Question 1 **20 Marks**

Pauline flips a fair coin 3 times, and records the outcomes.
She writes H for each head and T for each tail.

(i) Complete the table below to show all of the possible outcomes.
Two outcomes have already been filled in for you.

$H\ H\ H$	$T\ H\ H$
$H\ H\ T$	$T\ H\ T$
$H\ T\ H$	$T\ T\ H$
$H\ T\ T$	$T\ T\ T$

(ii) Find the probability of getting two heads and one tail.

$$\Pr(2\ H, 1\ T) = \frac{3}{8}$$

(iii) Jamie says: "You have the same probability of getting three heads as you do of getting two heads and one tail."
Do you agree with Jamie? Give a reason for your answer.

Answer: No

Reason: $\Pr(3\ H) = \frac{1}{8}$ but $\Pr(2\ H, 1\ T) = \frac{3}{8}$

Or:

Reason: There is only 1 way to get three heads. There are 3 ways to get two heads and one tail.

(iv) Max says: "You have the same probability of getting $H\ H\ H$ as you do of getting $H\ T\ H$."
Do you agree with Max? Give a reason for your answer.

Answer: Yes

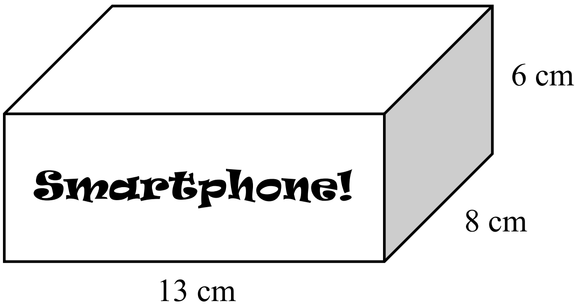
Reason: $\Pr(H\ H\ H) = \frac{1}{8}$ and $\Pr(H\ T\ H) = \frac{1}{8}$

Or:

Reason: There is only one way to get each event.

Question 2 **35 Marks**

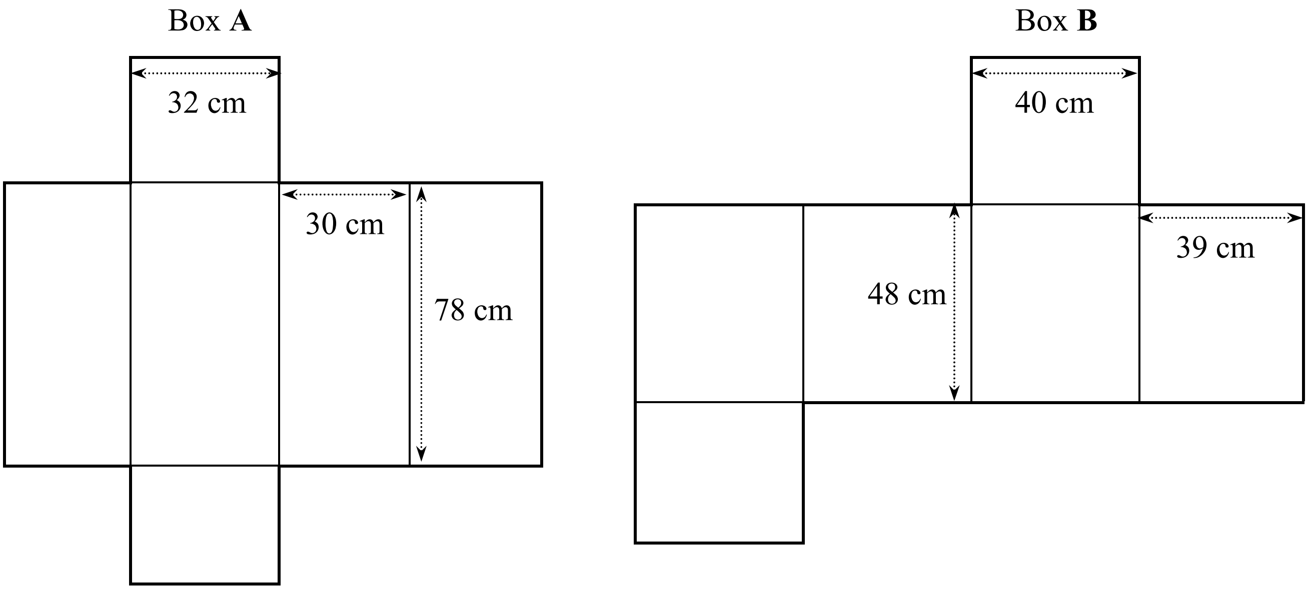
The box for an individual mobile phone is 13 cm long, 8 cm wide, and 6 cm high, as shown.



(i) Find the volume of an individual mobile phone box.

Volume $= 13 \times 8 \times 6 = 624 \text{ cm}^3$

These individual mobile phone boxes will be shipped in a large rectangular box.
Below are diagrams of the nets of two large boxes that could be used, Box **A** and Box **B**.



(ii) Show that Box **A** and Box **B** have the same volume.

Box **A**:	Box **B**:
Volume $= 32 \times 30 \times 78 = 74\ 880 \text{ cm}^3$	Volume $= 48 \times 40 \times 39 = 74\ 880 \text{ cm}^3$

(iii) What is the largest number of individual mobile phone boxes that will fit in each large box?

> Box **A**: $32 \div 8 = 4$; $30 \div 6 = 5$; $78 \div 13 = 6$; so Box **A** can be filled completely.
>
> Box **B**: $48 \div 6 = 8$; $40 \div 8 = 5$; $39 \div 13 = 3$; so Box **B** can be filled completely.
>
> Total: $4 \times 5 \times 6 = 120$ individual phone boxes.

(iv) Find the surface area of each large box.

Box **A**:	Box **B**:
Surface Area $= 2(32 \times 30 + 32 \times 78 + 30 \times 78)$ $= 11\ 592 \text{ cm}^2$.	Surface Area $= 2(48 \times 40 + 48 \times 39 + 40 \times 39)$ $= 10\ 704 \text{ cm}^2$.

(v) The large boxes are made from cardboard. The cardboard costs €0·67 per m^2. The cardboard just covers the net of a box. Find the cost of the box that uses the least amount of cardboard.

> *Use Box **B**. The cost is given per m^2, so convert surface area to m^2 (or cost to per cm^2).*
>
> $1 \text{ cm} = 0\cdot 01 \text{ m}$, so $1 \text{ cm}^2 = 0\cdot 01^2 \text{ m}^2 = 0\cdot 0001 \text{ m}^2$.
>
> Surface area $= 10\ 704 \text{ cm}^2 = 10\ 704 \times 0\cdot 0001 \text{ m}^2 = 1\cdot 0704 \text{ m}^2$.
>
> $$\begin{aligned} \text{Cost of box } &= €1\cdot 0704 \times 0\cdot 67 \\ &= €0\cdot 717168 \\ &= €0\cdot 72, \text{ to the nearest cent.} \end{aligned}$$

(vi) An average of 140 large boxes is produced each month. Find the saving, per annum, if you choose to make the box that uses the least amount of cardboard.

> $$\begin{aligned} \text{Cost of Box A } &= €(11592 \times 0\cdot 0001 \times 0\cdot 67) \\ &= €0\cdot 776664 \\ &= €0\cdot 78, \text{ to the nearest cent.} \end{aligned}$$
>
> $$\begin{aligned} \text{Saving per annum } &= €(0\cdot 78 - 0\cdot 72) \times 140 \times 12 \\ &= €(0\cdot 06) \times 1680 \\ &= €100\cdot 80. \end{aligned}$$
>
> *Or*:
>
> Difference in area $= (11592 - 10704) \text{ cm}^2 = 888 \text{ cm}^2 = 0\cdot 0888 \text{ m}^2$.
>
> Saving per annum $= €0\cdot 67 \times 0\cdot 0888 \times 140 \times 12 = €99\cdot 95$.

Question 3 **40 Marks**

All of the students in a class took *IQ Test 1* on the same day. A week later they all took *IQ Test 2*. Their scores on the two IQ tests are shown in the tables below.

IQ Test 1				
86	104	89	105	96
96	103	94	104	119
115	79	97	111	108

IQ Test 2				
83	120	105	111	114
99	111	108	106	97
97	102	94	108	117

(i) Draw a back-to-back stem-and-leaf plot below to display the students' scores.

IQ Test 1						*IQ Test 2*				
				9	7					
			9	6	8	3				
	7	6	6	4	9	4	7	7	9	
8	5	4	4	3	10	2	5	6	8	8
		9	5	1	11	1	1	4	7	
					12	0				

Key: $9\,|\,7 =$ a score of 97

(ii) Find the range of scores for each IQ test.

Range of *IQ Test 1* $= 119 - 79 = 40$.	Range of *IQ Test 2* $= 120 - 83 = 37$.

(iii) Find the median score for each IQ test.

15 data points in each set, so median is the $\frac{15+1}{2} = 8th$ *data point.*	
Median of *IQ Test 1* $= 103$.	Median of *IQ Test 2* $= 106$.

(iv) Find the mean score for each IQ test.

Mean of *IQ Test 1* $= \frac{1506}{15} = 100\cdot4$.	Mean of *IQ Test 2* $= \frac{1572}{15} = 104\cdot8$.

(v) Compare the scores on the two IQ tests. Refer to **at least one** measure of central tendency and **at least one** measure of variability (spread) in your answer.

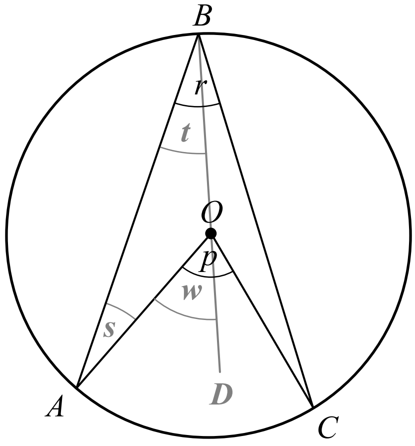
> In general, the scores in *IQ Test 2* are slightly higher than in *IQ Test 1*, as both the mean and median are higher for *IQ Test 2*.
>
> The scores are slightly more spread out in *IQ Test 1* than in *IQ Test 2*, as the range is bigger for *IQ Test 1*; *or* The spread of scores is very similar, as the two ranges are almost the same.

(vi) Marshall says that every student in the class must have done better on *IQ Test 2* than on *IQ Test 1*. Is Marshall correct? Explain your answer.

> Answer: No.
>
> Explanation: The person who got 119 on *IQ Test 1* could have got less, e.g. 94, on *IQ Test 2.*
>
> *Or*:
>
> Explanation: The maximum score on *IQ Test 1* is greater than the minimum score on *IQ Test 2*.

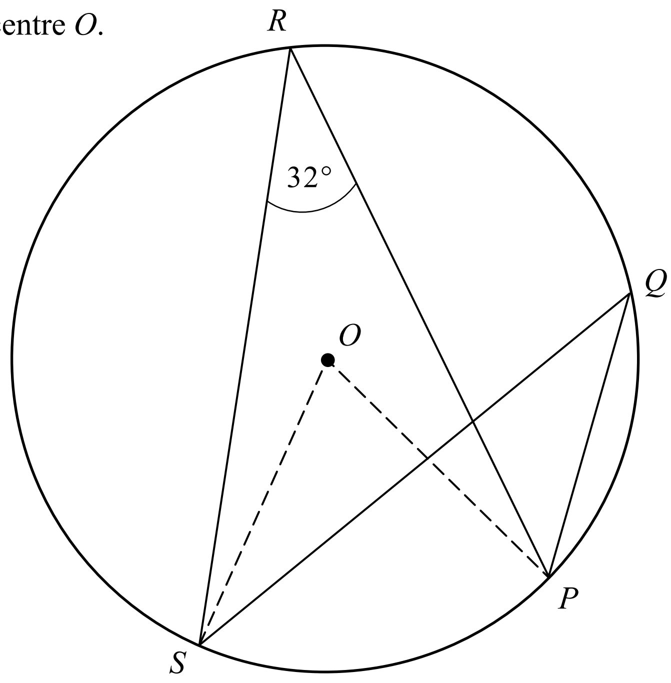
Question 4 **30 Marks**

(a) Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Diagram: 
Given: A circle with centre *O*. Points *A*, *B*, and *C* on the circle. Angles *p* and *r*, as shown.
To Prove: $p = 2r$.
Construction: Join *B* to *O*, and extend to *D*. Mark the angles *s*, *t*, and *w*.
Proof: $\lvert OA \rvert = \lvert OB \rvert$ radii of circle *Step 1* $\therefore s = t$ isosceles triangle *Step 2* $w = s + t$ exterior angle *Step 3* $\therefore w = 2t$ *Step 4* Similarly, $(p - w) = 2(r - t)$. So $p = (p - w) + w$ $= 2(r - t) + 2t$ $= 2r$ *Step 5*

(b) P, Q, R, and S are points on a circle with centre O.

$|\angle PRS| = 32°$, as shown.



(i) Find $|\angle SOP|$.

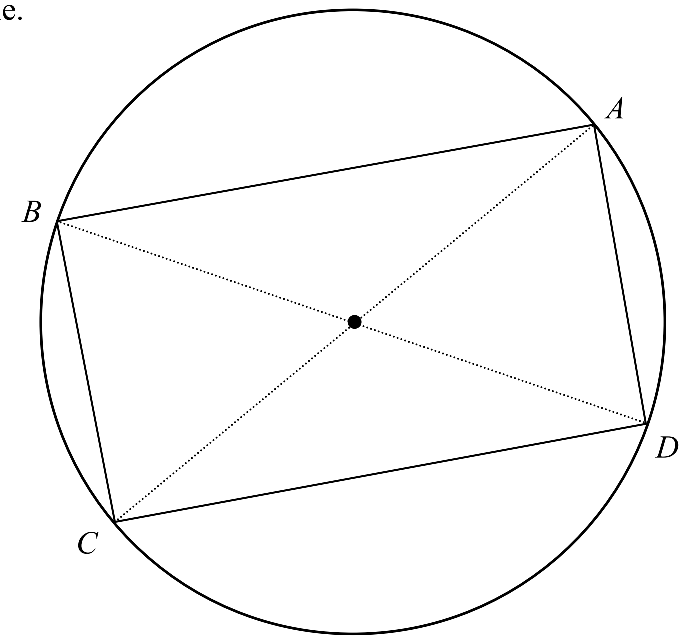
$|\angle SOP| = 2 \times 32° = 64°$.

(ii) Find $|\angle SQP|$.

$|\angle SQP| = |\angle SRP| = 32°$.

(c) A, B, C, and D are points on a circle, as shown below.
$[AC]$ and $[BD]$ are diameters of the circle.

Prove that $ABCD$ is a rectangle.



We just need to prove that the four angles are 90°.

$|\angle BAD| = |\angle BCD| = 90°$, as $[BD]$ is a diameter.

Similarly, $|\angle CBA| = |\angle CDA| = 90°$.

So $ABCD$ is a rectangle.

Question 5 **45 Marks**

Students in a class are investigating spending in their local area. They each carry out a different survey, and display the results.

(a) John is investigating whether people pay for their weekly shopping with Credit Card, Debit Card, Cash, or Cheque. When people tell him which one of these they usually use, he writes it in a table. His results are shown below.

Credit Card	Debit Card	Debit Card	Cash	Debit Card
Credit Card	Cash	Cash	Credit Card	Debit Card
Debit Card	Debit Card	Cheque	Cash	Cash
Cash	Cash	Debit Card	Cash	Credit Card

(i) What type of data has John collected? Put a tick (✓) in the correct box below.

Numerical Continuous	Numerical Discrete	Categorical Nominal	Categorical Ordinal
☐	☐	☑	☐

(ii) Fill in the frequency table below.

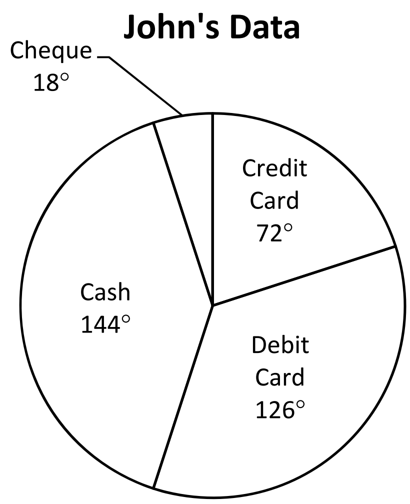
Method of Payment	Credit Card	Debit Card	Cash	Cheque
Frequency	4	7	8	1

(iii) What is the mode of John's data? Mode = Cash

(iv) John says that he cannot find the mean of his data. Explain why this is the case.

He cannot add up his values and divide by 20.

(v) Display John's data in a pie chart. Show all of your calculations clearly.

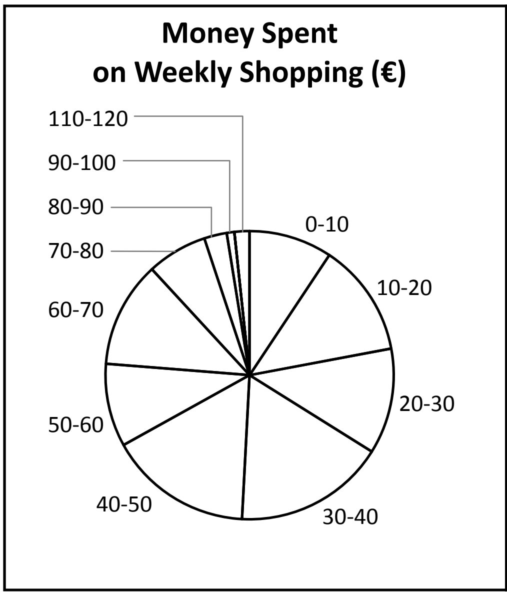
Credit Card: $\frac{4}{20}\times 360° = 72°$ Debit Card: $\frac{7}{20}\times 360° = 126°$ Cash: $\frac{8}{20}\times 360° = 144°$ Cheque: $\frac{1}{20}\times 360° = 18°$.	

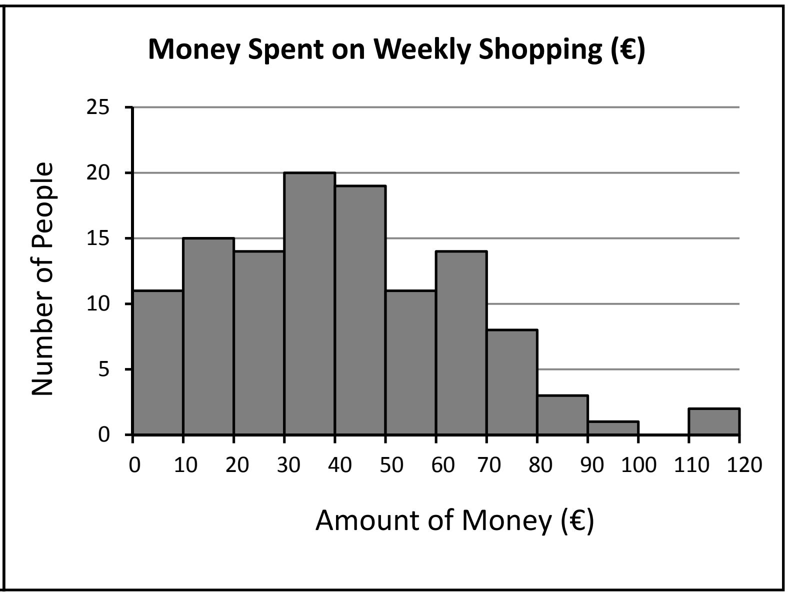
(b) Margaret wants to examine if people prefer to do their weekly shopping in *Tesco*, *Dunnes Stores*, *SuperValu*, or *Lidl*. She stands outside her local *Lidl* shop for one day, and asks everyone as they leave the shop where they prefer to do their weekly shopping.

Give one reason why Margaret's data may be biased.

Margaret's data may be biased because her sample is probably not representative. She will probably have a lot more people answering "Lidl" than she should.

(c) Mary is interested in the amount of money people spend on their weekly shopping.
She surveys people as they leave the local supermarket on a Saturday morning, and displays her results in the two graphs below.





(i) Mary wants to show that about half of her sample spent less than €40 on their weekly shopping. Which graph do you think she should use? Give a reason for your answer.

Answer:	Pie chart.
Reason:	It’s easy to see where half the pie chart is (180°).

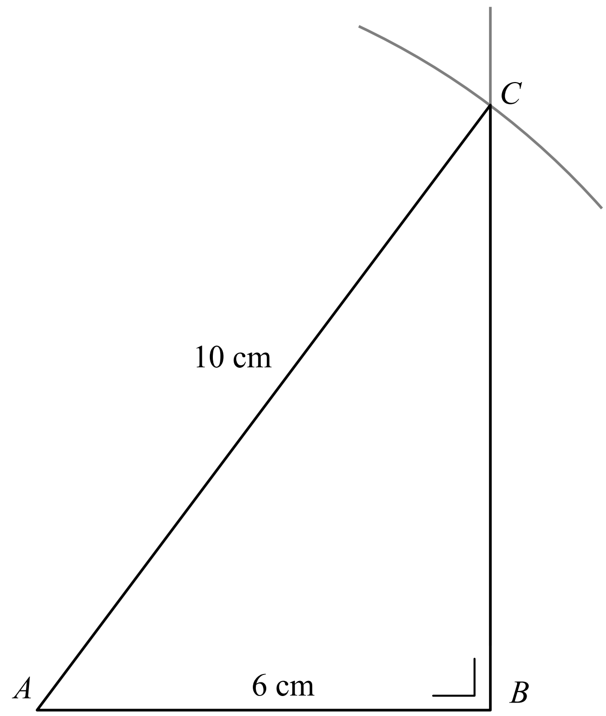
(ii) Mary wants to show that there were more people in the 30–40 group than in any other. Which graph do you think she should use? Give a reason for your answer.

Answer:	Bar chart / Histogram.
Reason:	It’s easy to see which bar is highest.

Question 6 **25 Marks**

(i) Construct a right-angled triangle $ABC,$ where:

$$|AB| = 6 \text{ cm}$$
$$|\angle ABC| = 90^\circ$$
$$|AC| = 10 \text{ cm.}$$



Note: *It is also possible to work out the length of the third side,* $[BC]$*, using the Theorem of Pythagoras, and then construct* $[BC]$ *and* $[AC]$.

(ii) On your diagram, measure the angle $\angle CAB$. Give your answer correct to the nearest degree.

$|\angle CAB| =$ $\boxed{53^\circ}$

(iii) Let X be the whole number you wrote as your answer to **(ii)**.

Use a calculator to find $\cos X$. Give your answer correct to 3 decimal places.

$\cos(53^\circ) = 0 \cdot 6018... = 0 \cdot 602$, correct to three decimal places.

(iv) Jacinta says that $\cos(\angle CAB)$ is exactly 0·6, because $\cos(\angle CAB) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$.

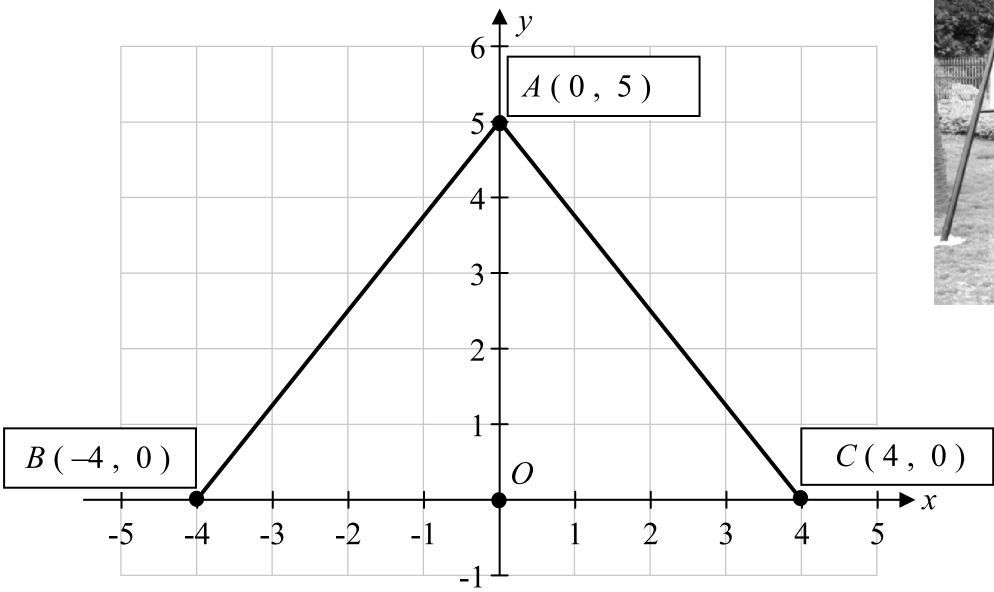
Explain why your answer in **(iii)** is **not** the same as Jacinta's.

They are not the same because $|\angle CAB| = \cos^{-1}\left(\dfrac{6}{10}\right) = 53 \cdot 1301...^\circ$.

So if X is a whole number then $\cos X$ can never be exactly 0·6.

Question 7 **45 Marks**

The diagram below shows part of the frame of a swing on a co-ordinate grid.
Each unit on the grid represents one metre.
The line segments $[AB]$ and $[AC]$ represent metal bars.



(i) Write the co-ordinates of the points A, B, and C in the spaces provided in the diagram.

(ii) Find the total length of metal bar needed to make this part of the swing.
Give your answer in metres, correct to one decimal place.

$|AB| = \sqrt{4^2 + 5^2} = \sqrt{41}.$

Similarly, $|AC| = \sqrt{41}.$

Total length of metal bar $= 2\sqrt{41} = 12 \cdot 80... = 12 \cdot 8$ m, correct to one decimal place.

(iii) Find the slope of AB and the slope of AC.

AB:	AC:
Slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{5}{4}$ or $1 \cdot 25$.	Slope $= \dfrac{5-0}{0-4} = -\dfrac{5}{4}$ or $-1 \cdot 25$.

(iv) Is AB perpendicular to AC? Give a reason for your answer.

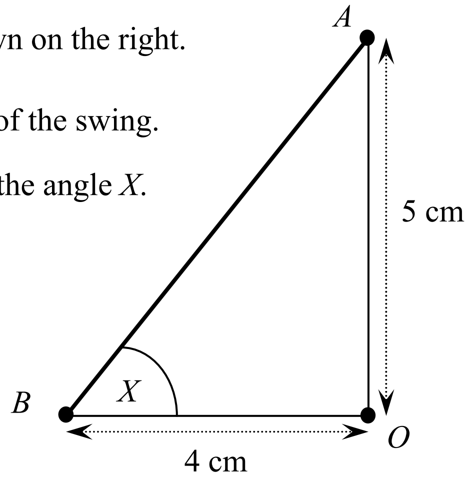
Answer: No

Reason: Product of slopes $= \dfrac{5}{4} \times -\dfrac{5}{4} = -\dfrac{25}{16} \neq -1.$

Or: Reason: When you invert one slope and change the sign, you don't get the other slope.

(v) Madison draws the scale diagram of the triangle OAB shown on the right. She marks in the angle X.
Recall that $[AB]$ is a metal bar, which is part of the frame of the swing.

Write down the value of $\tan X$, and hence find the size of the angle X. Give the size of the angle X correct to two decimal places.



$\tan X = \frac{5}{4}$
$\lvert\angle X\rvert = \tan^{-1}\left(\frac{5}{4}\right) = 51 \cdot 340... = 51 \cdot 34°$, correct to two decimal places.

In order to increase the height of the swing, it is decided to increase X by 20%.
The distance $|AB|$ will be kept the same.

(vi) Find the new height of the swing. Give your answer in metres, correct to one decimal place.

Recall from **(ii)** that $|AB| = \sqrt{41}$ m.

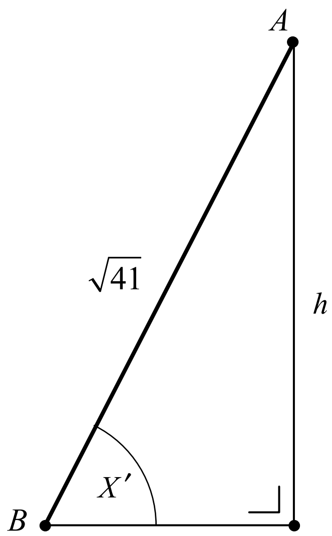
Increase $|\angle X|$ by 20% to get $|\angle X'|$:

$$|\angle X'| = 51 \cdot 34 \times 1 \cdot 2 = 61 \cdot 608°$$

From the diagram, $\sin X' = \sin 61 \cdot 608 = \frac{h}{\sqrt{41}}$.

$$\Rightarrow \quad h = \sqrt{41} \sin 61 \cdot 608 = 5 \cdot 632... = 5 \cdot 6 \text{ m},$$

correct to one decimal place.



Question 8 **20 Marks**

The equation of the line l is $x-3y-6=0$.

(i) Find the slope of the line l.

Method 1:	*Method 2*:
$-3y = -x+6$ *Step 1* $3y = x-6$ $y = \frac{1}{3}x-2$ *Step 2* $\Rightarrow$ Slope $= \frac{1}{3}$ *Step 3*	Slope $= -\frac{a}{b}$ $= -\frac{1}{-3}$ $= \frac{1}{3}$

(ii) Show that the point $(1,-2)$ is **not** on the line l.

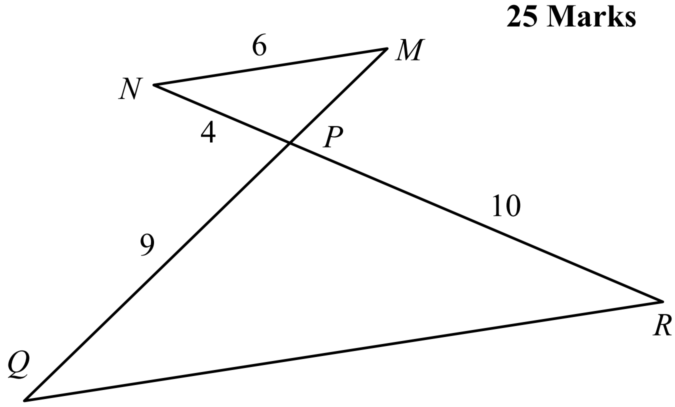
Sub in $(1,-2)$ to l: LHS $= 1-3(-2)-6 = 1 \neq 0 =$ RHS. Point not on l.

(iii) The line k passes through $(1,-2)$ and is parallel to the line l.
Find the equation of the line k.

Slope of $k = \frac{1}{3}$. Point on $k = (1,-2)$.	
Equation of k: $y-(-2) = \frac{1}{3}(x-1)$ $\Rightarrow$ $y = \frac{x}{3} - \frac{7}{3}$ *or* $x-3y-7 = 0$	*Or*: Equation of k: $x-3y+c = 0$ $\Rightarrow$ $1-3(-2)+c = 0$ $\Rightarrow$ $c = -7$ $\Rightarrow$ $x-3y-7 = 0$

Question 9 **25 Marks**

In the diagram below, $|\angle MNP| = |\angle PRQ|$.



(i) Prove that $\Delta\, MNP$ and $\Delta\, QRP$ are similar.

Proof:	$\lvert\angle MNP\rvert$	$=$	$\lvert\angle PRQ\rvert$	(given)
	$\lvert\angle NPM\rvert$	$=$	$\lvert\angle QPR\rvert$	(vertically opposite)
	$\lvert\angle NMP\rvert$	$=$	$\lvert\angle PQR\rvert$	(third angle)

$\Rightarrow$ Triangles are similar.

(ii) Is NM parallel to QR? Give a reason for your answer.

Answer: Yes

Reason: $|\angle MNP| = |\angle PRQ|$ *or* $|\angle NMP| = |\angle PQR|$ *or* alternate angles are equal.

Given $|MN| = 6,\ |NP| = 4,\ |QP| = 9,$ and $|PR| = 10,$ find:

(iii) $|QR|$

By similar triangles $\triangle MNP$ and $\triangle QRP$:

$$\frac{|QR|}{6} = \frac{10}{4}$$

$$\Rightarrow |QR| = 6\times\frac{10}{4} = 15.$$

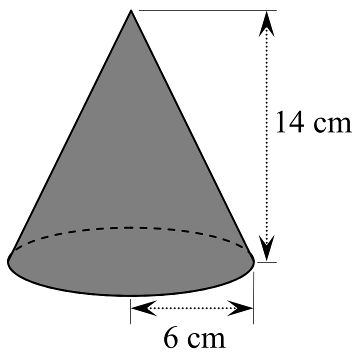
(iv) $|QM|$.

By similar triangles $\triangle MNP$ and $\triangle QRP$:	
$\frac{\vert PM\vert}{9} = \frac{6}{15}$ or $\frac{4}{10}$ $\Rightarrow \vert PM\vert = \frac{18}{5}$ or $3\cdot 6$ $\Rightarrow \vert QM\vert = 9 + 3\cdot 6 = \frac{63}{5}$ or $12\cdot 6$.	*Or*: $\frac{\vert PM\vert}{4} = \frac{9}{10}$ $\Rightarrow \vert PM\vert = 4\times\frac{9}{10} = \frac{18}{5}$ or $3\cdot 6$ $\Rightarrow \vert QM\vert = 9+3\cdot 6 = \frac{63}{5}$ or $12\cdot 6$.

Question 10 **15 Marks**

A solid cone has a radius of 6 cm and a height of 14 cm, as shown.

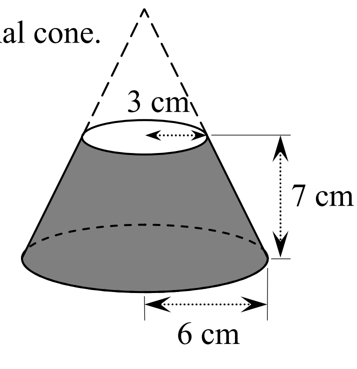
(i) Find the volume of the cone. Give your answer in terms of π.



Volume = $\frac{1}{3}\pi(6)^2\times 14 = 168\pi \text{ cm}^3$

The shape shown below is a *frustum*. This is made by taking the cone above, cutting it horizontally at a height of 7 cm, and removing the upper portion. The radius of the circular top of the frustum is 3 cm, as shown in the diagram.

(ii) Find the ratio of the volume of the frustum to the volume of the original cone.



Volume of upper (removed) portion:

$$\frac{1}{3}\pi(3)^2\times 7 = 21\pi \text{ cm}^3$$

Volume of frustum:

$$168\pi - 21\pi = 147\pi \text{ cm}^3$$

Or: Volume of frustum $= \frac{1}{3}\pi h\left[R^2 + Rr + r^2\right]$

$$= \frac{1}{3}\pi\times 7\left(6^2 + 6\times 3 + 3^2\right) = 147\pi \text{ cm}^3$$

Required ratio:

$$\frac{147\pi}{168\pi} = \frac{7}{8} \quad or \quad 7:8 \quad or \quad 0\cdot 875$$