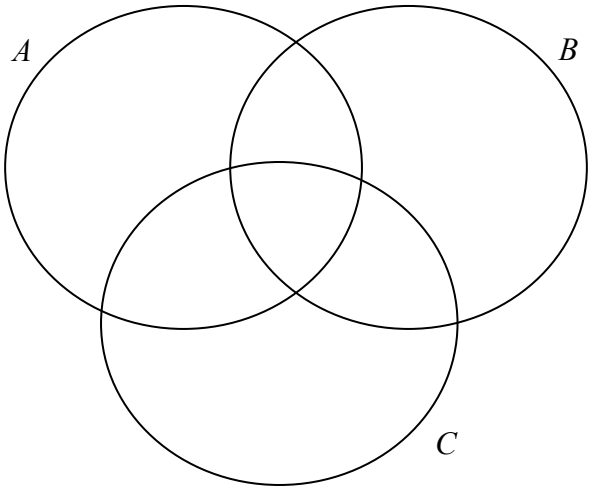
Question 1 **(Suggested maximum time: 10 minutes)**

The sets A, B, and C are as follows:

$$A = \{1, 2, 3, 5, 6, 7\} \qquad B = \{2, 3, 4, 5, 8, 9\} \qquad C = \{1, 4, 5, 10\}.$$

(a) Complete the Venn diagram below.



(b) List the elements of each of the following sets.

$A \cup B =$ _____

$A \setminus C =$ _____

$A \cup (B \cap C) =$ _____

(c) Complete the following identity.

$$A \cup (B \cap C) = (A \cup B) \cap (\_\_\_\_\_)$$

Question 2 **(Suggested maximum time: 5 minutes)**

(a) David weighs 88 kg. The average male triathlete of his height weighs 83 kg.

If David aims to reach this weight, what **percentage decrease** is required?
Give your answer correct to two decimal places.

(b) Mary's house was worth €200 000.
Mary increased the value of her house by 15% by building a conservatory.
She then increased its value by a further 10% by repaving the driveway.

Find the **total percentage** increase in value.

Question 3 **(Suggested maximum time: 10 minutes)**

Eleanor has a **gross** income of €38 500 for the year.
She has an annual tax credit of €3300.
The standard rate cut-off point is €33 800.
The standard rate of income tax is 20% and the higher rate is 40%.

(a) Find Eleanor's **net** income for the year (i.e. after tax is paid).

Eleanor receives a pay rise. As a result, her **net** income for the year is €34 780.

(b) Find Eleanor's new **gross** income for the year.

Question 4 **(Suggested maximum time: 5 minutes)**

Let $f(x) = 3x + 5$, for $x \in \mathbb{R}$.

(a) Find the value of $f(7)$.

(b) Write $f(k)$ in terms of k.

(c) Using your answer to part **(b)**, or otherwise, find the value of k for which $f(k) = k$.

Question 5 **(Suggested maximum time: 10 minutes)**

The Kelvin scale is one way of measuring temperature.
To convert a temperature from degrees Fahrenheit (F) to kelvin (K), you:

add $459{\cdot}67$ to F, then multiply your answer by 5 and divide by 9.

(a) Convert 212 degrees Fahrenheit (F) to kelvin (K).

(b) Write an algebraic formula to express K in terms of F.

$K =$

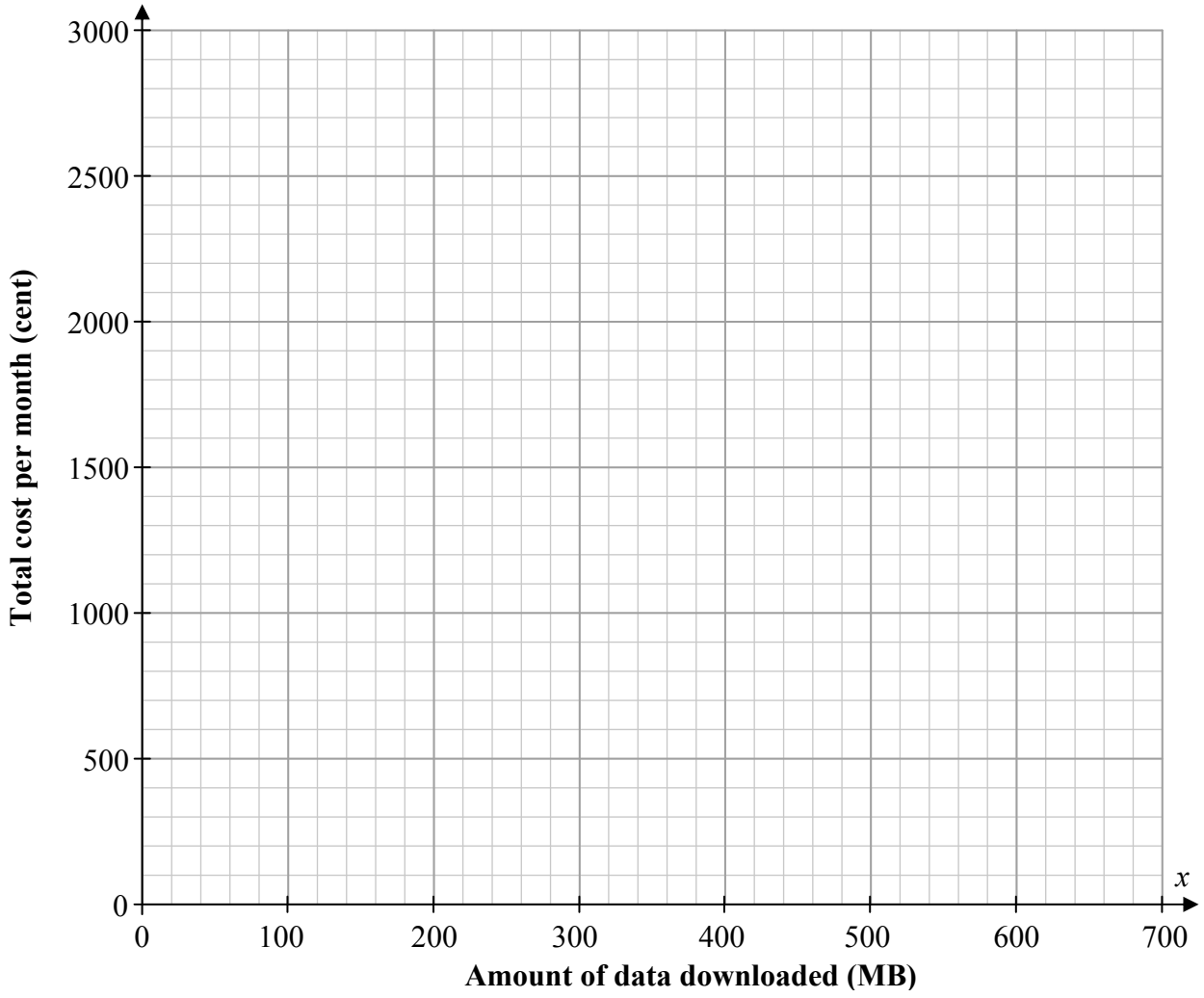
(c) Hence, or otherwise, convert 400 kelvin (K) to degrees Fahrenheit (F).

Question 6 **(Suggested maximum time: 15 minutes)**

Two mobile phone companies, *Cellulon* and *Mobil*, offer price plans for mobile internet access. A formula, in x, for the total cost per month for each company is shown in the table below. x is the number of MB of data downloaded per month.

Phone company	Total cost per month (cent)
Cellulon	$c(x) = 4x$
Mobil	$m(x) = 1000 + 2x$

(a) **Draw** the graphs of $c(x)$ and $m(x)$ on the co-ordinate grid below to show the total cost per month for each phone company, for $0 \leq x \leq 700$. **Label** each graph clearly.



(b) Which company charges **no** fixed monthly fee?
Justify your answer, with reference to the relevant **formula** or **graph**.

Answer:

Reason:

(c) Write down the **point of intersection** of the two graphs.

Fergus wants to buy a mobile phone from one of these two companies, and wants his mobile internet bill to be as low as possible.

(d) **Explain** how your answer to part **(c)** would help Fergus choose between *Cellulon* and *Mobil*.

Question 7 **(Suggested maximum time: 5 minutes)**

(a) Multiply out and simplify $(x + 5)(x^2 - 2x + 6)$.

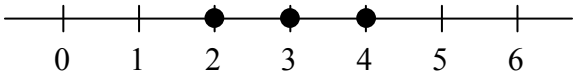
(b) Factorise fully $ac - ad - bd + bc$.

(c) Write the following as a single fraction in its simplest form.

$$\frac{x+2}{3} - \frac{x-3}{4}$$

Question 8 **(Suggested maximum time: 5 minutes)**

(a) **Complete** the inequality in n below so that it has the solution set shown.

Inequality	**Solution Set**
$\square \leq n \leq \square$, $n \in \mathbb{N}$.	

(b) **Complete** the inequality in x below so that there is only **one** possible value of x, where $x \in \mathbb{R}$.

$\square \leq x \leq \square$, $x \in \mathbb{R}$.

Question 9 **(Suggested maximum time: 10 minutes)**

(a) **(i)** Factorise $x^2 + 7x - 30$.

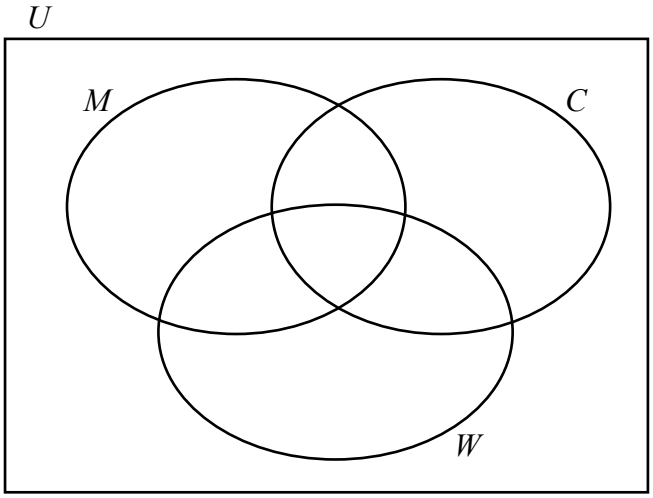
(ii) Hence, or otherwise, solve the equation $x^2 + 7x - 30 = 0$.

(b) Solve the equation $2x^2 - 7x - 10 = 0$.
Give each answer correct to two decimal places.

Question 10 **(Suggested maximum time: 10 minutes)**

A researcher has found old census data about Measles (M), Chickenpox (C), and Whooping cough (W) among 12-year-old children. In a group of 100 children:

31 had **none** of these diseases
2 had **all three** diseases
2 had Measles **and** Chickenpox, but **not** Whooping cough
6 had Whooping cough **and** Chickenpox
11 had **at least two** diseases
18 had Measles
40 had Chickenpox.



(a) Use this data to **fill in** the Venn diagram.

(b) Find the **probability** that a child chosen at random from the group had Chickenpox.

The table below shows 3 statements. Each statement is written in English and in set notation.

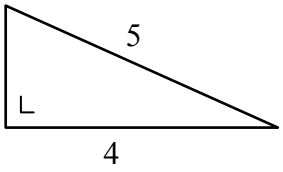
(c) Complete the table.

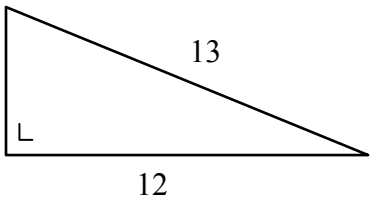
	English	**Set notation**
Statement 1	6 had Whooping cough **and** Chickenpox	$6 = \#(W \cap C)$
Statement 2		$36 = \#(C \setminus M)$
Statement 3	2 had Measles **and** Chickenpox but **not** Whooping cough	

Question 11 **(Suggested maximum time: 20 minutes)**

Two right-angled triangles are shown below.

(a) Find the height of each triangle.
Write each answer in the box below the appropriate diagram.



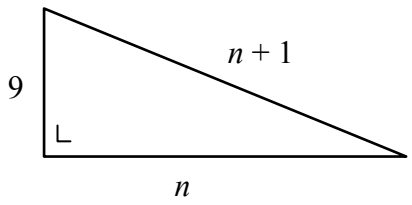


Height = ☐ Height = ☐

The triangles above are the first two triangles (with sides of integer lengths) where the hypotenuse is 1 unit longer than the base.

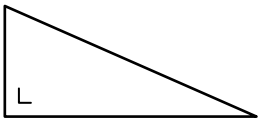
(b) Another such triangle is shown on the right.
It has a height of 9 units.

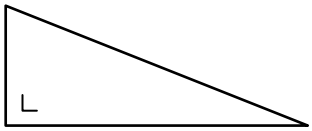
Use the Theorem of Pythagoras to find the value of n, the length of the base of this triangle.

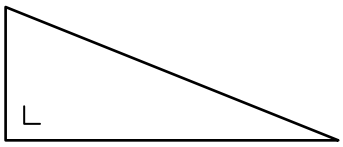


These triangles can be put in a sequence of increasing size.
The lengths of the bases of the triangles in this sequence follow a **quadratic** pattern.
Three consecutive triangles in this sequence are shown below.

(c) Use this information to find the length of the base of the next triangle in the sequence.







The length of the hypotenuse, h, of triangle x in this sequence is given by the function below, where b and c are integers.

$$h(x) = 2x^2 + bx + c$$

Also, $h(1) = 5$ and $h(2) = 13$.

(d) **(i)** Use this information to write two equations in b and c.

Equation 1:

Equation 2:

(ii) Solve these simultaneous equations to find the value of b and the value of c.

$b =$ _____ $c =$ _____

Question 12 **(Suggested maximum time: 10 minutes)**

(a) **(i)** Factorise $n^2 - 1$.

Hence, or otherwise, answer the following question.

(ii) The **product** of two **consecutive odd** positive numbers is 399. Find the two numbers.

(b) Divide $x^3 + 5x^2 - 29x - 105$ by $x + 3$.

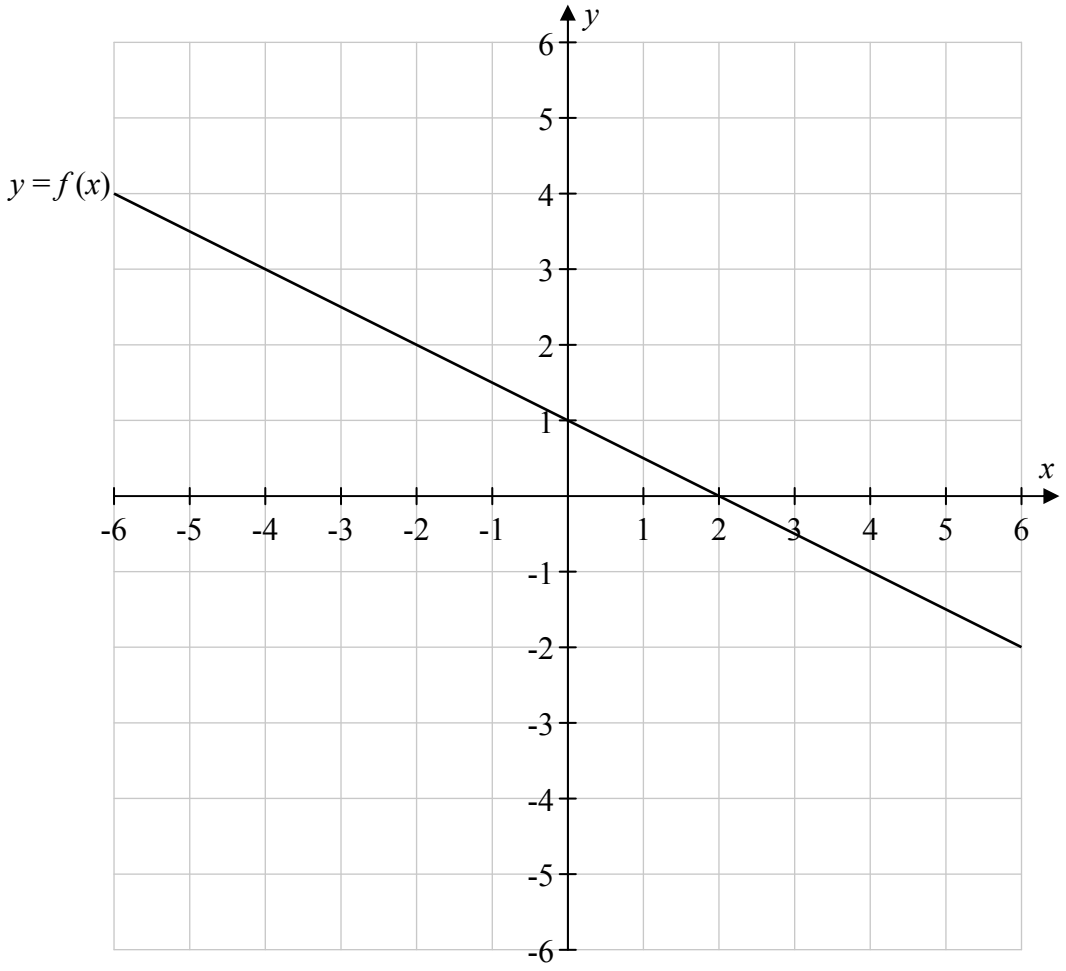
Question 13 **(Suggested maximum time: 5 minutes)**

The graph of the linear function $y = f(x)$ is drawn on the co-ordinate grid below.

Using the same axes, draw the graph of each of the following functions, where $-6 \leq x \leq 6,\ x \in \mathbb{R}$. Label each graph clearly.

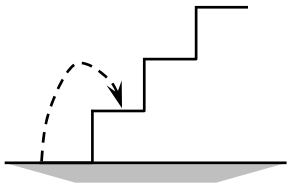
(a) $y = f(x) + 2$

(b) $y = -f(x)$

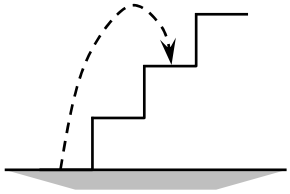


Question 14 **(Suggested maximum time: 20 minutes)**

A boxer runs up stairs as part of her training.
She can go up 1 step or 2 steps with each stride, as shown.



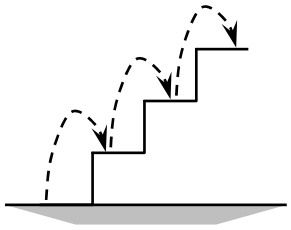
Up 1 step

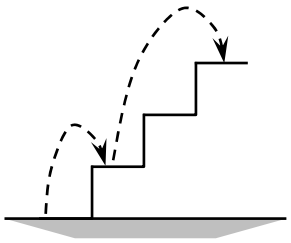


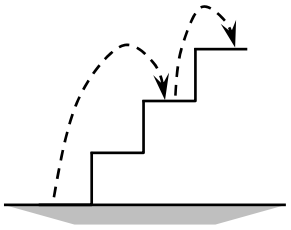
Up 2 steps

The boxer wants to count how many different ways she can reach the nth step.
She calls this T_n, the nth Taylor number.

For example, she has **3** different ways to reach the 3rd step, as shown in the tables below.
So $\boldsymbol{T_3 = 3}$.

3rd step: way 1
Up 1 step, then 1 step, then 1 step
1 + 1 + 1


3rd step: way 2
Up 1 step, then 2 steps
1 + 2


3rd step: way 3
Up 2 steps, then 1 step
2 + 1


(a) Find the value of T_1 and T_2.

T_1 = _____ T_2 = _____

(b) **List** all the different ways that she can reach the 4th step; one way is already done for you.
Hence **write down** the value of T_4.

Different ways to reach the 4th step: **1 + 1 + 1 + 1**

Answer: T_4 = _____

Some of the ways to reach the nth step start by going up **1 step**; others start by going up **2 steps**.

(c) **(i)** **List** the different ways that she can reach the 5th step, if she starts by going up **1 step**.

(ii) **List** the different ways that she can reach the 5th step, if she starts by going up **2 steps**.

(d) **Explain** why $T_{100} = T_{99} + T_{98}$.