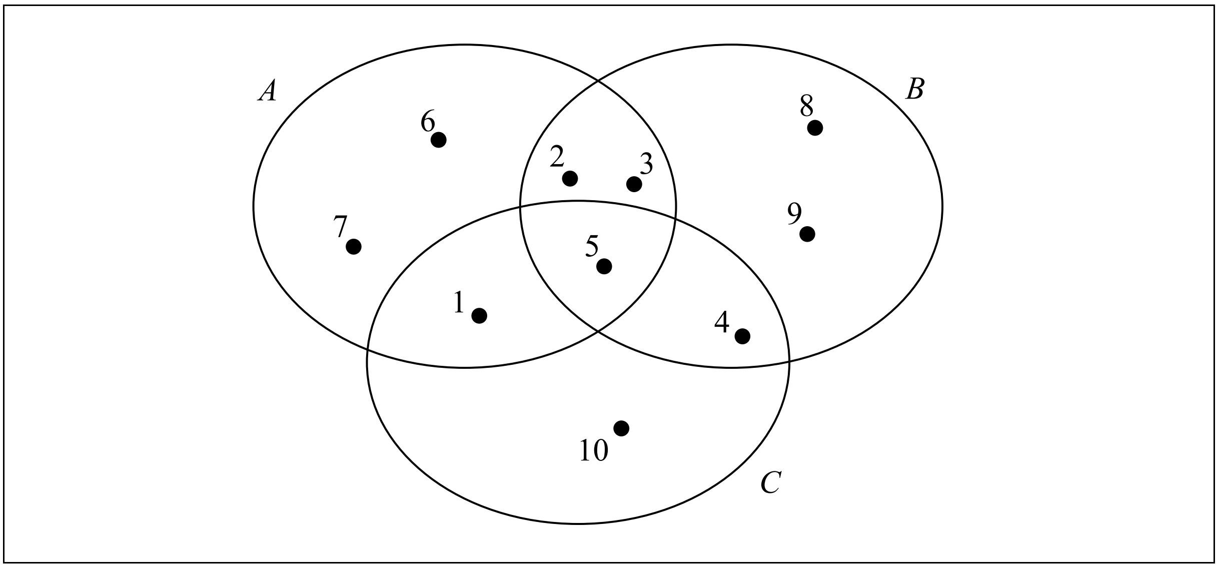
Question 1 **15 Marks**

The sets A, B, and C are as follows:

$A = \{1, 2, 3, 5, 6, 7\}$ $B = \{2, 3, 4, 5, 8, 9\}$ $C = \{1, 4, 5, 10\}$.

(a) Complete the Venn diagram below.



(b) List the elements of each of the following sets.

$A \cup B =$ $\{1, 2, 3, 5, 6, 7, 4, 8, 9\}$

$A \setminus C =$ $\{2, 3, 6, 7\}$

$A \cup (B \cap C) =$ $\{1, 2, 3, 5, 6, 7, 4\}$

(c) Complete the following identity.

$$A \cup (B \cap C) = (A \cup B) \cap \big(A \cup C \big)$$

Question 2 **15 Marks**

(a) David weighs 88 kg. The average male triathlete of his height weighs 83 kg.

If David aims to reach this weight, what **percentage decrease** is required?
Give your answer correct to two decimal places.

Decrease	$= 88 - 83 = 5$ kg
% Decrease	$= \dfrac{5}{88} \times 100 = 5{\cdot}681... = 5{\cdot}68\%$ (2 decimal places)

(b) Mary's house was worth €200 000.
Mary increased the value of her house by 15% by building a conservatory.
She then increased its value by a further 10% by repaving the driveway.

Find the **total percentage** increase in value.

	€200 000 add 15%	=	€230 000		
	€230 000 add 10%	=	€253 000		
⇒	Total Increase	=	€53 000		
⇒	Percentage Increase	=	$\frac{53\ 000}{200\ 000} \times 100$	=	26·5%
			OR		
	100% add 15%	=	1·15		
	115% add 10%	=	$1{\cdot}10 \times 1{\cdot}15$	=	1·265
⇒	Percentage Increase	=	26·5%		
			OR		
	10% of 15% = 1·5%				
	15% + 10% + 1·5%	=	26·5%		

Question 3 **25 Marks**

Eleanor has a **gross** income of €38 500 for the year.
She has an annual tax credit of €3300.
The standard rate cut-off point is €33 800.
The standard rate of income tax is 20% and the higher rate is 40%.

(a) Find Eleanor's **net** income for the year (i.e. after tax is paid).

	€33 800 at 20%	=	€6760		
	€4 700 at 40%	=	€1880		
$\Rightarrow$	Gross Tax	=	6760 + 1880	=	€8640
	Tax Credit	=	€3300		
$\Rightarrow$	Net Tax	=	8640 – 3300	=	€5340
$\Rightarrow$	Net Income	=	38 500 – 5340		
		=	€33 160.		

Eleanor receives a pay rise. As a result, her **net** income for the year is €34 780.

(b) Find Eleanor's new **gross** income for the year.

I

$\Rightarrow$ Increase in net income $= 34\,780 - 33\,160$

$= €1620 = 60\%$ of increase in gross income

$\Rightarrow$ 1% of increase in gross income $= \dfrac{1620}{60} = €27$

$\Rightarrow$ 100% of increase in gross income $= €27 \times 100 = €2700$

$\Rightarrow$ New gross income $= 38\,500 + 2700$

$= €41\,200.$

OR

II

New gross income $= 38\,500 + x$

$\Rightarrow$ New net tax $= 5340 + 0{\cdot}4\,x$

$\Rightarrow$ New net income $= (38\,500 + x) - (5340 + 0{\cdot}4\,x) = 33\,160 + 0{\cdot}6\,x$

$\Rightarrow$ $33\,160 + 0{\cdot}6\,x = 34\,780$

$\Rightarrow$ $0{\cdot}6\,x = 1620$ so $x = \dfrac{1620}{0{\cdot}6} = 2700$

$\Rightarrow$ New gross income $= 38\,500 + 2700 = €41\,200$

OR			
III	€34780 − €3300	=	€31 480
	Net income at standard rate	=	€33 800 × 0·8 = €27 040
	Net income at higher rate	=	€31 480 − €27 040 = €4440
	60% of Gross income at higher rate	=	€4440
	Gross income at higher rate	=	€4440 ÷ 60 × 100
	Total Gross Income	=	€33 800 + €7400 = €41 200

Question 4 **10 Marks**

Let $f(x) = 3x + 5$, for $x \in \mathbb{R}$.

(a) Find the value of $f(7)$.

$$f(7) = 3(7) + 5 = 26.$$

(b) Write $f(k)$ in terms of k.

$$f(k) = 3k + 5$$

(c) Using your answer to part **(b)**, or otherwise, find the value of k for which $f(k) = k$.

$$3k + 5 = k$$

$$\Rightarrow \quad 2k = -5$$

$$\Rightarrow \quad k = -\frac{5}{2}$$

Question 5 **15 Marks**

The Kelvin scale is one way of measuring temperature.
To convert a temperature from degrees Fahrenheit (F) to kelvin (K), you:

add 459·67 to F, then multiply your answer by 5 and divide by 9.

(a) Convert 212 degrees Fahrenheit (F) to kelvin (K).

$$(212 + 459{\cdot}67) \times 5 \div 9 = 373{\cdot}15.$$

(b) Write an algebraic formula to express K in terms of F.

$$K = \frac{(F + 459{\cdot}67)\times 5}{9}$$

(c) Hence, or otherwise, convert 400 kelvin (K) to degrees Fahrenheit (F).

$$400 = \frac{(F + 459{\cdot}67)\times 5}{9}$$

$$\Rightarrow \quad 3600 = (F + 459{\cdot}67) \times 5$$

$$\Rightarrow \quad 720 = F + 459{\cdot}67$$

$$\Rightarrow \quad F = 720 - 459{\cdot}67$$

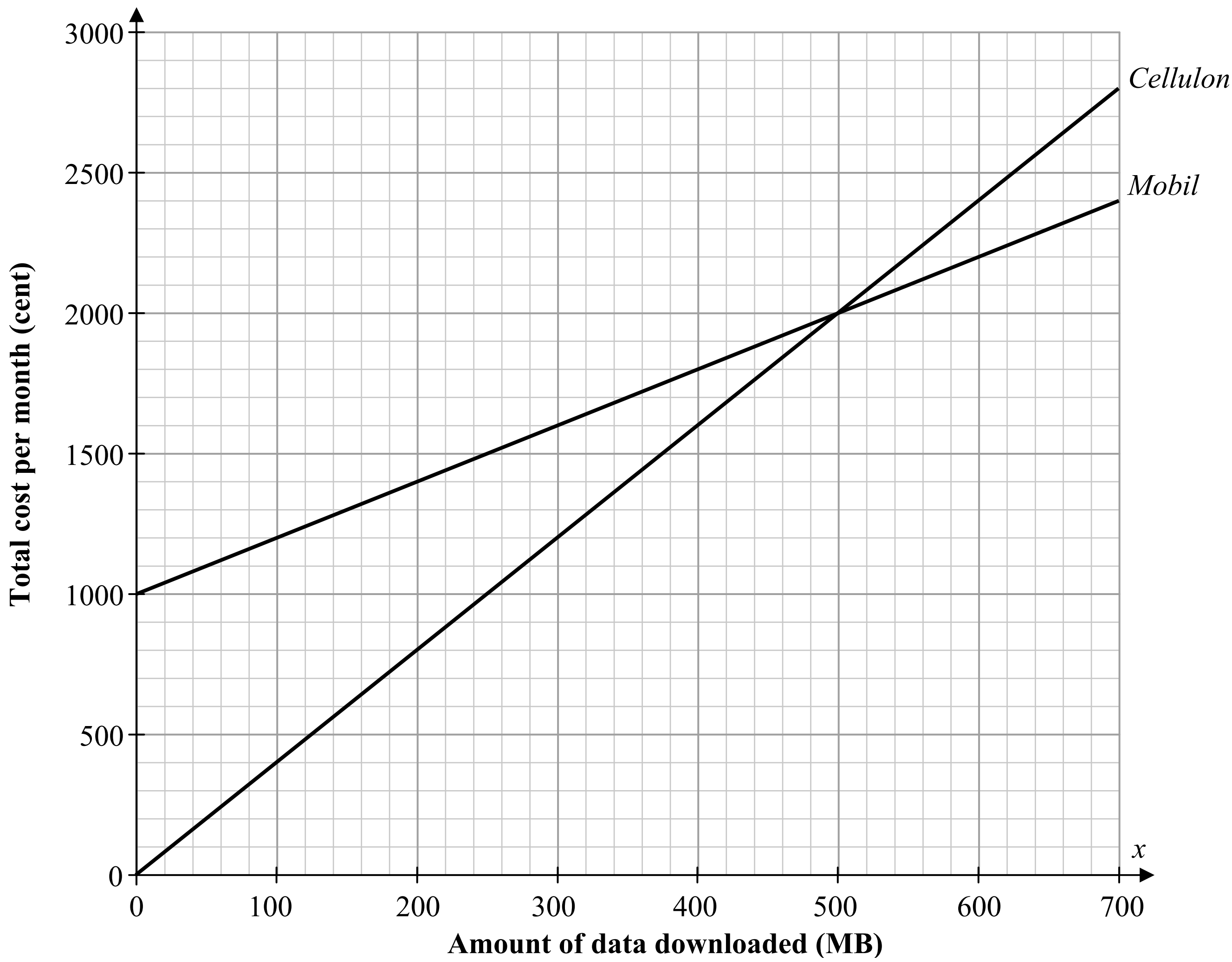
$$= 260{\cdot}33.$$

Question 6 **30 Marks**

Two mobile phone companies, *Cellulon* and *Mobil*, offer price plans for mobile internet access. A formula, in x, for the total cost per month for each company is shown in the table below. x is the number of MB of data downloaded per month.

Phone company	Total cost per month (cent)
Cellulon	$c(x) = 4x$
Mobil	$m(x) = 1000 + 2x$

(a) **Draw** the graphs of $c(x)$ and $m(x)$ on the co-ordinate grid below to show the total cost per month for each phone company, for $0 \leq x \leq 700$. **Label** each graph clearly.



$c(0) = 0$;	$c(700) = 2800$.
$m(0) = 1000$;	$m(700) = 2400$.

(b) Which company charges **no** fixed monthly fee?
Justify your answer, with reference to the relevant **formula** or **graph**.

Answer:	*Cellulon*
Justification:	$c\,(0) = 0$, so no cost if no data used. **(formula)**
	OR
	The line goes through (0, 0), so no cost if no data used. **(graph)**

(c) Write down the **point of intersection** of the two graphs.

(500, 2000)

Fergus wants to buy a mobile phone from one of these two companies, and wants his mobile internet bill to be as low as possible.

(d) **Explain** how your answer to part **(c)** would help Fergus choose between *Cellulon* and *Mobil*.

If the data is less than 500 MB per month, *Cellulon* is cheaper.

If the data is more than 500 MB per month, *Mobil* is cheaper.

Question 7 **20 Marks**

(a) Multiply out and simplify $(x + 5)(x^2 - 2x + 6)$.

$$(x+5)(x^2-2x+6) = x^3 - 2x^2 + 6x + 5x^2 - 10x + 30$$
$$= x^3 + 3x^2 - 4x + 30.$$

OR

	x^2	$-2x$	$+6$
x	x^3	$-2x^2$	$+6x$
$+5$	$+5x^2$	$-10x$	$+30$

$$= x^3 + 3x^2 - 4x + 30$$

(b) Factorise fully $ac - ad - bd + bc$.

$$ac - ad - bd + bc = a(c-d) + b(c-d)$$
$$= (c-d)(a+b).$$

OR

$$ac + bc - ad - bd = c(a+b) - d(a+b)$$
$$= (c-d)(a+b).$$

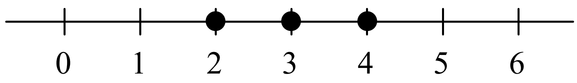
(c) Write the following as a single fraction in its simplest form.

$$\frac{x+2}{3} - \frac{x-3}{4}$$

$$\frac{x+2}{3} - \frac{x-3}{4} = \frac{4(x+2)-3(x-3)}{12}$$
$$= \frac{4x+8-3x+9}{12}$$
$$= \frac{x+17}{12}.$$

Question 8 **15 Marks**

(a) **Complete** the inequality in n below so that it has the solution set shown.

Inequality	**Solution Set**
$\boxed{2} \leq n \leq \boxed{4 \cdot 7}$, $n \in \mathbb{N}$.	

(b) **Complete** the inequality in x below so that there is only **one** possible value of x, where $x \in \mathbb{R}$.

$$\boxed{17 \cdot 3} \leq x \leq \boxed{17 \cdot 3}, \quad x \in \mathbb{R}.$$

Question 9 **20 Marks**

(a) **(i)** Factorise $x^2 + 7x - 30$.

$$x^2 + 7x - 30 = (x + 10)(x - 3).$$

OR

$$\begin{aligned} x^2 + 7x - 30 &= x^2 + 10x - 3x - 30 \\ &= x(x + 10) - 3(x + 10) \\ &= (x + 10)(x - 3). \end{aligned}$$

(ii) Hence, or otherwise, solve the equation $x^2 + 7x - 30 = 0$.

$(x + 10)(x - 3) = 0$

$\Rightarrow$ $x + 10 = 0$ or $x - 3 = 0$

$\Rightarrow$ $x = -10$ or $x = 3$.

OR

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-30)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{169}}{2}$$

$= -10$ or 3.

(b) Solve the equation $2x^2 - 7x - 10 = 0$.
Give each answer correct to two decimal places.

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-10)}}{2 \times 2}$$

$$= \frac{7 \pm \sqrt{129}}{4}$$

$= 4{\cdot}589\ldots$ or $-1{\cdot}089\ldots$

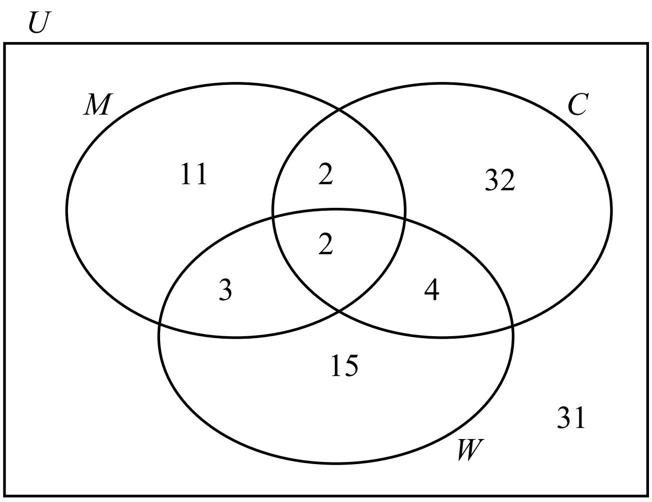
$= 4{\cdot}59$ or $-1{\cdot}09$ (2 decimal places)

Question 10 **25 Marks**

A researcher has found old census data about Measles (M), Chickenpox (C), and Whooping cough (W) among 12-year-old children. In a group of 100 children:

31 had **none** of these diseases
2 had **all three** diseases
2 had Measles **and** Chickenpox, but **not** Whooping cough
6 had Whooping cough **and** Chickenpox
11 had **at least two** diseases
18 had Measles
40 had Chickenpox.

(a) Use this data to **fill in** the Venn diagram.



(b) Find the **probability** that a child chosen at random from the group had Chickenpox.

$P(\text{C}) = \frac{40}{100}$ or $\frac{2}{5}$.

The table below shows 3 statements. Each statement is written in English and in set notation.

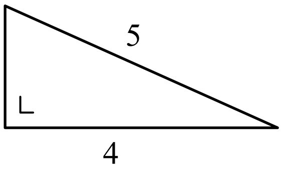
(c) Complete the table.

	English	Set notation
Statement 1	6 had Whooping cough **and** Chickenpox	$6 = \#(W \cap C)$
Statement 2	36 had Chickenpox but **not** Measles	$36 = \#(C \setminus M)$
Statement 3	2 had Measles **and** Chickenpox but **not** Whooping cough	$2 = \#[\,(M \cap C) \setminus W\,]$ or $2 = \#[\,M \cap (C \setminus W)\,]$

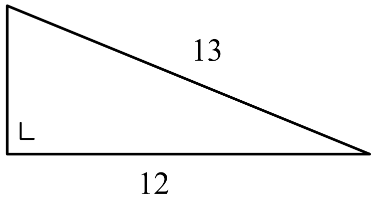
Question 11 **40 Marks**

Two right-angled triangles are shown below.

(a) Find the height of each triangle.
Write each answer in the box below the appropriate diagram.



Height = $\boxed{3}$



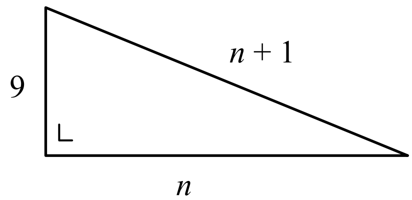
Height = $\boxed{5}$

$x^2 + 4^2 = 5^2 \Rightarrow x^2 + 16 = 25$ $\Rightarrow x^2 = 9$ $\Rightarrow x = 3.$	$y^2 + 12^2 = 13^2 \Rightarrow y^2 + 144 = 169$ $\Rightarrow y^2 = 25$ $\Rightarrow y = 5.$

The triangles above are the first two triangles (with sides of integer lengths) where the hypotenuse is 1 unit longer than the base.

(b) Another such triangle is shown on the right.
It has a height of 9 units.

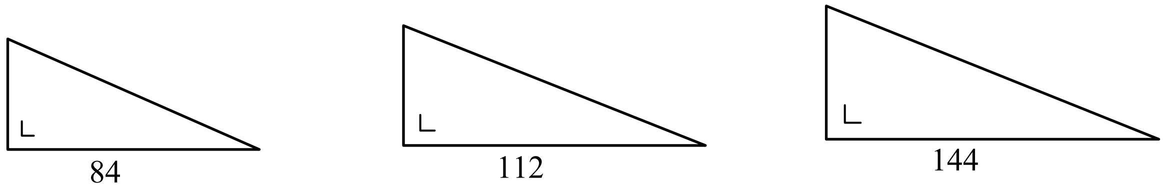
Use the Theorem of Pythagoras to find the value of n, the length of the base of this triangle.



$$9^2 + n^2 = (n+1)^2$$
$$\Rightarrow 81 + n^2 = n^2 + 2n + 1$$
$$\Rightarrow 2n = 80$$
$$\Rightarrow n = 40.$$

These triangles can be put in a sequence of increasing size.
The lengths of the bases of the triangles in this sequence follow a **quadratic** pattern.
Three consecutive triangles in this sequence are shown below.

(c) Use this information to find the length of the base of the next triangle in the sequence.



1st difference	$112 - 84 = 28$	$144 - 112 = 32$
2nd difference	$32 - 28 = 4$	
$\Rightarrow$ next 1st difference	$= 32 + 4 = 36.$	
$\Rightarrow$ next base	$= 144 + 36 = 180.$	

The length of the hypotenuse, h, of triangle x in this sequence is given by the function below, where b and c are integers.

$$h(x) = 2x^2 + bx + c$$

Also, $h(1) = 5$ and $h(2) = 13$.

(d) **(i)** Use this information to write two equations in b and c.

Equation 1:	Equation 2:
$h(1) = 2(1)^2 + b(1) + c = 5$	$h(2) = 2(2)^2 + b(2) + c = 13$
$\Rightarrow \quad 2 + b + c = 5$	$\Rightarrow \quad 8 + 2b + c = 13$
$\Rightarrow \quad b + c = 3$	$\Rightarrow \quad 2b + c = 5$

(ii) Solve these simultaneous equations to find the value of b and the value of c.

Equation $1 \times (-1)$:	$-b - c = -3$	
Equation 2 :	$\underline{2b + c = 5}$.	
$\Rightarrow$	$b = 2$	$\Rightarrow \quad c = 1.$
OR		
Equation 1 $\Rightarrow$	$c = 3 - b$	
Equation 2 $\Rightarrow$	$2b + 3 - b = 5$	
$\Rightarrow$	$b = 2$	$\Rightarrow \quad c = 1.$

Question 12 **20 Marks**

(a) **(i)** Factorise $n^2 - 1$.

$$n^2 - 1 = (n - 1)(n + 1)$$

Hence, or otherwise, answer the following question.

(ii) The **product** of two **consecutive odd** positive numbers is 399. Find the two numbers.

$(n - 1)(n + 1) = 399$

$\Rightarrow \quad n^2 - 1 = 399$

$\Rightarrow \quad n^2 = 400 \quad \Rightarrow \quad n = 20$

$\Rightarrow$ Two numbers are 19 and 21.

OR

$\sqrt{399} = 19{\cdot}97...$

$\Rightarrow$ Two numbers are 19 and 21.

(b) Divide $x^3 + 5x^2 - 29x - 105$ by $x + 3$.

$$\begin{array}{r}
x^2 + 2x - 35 \\
x + 3 \overline{\big)\, x^3 + 5x^2 - 29x - 105} \\
\underline{x^3 + 3x^2} \qquad\qquad\quad \\
2x^2 - 29x - 105 \\
\underline{2x^2 + 6x} \qquad\quad\; \\
-35x - 105 \\
\underline{-35x - 105} \\
0
\end{array}$$

Answer: $x^2 + 2x - 35$

OR

	x^2	$2x$	-35
x	x^3	$2x^2$	$-35x$
3	$3x^2$	$6x$	-105

Answer: $x^2 + 2x - 35$

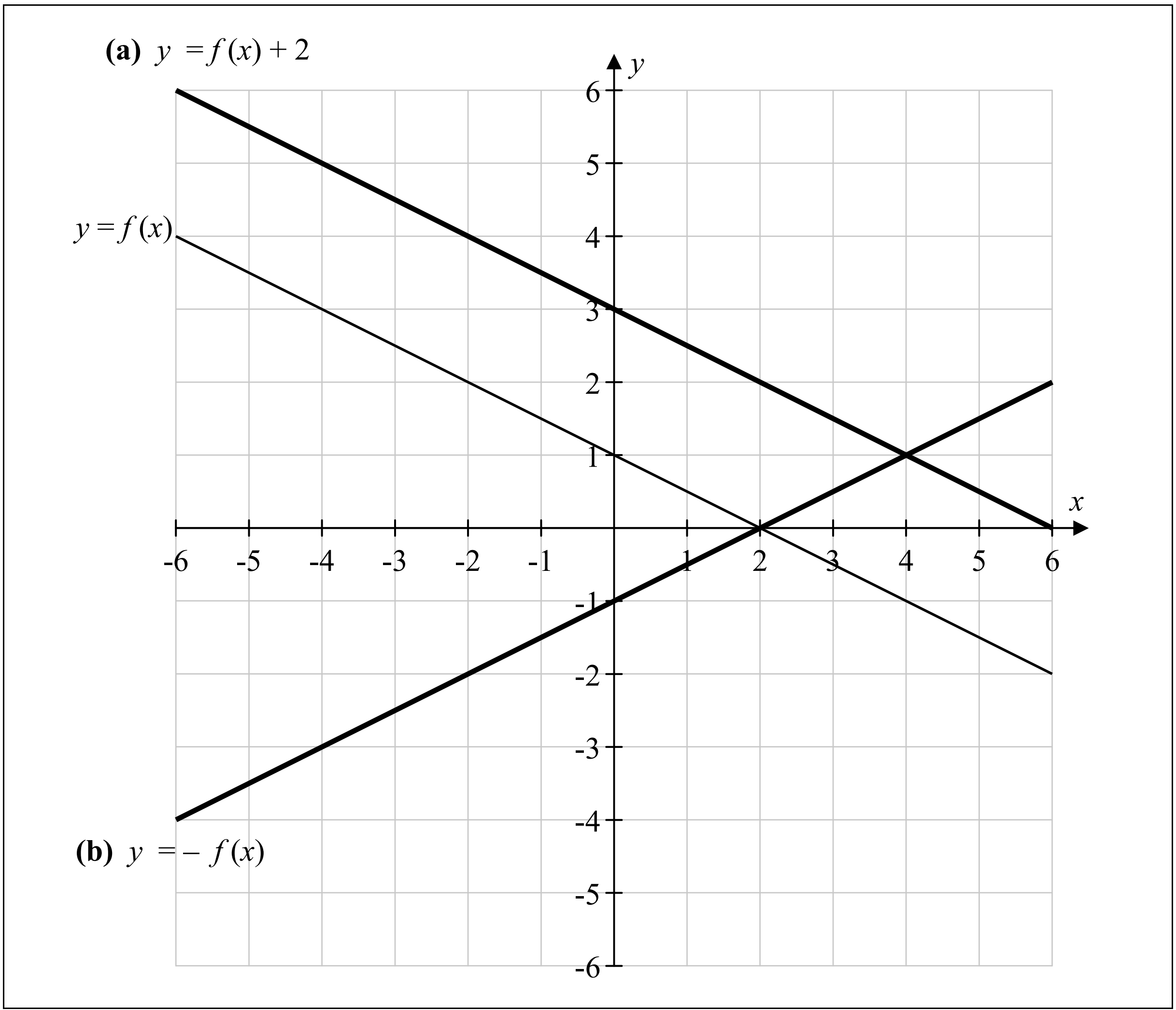
Question 13 **20 Marks**

The graph of the linear function $y = f(x)$ is drawn on the co-ordinate grid below.

Using the same axes, draw the graph of each of the following functions, where $-6 \leq x \leq 6$, $x \in \mathbb{R}$. Label each graph clearly.

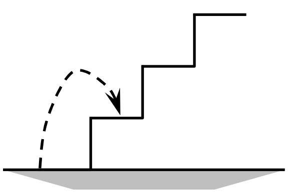
(a) $y = f(x) + 2$

(b) $y = -f(x)$

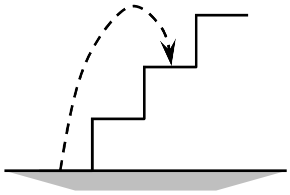


Question 14 **30 Marks**

A boxer runs up stairs as part of her training.
She can go up 1 step or 2 steps with each stride, as shown.



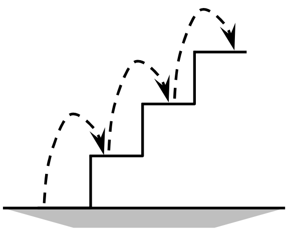
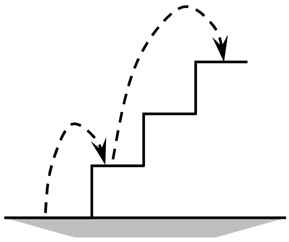
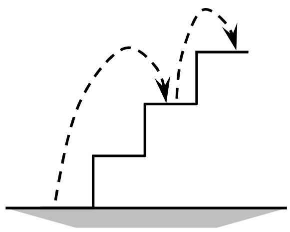
Up 1 step



Up 2 steps

The boxer wants to count how many different ways she can reach the nth step.
She calls this T_n, the nth Taylor number.

For example, she has **3** different ways to reach the 3rd step, as shown in the tables below.
So $\boldsymbol{T_3 = 3}$.

3rd step: way 1	**3rd step: way 2**	**3rd step: way 3**
Up 1 step, then 1 step, then 1 step	Up 1 step, then 2 steps	Up 2 steps, then 1 step
1 + 1 + 1	1 + 2	2 + 1
		

(a) Find the value of T_1 and T_2.

$T_1 = 1$ [way] [1 step]	$T_2 = 2$ [ways] [1 step + 1 step or 2 steps]

(b) **List** all the different ways that she can reach the 4th step; one way is already done for you.
Hence **write down** the value of T_4.

Different ways to reach the 4th step: **1 + 1 + 1 + 1**

1 + 1 + 2

1 + 2 + 1

2 + 1 + 1

2 + 2 [steps]

$\Rightarrow$ $T_4 = 5.$ [ways]

Some of the ways to reach the nth step start by going up **1 step**; others start by going up **2 steps**.

(c) **(i)** **List** the different ways that she can reach the 5th step, if she starts by going up **1 step**.

1 + 1 + 1 + 1 + 1
1 + 1 + 1 + 2
1 + 1 + 2 + 1
1 + 2 + 1 + 1
1 + 2 + 2 [steps]

[5 ways]

(ii) **List** the different ways that she can reach the 5th step, if she starts by going up **2 steps**.

2 + 1 + 1 + 1
2 + 1 + 2
2 + 2 + 1 [steps]

[3 ways]

(d) **Explain** why $T_{100} = T_{99} + T_{98}$.

To get to the 100th step, you must start by going up either 1 step or 2 steps.

If you start by going up 1 step, there are T_{99} ways to finish.

If you start by going up 2 steps, there are T_{98} ways to finish.