Question 1

(a) Give two reasons why − 7·3 is not a natural number.

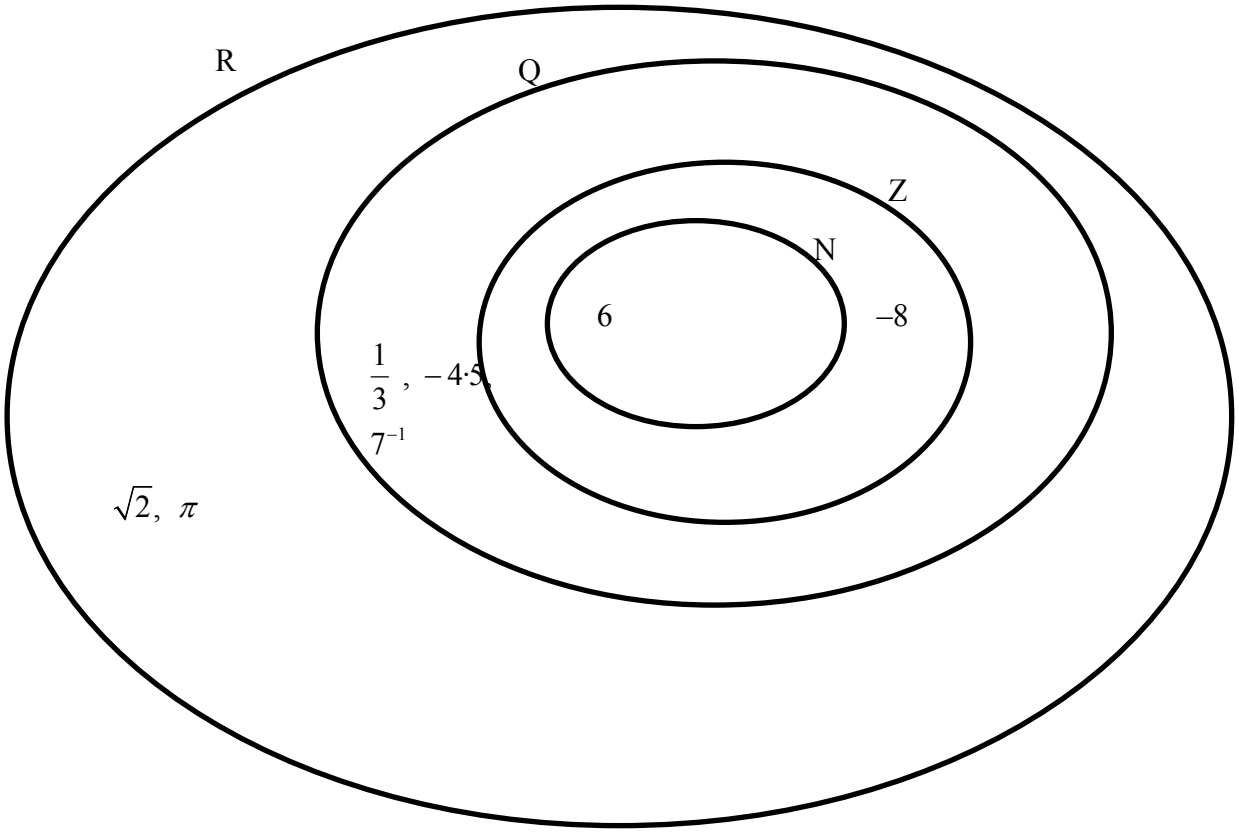
Reason 1:

–7·3 is not a positive number OR –7·3 is a minus number

Reason 2:

–7·3 is not a whole number OR –7·3 is a decimal

(b) The diagram represents the sets:
Natural Numbers N
Integers Z
Rational Numbers Q
Real Numbers R

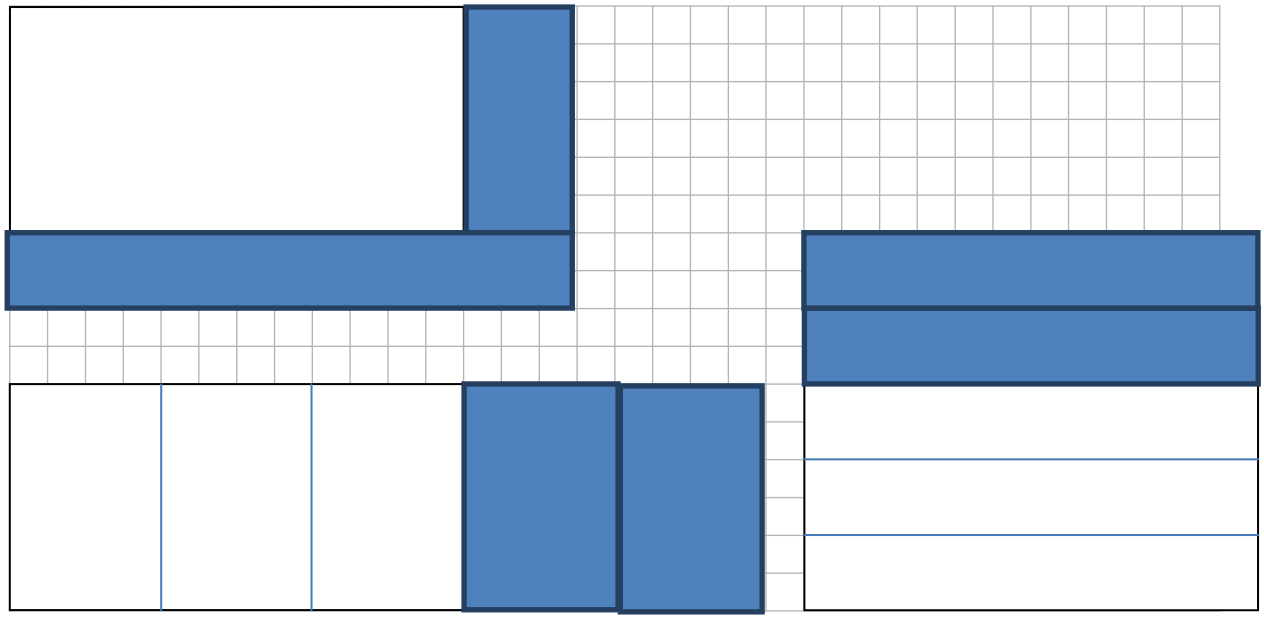


Insert each of the following numbers in the correct place on the diagram:

$-8,\ \pi,\ \frac{1}{3}, 6, \sqrt{2},\ -4{\cdot}5$ and 7^{-1}.

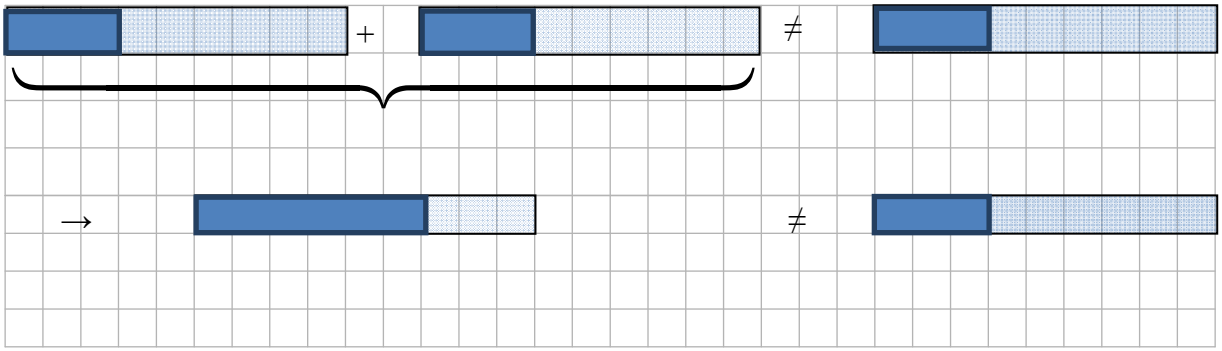
Question 2

(a) The diagram below shows three fifths of a rectangle. Complete the rectangle on the grid.



(b) By shading appropriate sections of the strips below, show that

$$\frac{1}{3}+\frac{2}{6}\neq\frac{3}{9}$$



Question 3

The value of one euro against other currencies on a particular day is shown in the table below.

Currency	**Rate (€)**
US Dollar	1·4045
Pound Sterling	0·87315
Lithuanian Litas	3·4528
Latvian Lats	0·7093
Polish Zloty	4·0440

(a) Mary was going to America for a few months. She changed €1200 into US Dollars using the exchange rate in the table.

(i) How many dollars should she receive at this exchange rate?

€1200 × 1·405 = $1685·4

(ii) The bank charged 3% commission on the transaction. How many dollars did she receive?

$1685 × 0·97 = $1634·84

(b) On returning to Ireland Mary had $3060. She changed this amount into euro. The bank again charged her 3% commission on the transaction. She received €2047.
Find the exchange rate on that day, correct to two decimal places.

$$\frac{2047}{97} \times 100 = 2110{\cdot}3$$
$$2110{\cdot}3 \times R = 3060$$
$$R = 1{\cdot}45$$

$$360 \times 0{\cdot}03 = 91{\cdot}8$$
$$3060 - 91{\cdot}8 = 2968{\cdot}2$$
$$\frac{2968{\cdot}2}{2047} = 1{\cdot}45$$

(c) David changed a certain amount of sterling into euro at the exchange rate in the table above. A few days later he again changed the same amount of sterling into euro at a different exchange rate. He received fewer euro this time. No commission was charged on these transactions. Write down one possible value for the exchange rate for the second transaction.

1 Euro = £ 0·87315
=> £1 = 1·1453 Euro

€1 = anything greater than 0·87315

£1 = anything less than €1·1453

Question 4

A soccer team has three strikers John, Paul and Michael. The number of minutes each had played by the end of a particular season is shown on the table. The team divided a bonus of €150 000 between its strikers in proportion to the time each had played.

Name	**Minutes Played**
John	2250
Paul	2600
Michael	150

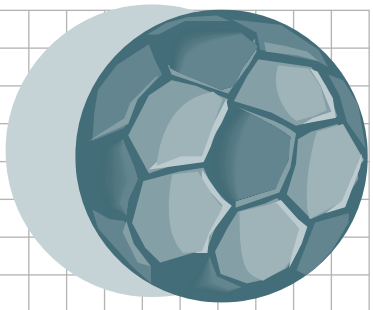
(a) Calculate the amount each player received.

$2250 + 2600 + 150 = 5000$

$\frac{150000}{5000} \times 2250 = €67\,500$

$\frac{150000}{5000} \times 600 = €78\,000$

$\frac{150000}{5000} \times 150 = €4500$



(b) At the end of the following season a larger total bonus was paid. At that time, John said: "The bonus should be paid according to the number of goals scored by the striker. Paul scored 50% more goals than Michael. I scored as many as both of them together. I would get €140 000 if the team used this method."

(i) Calculate the total bonus on offer that season.

Michael	Paul	John
1	1·5	2·5

$$\frac{140\,000}{2{\cdot}5} \times 5 = €280\,000$$

50% = 140 000
=> 100% = €280 000

(ii) How much each would Paul and Michael get under John's system?

$$\frac{280\,000}{5} = 56\,000 \text{ (one part)}$$

Michael $56\,000 \times 1 = €56\,000$
Paul $56\,000 \times 1{\cdot}5 = €84\,000$

Question 5

The USC (Universal Social Charge) is calculated on gross income. The rates of the USC are:

- 2% on the first €10 036 of gross annual income
- 4% on the next €5980
- 7% on the balance.

(a) Niamh earned €45 000 in 2011. Find her USC for that year.

$10\,036 \times 0{\cdot}02 = 200{\cdot}72$ (2% USC charge)

$5980 \times 0{\cdot}04 = 239{\cdot}2$ (4% USC charge)

$[45\,000 - (10\,036 + 5980)] \times 0{\cdot}07 = 2028{\cdot}88$ (7% USC charge)

$2028{\cdot}88 + 239{\cdot}2 + 200{\cdot}72 =$ €2468·8 Total USC Charge

The table shows a selection of the tax credits available in Ireland in 2011.

Individual's Tax Credits	**Tax Credit 2011**
Single Person	€1650
Married or Civil Partner	€3300
Widowed or Surviving Civil Partner	€2190
Home Carer	€810
PAYE	€1650
One Parent Family	€1650

(b) Niamh is a single person who is a PAYE worker. Calculate her total tax credits for 2011.

$1650 + 1650 =$ €3300

(c) The standard rate of tax is 20% and the higher rate is 41%. The Standard Rate Cut-off Point for a single person is €32 800. Calculate Niamh's tax bill for 2011.

32 800 × 0·2 = 6560 Tax at lower rate

45 000 − 32 800 = 12 200
12 200 × 0·41 = 5002 Tax at upper rate

6560 + 5002 = 11 562 Total gross Tax

11 562 − 3300 = €8262 Total Net Tax

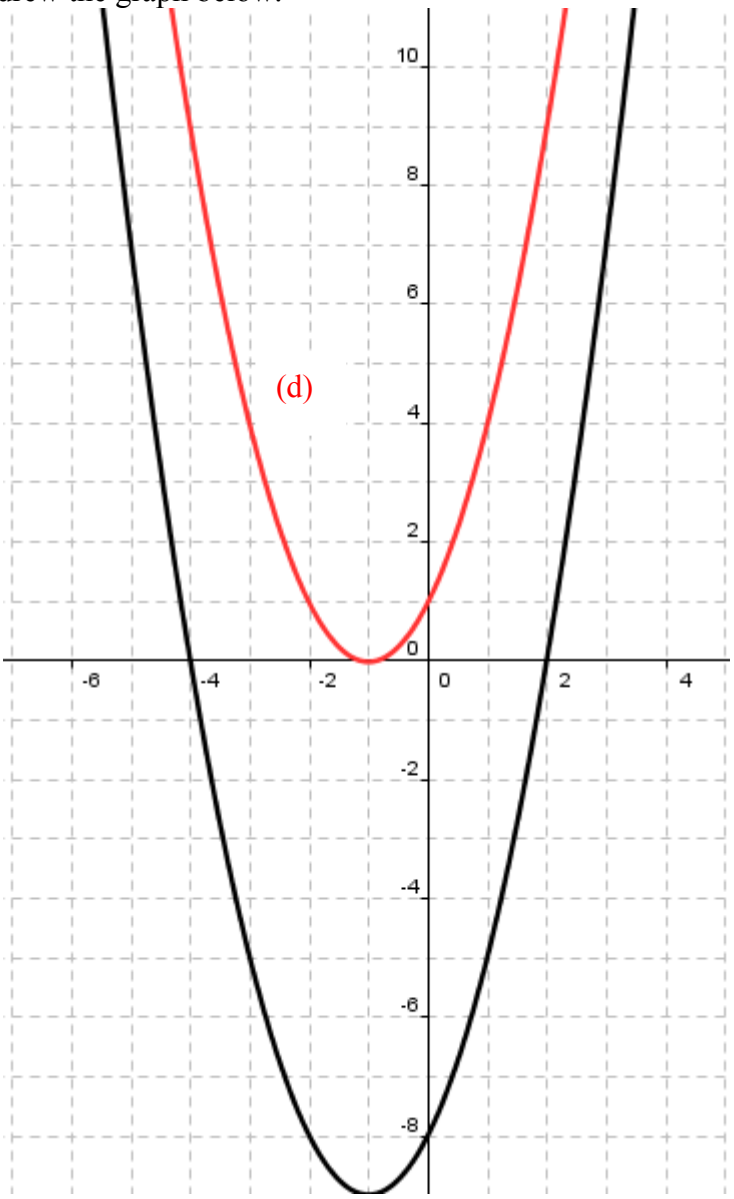
(d) Calculate Niamh's net pay for the year, after tax and USC are paid.

45 000 − (2468·8 + 8262) = €34 269·2

2468·8 + 8262 = 10 730·8
45 000 – 10 730·8 = €34 269·2

Question 6

A group of four students is studying graphs of functions of the form $f : x \mapsto x^2 + 2x + k, \quad x \in \mathbb{R}$.
Each takes an integer value of k and draws the graph of their function in a suitable domain.
Maria took $k = -8$ and drew the graph below.



(a) Use the graph to write down the roots of the equation $x^2 + 2x - 8 = 0$.

Roots 2 and -4

(b) Keith's graph passes through the point (3, 2). Find the value of k that Keith used.

$$f(x) = x^2 + 2x + k$$
$$2 = 3^2 + 2(3) + k$$
$$2 = 9 + 6 + k$$
$$k = -13$$

(c) On Alice's graph, the two roots of the function are the same.
Find the value of k that Alice used.

$$(x+t)^2 = x^2 + 2x + k$$
$$x^2 + 2tx + t^2 = x^2 + 2x + k$$
$$\Rightarrow 2t = 2$$
$$t = 1$$

(d) Draw a sketch of Alice's function on the diagram shown in part **(a)**.

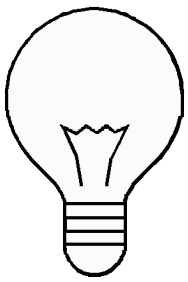
(e) Emma's graph shows that the roots of her function are – 5 and 3.
Find the value of k that she used.

$$(x+5)(x-3) = 0$$
$$x^2 + 2x - 15 = 0$$
$$\Rightarrow k = -15$$

Constant is product of roots
$$-5 \times 3 = -15$$
$$\Rightarrow k = -15$$

Question 7

Lisa is on a particular payment plan called "Plan A" for her electricity. She pays a standing charge each month even if no electricity is used. She also pays a rate per unit used. The table shows the cost, including the standing charge, of using different amounts of units, in a month.

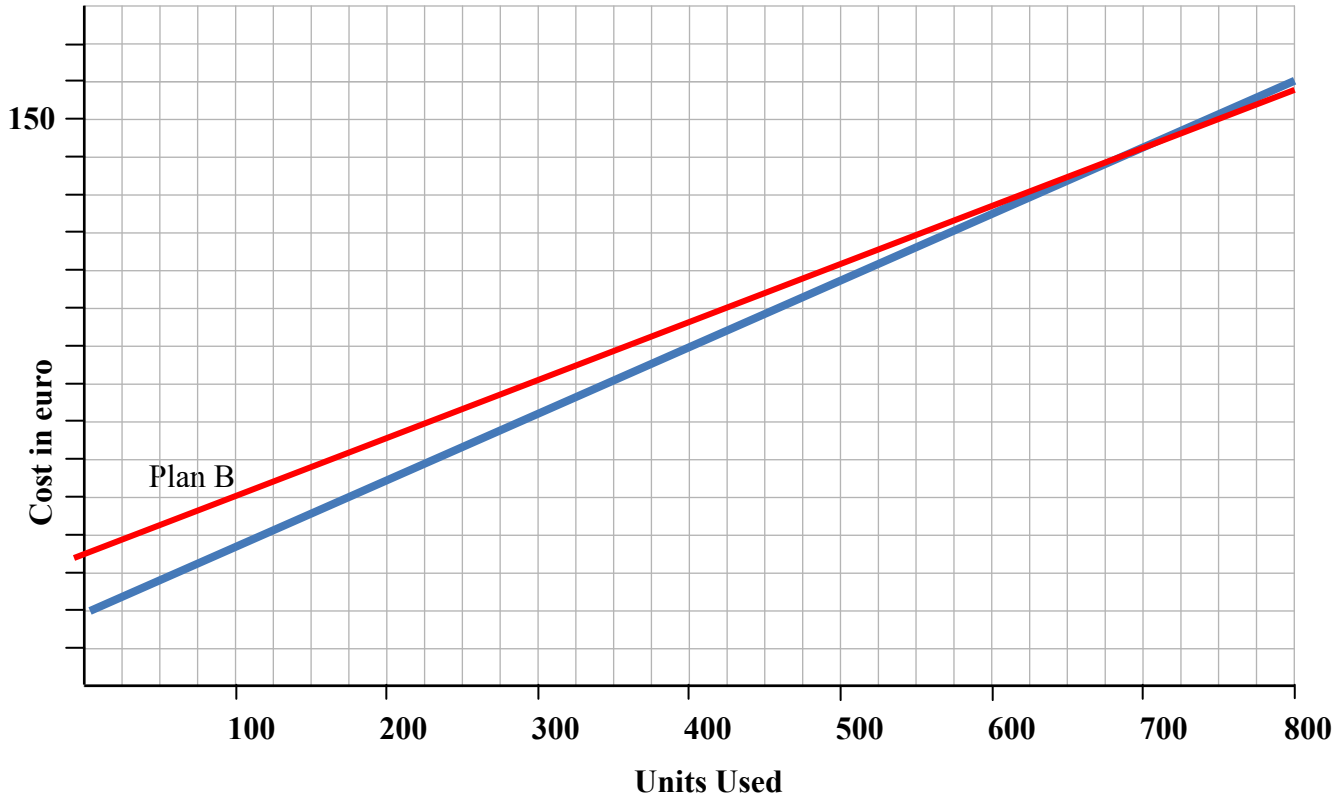


Units Used	Plan A Cost in euro
100	38
200	56
300	74
400	92
500	110
600	128
700	146
800	164

(a) Use the data in the table to show that the relationship between the number of units used and the cost is linear.

$56-38=18,\ 74-56=18, 92-74=18,$
$110-92=18,\ 128-110=18,$
$146-128=18,\ 164-146=18$
Common first difference of 18

(b) Draw a graph to show the relationship between the number of units used and the cost of electricity.



(c) Use your graph to estimate the standing charge. _____€20_____

(d) Write down a different method of finding the standing charge.
Find the standing charge using your method.

Method:

When the units used go down by 100 then the cost goes down by 18.
=> 38 – 18 = 20

$$m = \frac{56-38}{200-100} = 0{\cdot}18 \left(\text{or } \frac{9}{50} \right)$$
$$y - 38 = 0{\cdot}18(x - 100)$$
$$0{\cdot}18x - y + 20 = 0$$
$$sub\ x = 0$$
$$=> y = 20$$

Standing charge:€20

(e) Write down a formula to represent the relationship between the number of units used and the cost for any given number of units.

$$\text{Cost} = 20 + 0{\cdot}18x$$

(f) The table above does not include VAT. One month Lisa used 650 units.
Her total bill for that month, including VAT, was €155·50.
Find the VAT rate on electricity, correct to one decimal place.

$650 \times 0{\cdot}18 + 20 = 137$

$155{\cdot}5 - 137 = 18{\cdot}5$

$\frac{18{\cdot}5}{137} \times 100 = 13{\cdot}5\%$ VAT

$650 \times 18 + 2000 = 13\,700$

$15550 - 13700 = 1850$

$\frac{1850}{13700} \times 100 = 13{\cdot}5\%$ VAT

(g) Lisa is offered a new plan, "Plan B", where the standing charge is €36 and the rate per unit used is 15·5 cent. Complete the following table for Plan B.

Units Used	Plan B Cost in euro
100	€51·50
200	€67·00
300	€82·50
400	€98·00
500	€113·50
600	€129·00
700	€144·50
800	€160·00

(h) Which plan do you think Lisa should choose? Give a reason for your answer.

Scenario 1: Concentrates on 650 units

[36 + 0·155 × 650 = €136·75]
The cost of Plan A and Plan B are very similar therefore it doesn't really matter which plan Lisa chooses
OR
Lisa should choose plan B as it is 25c cheaper

Scenario 2: Concentrates on low and/or high usage

If Lisa tends to use a low number of units on average then plan A is better but if she uses a high number of units on average then Plan B is better.

(i) On your diagram for part **(b)**, draw a graph to show the relationship between the number of units used and the cost of electricity for Plan B. Label this graph "Plan B".

(j) Use your diagram to find the number of units for which both plans have the same cost.

640 units

Question 8

A capacitor is a device which stores electricity. The formula $W = \frac{1}{2}CV^2$ gives the energy stored in the capacitor, where W is the energy, C is the capacitance and V is the voltage, and standard units are used throughout.

(a) Find the amount of energy stored in a capacitor when $C = 2500$ and $V = 32$.

$$W = \frac{1}{2}CV^2$$
$$W = \frac{1}{2}(2500)(32)^2$$
$$W \ 1\,280\,000$$

(b) Write V in terms of W and C.

$$W = \frac{1}{2}CV^2$$
$$2W = CV^2$$
$$\frac{2W}{C} = V^2$$
$$\sqrt{\frac{2W}{C}} = V$$

Question 9

Consideration is being given to changing the number of points a team gets for a win and also the number of points a team gets for a draw in a soccer league. No points will be awarded for a loss. The table below shows the standing of two teams after six games under the proposed new system.

Team	Played	Won	Drawn	Lost	Points (new system)
Team A	6	2	2	2	12
Team B	6	1	5	0	10

(a) Find the number of points which would be awarded for **(i)** a win and **(ii)** a draw under this proposed system.

2w+2d=12
1w+5d=10

2w+2d=12
–2w–10d=–20

–8d=–8
8d=8
⇨ D= 1 point

Substitute d = 1
2w = 12 –2
2w = 10
W = 5 points

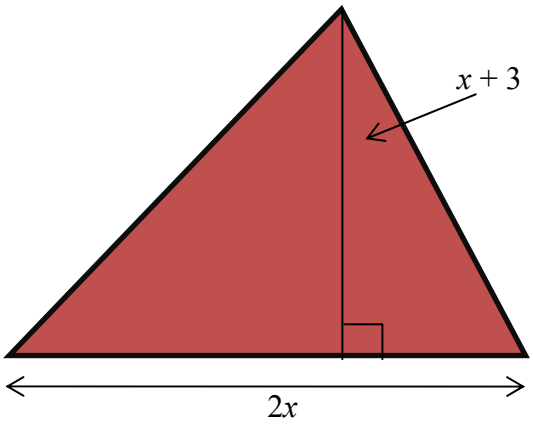
Trial and error with verification of both solutions is awarded full marks.

(b) The current system awards 3 points for a win and 1 point for a draw. Suggest one reason why it might be preferable to change to the system proposed in part **(a)**.

With new system the ratio of win:draw is higher which rewards a victory more and might encourage a team to go for a win.

Question 10

A triangle has a base length of $2x$ cm and a perpendicular height of $(x + 3)$ cm.
The area of the triangle is 10 cm^2. Find the distance x.



$\frac{1}{2}(2x)(x+3) = 10$

$x^2+3x-10 = 0$

$(x+5)(x-2) = 0$
$\Rightarrow x = -5$ (not possible) and $x = 2$cm

Question 11

Factorise fully each of the following expressions:

(i) $5x^3 - 10x^2$

$5x^2(x-2)$

(ii) $4x^2 - 81y^2$

$(2x-9y)(2x+9y)$

(iii) $a^2 - ab + 3a - 3b$

$a(a-b) + 3(a-b)$

$(a+3)(a-b)$

Question 12

(a) **Solve** each of the following equations:

(i) $x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$\Rightarrow x = 6$ or $x = -1$

$a = 1,\ b = -5,\ c = -6$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 + 24)}}{2}$$

$$x = \frac{5 \pm 7}{2}$$

$x = 6, x = -1$

(ii) $8x^2 - 14x + 3 = 0$

$(4x-1)(2x-3) = 0$

$\Rightarrow x = 1/4$ or $x = 3/2$

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(8)(3)}}{2(8)}$$

$$x = \frac{14 \pm \sqrt{196 - 96}}{2(8)}$$

$$x = \frac{1}{4}, x = \frac{3}{2}$$

(iii) $\dfrac{2x+5}{3} - \dfrac{4x-1}{2} = -\dfrac{1}{2}$

$2(2x+5) - 3(4x-1) = 3(-1)$
$4x + 10 - 12x + 3 = -3$
$8x = 16$
$x = 2$

(b) Find the roots of the equation $2x^2 - 7x - 6 = 0$.
Give your answers correct to two decimal places.

$$x = \frac{7 \pm \sqrt{49 - 4(2)(-6)}}{4}$$

$$x = \frac{7 \pm \sqrt{49 + 48}}{4}$$

$$x = \frac{7 \pm \sqrt{97}}{4}$$

$$x = \frac{7 \pm 9{\cdot}848857}{4}$$

$x = 4{\cdot}21$ or $x = -0{\cdot}71$

Question 13

For real numbers a, b and c, complete the table below. Indicate whether each statement is always true, never true or sometimes true.

Statement	Always true	Never true	Sometimes true
If $a > b$ and $b > c$, then $a > c$	√		
If $-a < 4$ and $b < -4$, then $a < b$		√	
If $a > b$, then $-a > -b$		√	
If $a > b$ and $b < c$, then $a < c$			√
If $3a + 1 > 2$, then $a > 0$	√		
If $2b - 4 < 3b - 8$, then $b > 4$	√		
If a and b are both positive and $a < b$, then $\frac{1}{a} < \frac{1}{b}$		√	

Question 14

Let g be the function $g : x \mapsto 2^{x-3}$.

(a) Find the value of $g(3)$.

$g(3) = 2^0 = 1$

(b) Let h be the function $h : x \mapsto x^2 - 3x$.

(i) Express $h(t)$ and $h(2t + 1)$ in terms of t.

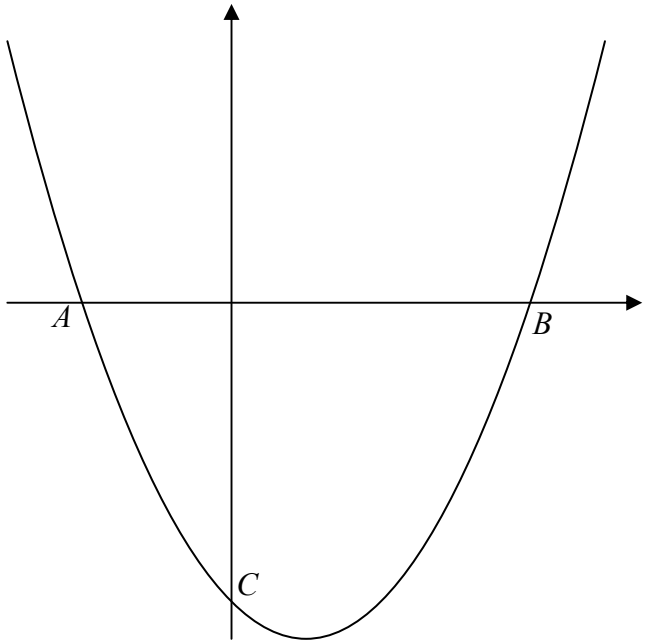
$h(t) = t^2 - 3t$

$h(2t+1) = (2t+1)^2 - 3(2t+1)$
$= 4t^2 + 4t + 1 - 6t - 3$

(ii) Hence, find the values of t for which $h(t) = h(2t + 1)$.

$$4t^2 - 2t - 2 = t^2 - 3t$$
$$3t^2 + 2t - 2 = 0$$
$$(3t - 2)(t + 1) = 0$$
$$t = \frac{2}{3}, t = -1$$

(c) The diagram shows part of the graph of the function $f : x \mapsto x^2 - 2x - 8,\ x \in \mathbb{R}$.



The graph intersects the x-axis at A and B, and the y-axis at C.

(i) Find the co-ordinates of A, B and C.

C
Sub $x = 0$
y = –8
C(0, –8)

A, B
f(x) = 0
(x–4)(x+2) = 0

=> $x = 4$ or $x = -2$

(ii) Hence, write down the range of values of x for which $x^2 - 2x - 8 \leq 0$.

$-2 \leq x \leq 4$