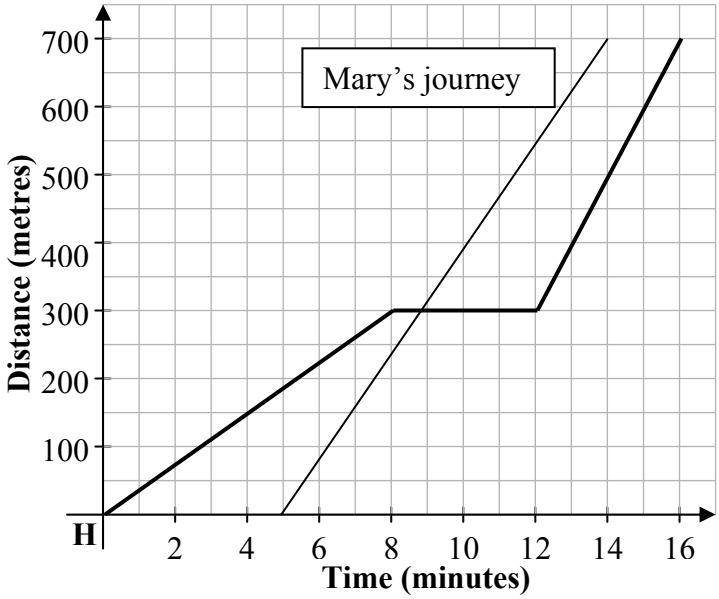
Car B: Distance travelled 45 × 1·4 = 63 km

[8]

Question 7 **(20 marks)**

Angela leaves home (H) at 5 pm to go to football practice, which is 700 m away. The graph shows her journey, on foot, to football practice.

(a) One of the stories below matches Angela's journey.
Place a tick in the box beside the correct matching story.
(**Note:** Only **one** story matches Angela's journey).



Story	Tick one story (√)
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a slower pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.12 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for five minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	√
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at the same pace and arrives at practice at 5.16.	

(b) Mary also lives 700 m from football practice, but cycles to practice. She leaves home five minutes after Angela. She cycles at a constant pace and arrives at practice two minutes before Angela.
Represent Mary's journey on the graph above.

[9]

Question 8 **(30 marks)**

(a) Express in its simplest form: $\frac{5-x}{5} + \frac{x-4}{4}$.

$$\frac{4(5-x)+5(x-4)}{20} = \frac{x}{20}$$

(b) Solve for x: $3x^2 + 11x = 4$.

$$3x^2+11x-4=0$$
$$(3x-1)(x+4)=0$$
$$x=\frac{1}{3} \qquad x=-4$$

$$3x^2+11x-4=0$$
$$x=\frac{-11\pm\sqrt{11^2-4(3)(-4)}}{2(3)}$$
$$x=\frac{-11\pm 13}{6}$$
$$x=\frac{1}{3} \qquad x=-4$$

(c) Divide $2x^3 + x^2 - 13x + 6$ by $x+3$.

Method A

$$\begin{array}{r} 2x^2-5x+2 \\ x+3\overline{)\,2x^3+x^2-13x+6} \\ \underline{2x^3+6x^2} \\ -5x^2-13x \\ \underline{-5x^2-15x} \\ 2x+6 \\ \underline{2x+6} \end{array}$$

Method B

	ax^2	bx	c
x	ax^3	bx^2	cx
+3	$3ax^2$	3bx	3c

$ax^3 = 2x^3 \Rightarrow a = 2$

$x^2(3a+b) = -5 \Rightarrow 3a+b = -5$

$\Rightarrow 6=b = -5 \Rightarrow b = -11$

$3c = 6 \Rightarrow c = 2$

[10]

(d) A company employs two drivers, John and David. Each has use of a company car and small van. The company buys €30 worth of Toll Tags for each driver. Each time that a vehicle goes through the M50 Toll, a charge will be deducted from the Toll Tags.

John goes through the M50 toll five times in his car and four times in his small van. He then has €7·90 remaining on his Toll Tags. David goes through the M50 Toll twice in his car and six times in his small van. He then has €8·40 left on his Toll Tags.

Calculate how much it costs for a car and for a small van to go through the M50 Toll.

$5x + 4y = 30 - 7 \cdot 90 = 22 \cdot 10$

$2x + 6y = 30 - 8 \cdot 0 = 21 \cdot 60$

$\Rightarrow x = €2 \cdot 10$

$y = €2 \cdot 90$

Question 9 **(15 marks)**

The 'Multiplier' is a variable used by economists to measure the affect of an increase in spending in an economy.

One version of the Multiplier is $M = \frac{1}{S+P}$ where M is the Multiplier, S relates to savings and P relates to imports.

(a) Calculate the value of the M, the Multiplier, if $S = 0{\cdot}2$ and $P = 0{\cdot}1$.

$\left(\frac{1}{0{\cdot}2 + 0{\cdot}1}\right) = 3\frac{1}{3}$ or $\frac{10}{3}$ or $3 \cdot 3$

(b) Explain the effect on the size of M if the value of P increases.

Denominator increases so value of fraction **decreases.**

(c) Sometimes the above formula is used to calculate P. Rearrange the formula to make P its subject.

$$M = \frac{1}{S+P}$$

$MS + MP = 1$
$MP = 1 - MS$
$P = \frac{1-MS}{M}$ or P = $\frac{1}{M} - S$

Question 10 **(20 marks)**

(a) If $n = 7$ find the value of $2n$ and also the value of $2n + 1$.

$2(7) = 14$
$2(7) + 1 = 15$

(b) **(i)** x represents an even number. Explain why $x + 2$ is the next even number.

2 is the lowest even no. so adding 2 on to an even no. will give the next even no.

(ii) If one third of the smaller even number is subtracted from half of the larger even number the result is 8. Find the value of x.

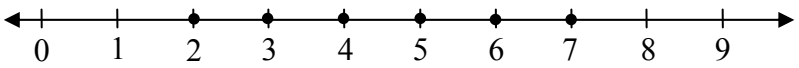
$$\frac{x+2}{2} - \frac{x}{3} = 8$$
$x = 42$

Question 11 **(20 marks)**

(a) Solve the following inequality and show the solution on the number line.

$$-2 \leq \frac{1}{2}x - 3 < 1, \; x \in \mathbb{N}.$$

$2 \leq x < 8$



(b) Josephine hopes to go to college. She has saved €3000. She will attend college for 32 weeks in her first year. She plans to have at least €800 left at the end of the year.

(i) If she spends €x each week, write an inequality to represent her spending during the year.

$32x \leq 3000 - 800$ or similar

(ii) Hence, or otherwise, find the maximum amount Josephine can spend each week.

$32x \leq 2200$

$x \leq$ €68·75

Question 12 **(10 marks)**

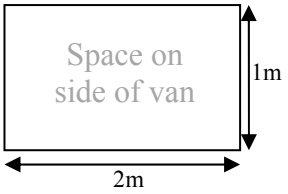
Irish Sport Promotions has designed a company logo. The actual size of the logo is shown here.

(a) Write the dimensions of the logo, to the nearest mm, below.

Width __27__mm **Height** _15_ mm

(b) The company wants to enlarge this logo in order to put it on the side of its vans.

The space available for the logo on the side of each van is: width 2 m and height 1 m.



If the company wants to keep the same width to height ratio as in the original logo, calculate the dimensions of the largest logo that will fit onto the side of the van.

Make height of logo = 1000 mm

$\frac{15}{27} = \frac{1000}{x}$

x = 1800 mm (= width) (or 1·8 m) (or 180 cm)

OR

Make height of logo = 1 m

$\frac{15}{27} = \frac{1}{x}$

x = 1·8 m

Scale Factor = $\frac{1000}{15} = 66\dot{6}$

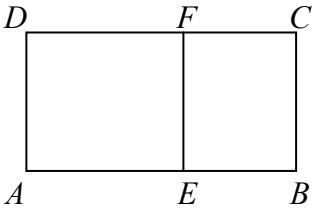
$27 \times 66{\cdot}66 = 1799{\cdot}82$ mm

Question 13 **(10 marks)**

$ABCD$ is a rectangle. $AEFD$ is a square. $|AD| = 1$ cm. $|DC| = x$ cm.

(i) Write $|EB|$ in terms of x.

$x - 1$



(ii) If $\frac{|AB|}{|AD|} = \frac{|EF|}{|EB|}$, find the distance x. Give your answer correct to two decimal places.

$\frac{x}{1} = \frac{1}{x-1}$

$x(x-1) = 1 \Rightarrow x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{5}}{2}$;

x = 1·618…= 1·62cm (discard neg. value)

Question 14 **(15 marks)**

Investigate whether the pattern in the table below is linear, quadratic or exponential.
Explain your conclusion.

Term 1	Term 2	Term 3	Term 4	Term 5
$2a - b + 2c$	$8a - 2b + 2c$	$18a - 3b + 2c$	$32a - 4b + 2c$	$50a - 5b + 2c$

$2a - b + 2c$

$8a - 2b + 2c$ Diff = $6a - b$

$18a - 3b + 2c$ Diff = $10a - b$ Diff = $4a$

$32a - 4b + 2c$ Diff = $14a - b$ Diff = $4a$

$50a - 5b + 2c$ Diff = $18a - b$ Diff = $4a$

2^{nd} difference is constant therefore the relationship is quadratic

Question 15 **(20 marks)**

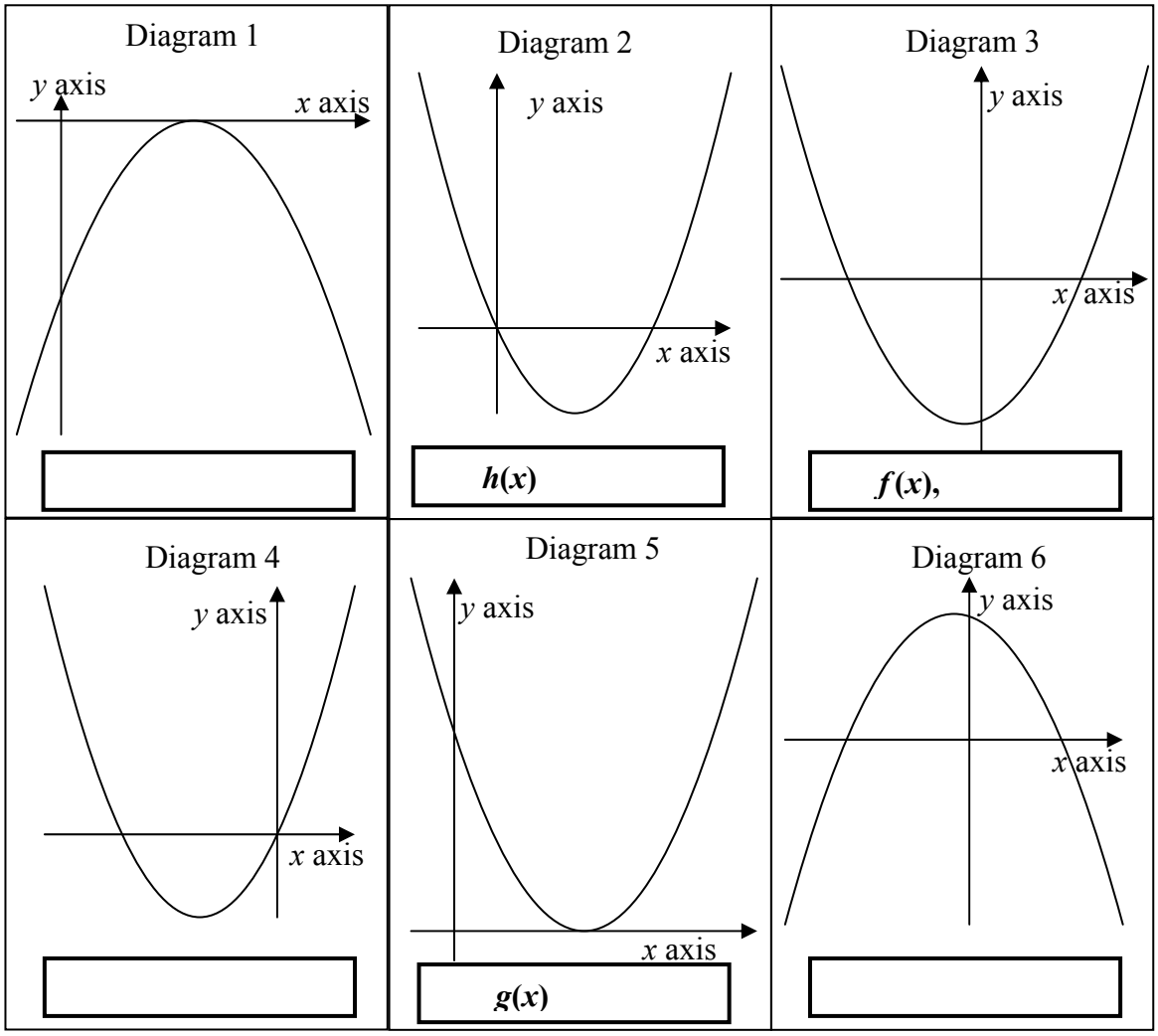
(a) Three functions: $f(x)$, $g(x)$ and $h(x)$ are defined as follows:

$f(x) = 2x^2 + x - 6$, $g(x) = x^2 - 6x + 9$ and $h(x) = x^2 - 2x$.

Solve $f(x) = 0$	Solve $g(x) = 0$	Solve $h(x) = 0$
$(2x - 3)(x + 2) = 0$ $x = \frac{3}{2}$, $x = -2$	$(x - 3)(x - 3) = 0$ $x = 3$	$x(x - 2) = 0$ $x = 0$, $x = 2$

(b) The table below shows the sketches of six different functions. Three of the sketches belong to the three functions from part **(a)**.

Write $f(x)$, $g(x)$ or $h(x)$ into the box underneath the correct sketch for each of the three functions.



[16]