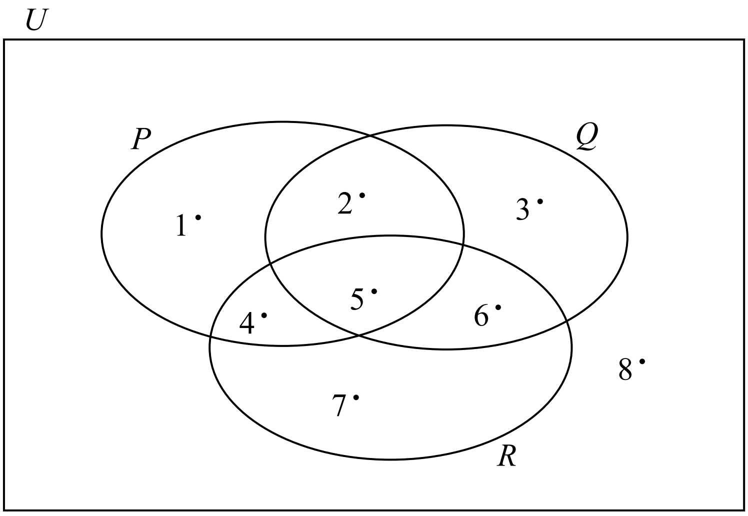
Question 1 **(Suggested maximum time: 5 minutes)**

The sets U, P, Q, and R are shown in the Venn diagram below.



(a) Use the Venn diagram to list the elements of:

$P \cup Q$ ____________________________

$Q \cap R$ ____________________________

$P \cup (Q \cap R)$ ____________________________

(b) Miriam says: "For all sets, union is distributive over intersection."

Name a set that you would use along with $P \cup (Q \cap R)$ to show that Miriam's claim is true for the sets P, Q, and R in the Venn diagram above.

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Question 2 **(Suggested maximum time: 5 minutes)**

The sets U, A, and B are defined as follows, where U is the universal set:

$U = \{2,\ 3,\ 4,\ 5,\ \ldots,\ 30\}$
$A = \{\text{multiples of } 2\}$
$B = \{\text{multiples of } 3\}$
$C = \{\text{multiples of } 5\}$.

(a) Find $\#\left[\left(A \cup B \cup C\right)'\right]$, the number of elements in the complement of the set $A \cup B \cup C$.

(b) How many divisors does each of the numbers in $\left(A \cup B \cup C\right)'$ have?

(c) What name is given to numbers that have exactly this many divisors?

Question 3 **(Suggested maximum time: 10 minutes)**

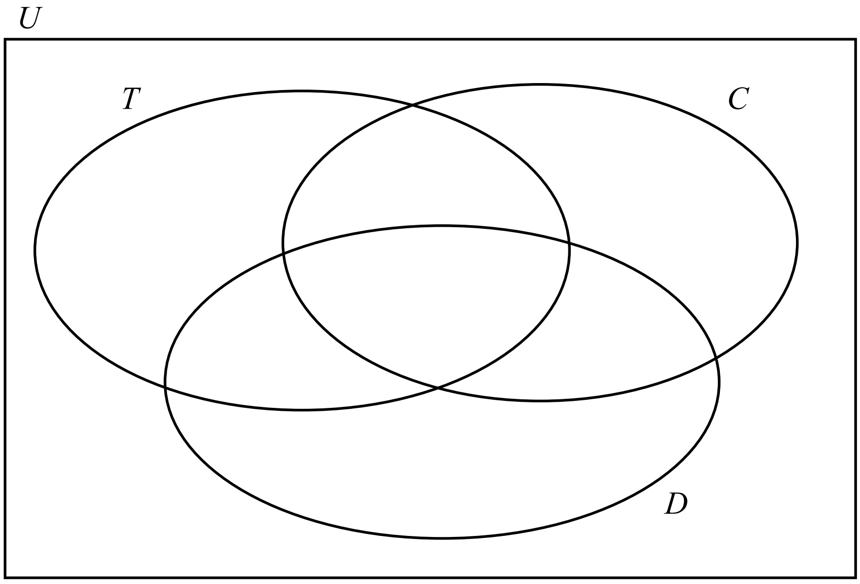
A group of 100 students were surveyed to find out whether they drank tea (T), coffee (C), or a soft drink (D) at any time in the previous week. These are the results:

24 had not drunk any of the three
51 drank tea or coffee, but not a soft drink
41 drank tea

8 drank tea and a soft drink, but not coffee
9 drank a soft drink and coffee

20 drank at least two of the three
4 drank all three.

(a) Represent the above information on the Venn diagram.



(b) Find the probability that a student chosen at random from the group had drunk tea **or** coffee.

(c) Find the probability that a student chosen at random from the group had drunk tea **and** coffee but **not** a soft drink.

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Question 4 **(Suggested maximum time: 10 minutes)**

Dermot has €5000 and would like to invest it for two years. A special savings account is offering a rate of 3% for the first year and a higher rate for the second year, if the money is retained in the account. Tax of 41% will be deducted each year from the interest earned.

(a) How much will the investment be worth at the end of one year, after tax is deducted?

(b) Dermot calculates that, after tax has been deducted, his investment will be worth €5223·60 at the end of the second year. Calculate the rate of interest for the second year.
Give your answer as a percentage, correct to one decimal place.

Question 5 **(Suggested maximum time: 10 minutes)**

A meal in a restaurant cost Jerry €30·52. The price included VAT at 9%.
Jerry wanted to know the price of the meal before the VAT was included.
He calculated 9% of €30·52 and subtracted it from the cost of the meal.

(a) Explain why Jerry will not get the correct answer using this method.

(b) Suppose that the rate of VAT was 13·5% instead of 9%.
How much would Jerry have paid for the meal in that case?

Question 6 **(Suggested maximum time: 5 minutes)**

Niamh is in a clothes shop and has a voucher which she **must** use.
The voucher gives a €10 reduction when buying goods to the value of at least €35.
She also has €50 cash.

(a) Write down an inequality in x to show the range of cash that she could spend in the shop.

$\boxed{} \le x \le \boxed{}$

(b) Niamh buys one item of clothing in the shop, using the voucher as she does so. Write an inequality in y to show the range of possible prices that this item could have, before the €10 reduction is applied.

$\boxed{} \le y \le \boxed{}$

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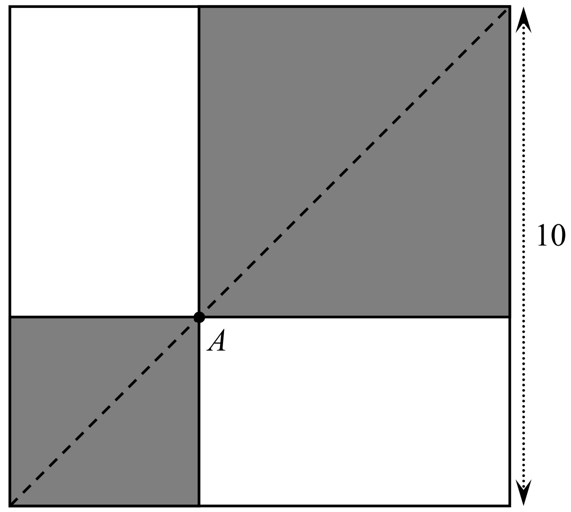
Question 7 **(Suggested maximum time: 15 minutes)**

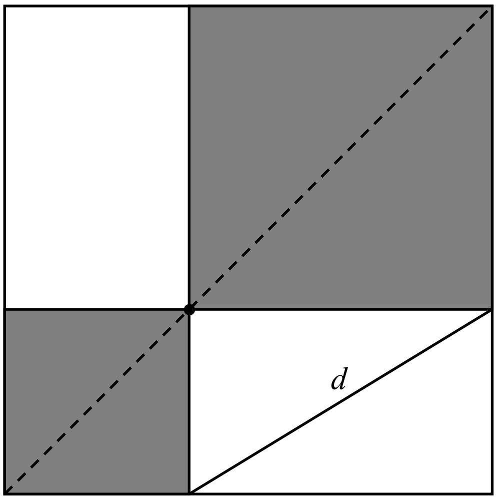
A square with sides of length 10 units is shown in the diagram. A point A is chosen on a diagonal of the square, and two shaded squares are constructed as shown. By choosing different positions for A, it is possible to change the value of the total area of the two shaded squares.



(a) Find the **minimum** possible value of the total area of the two shaded squares. Justify your answer fully.

(b) The diagram below shows the same square.
The diagonal of one of the rectangles is also marked. The length of this diagonal is d.

Show that the value of the total area of the two shaded squares is equal to d^2.



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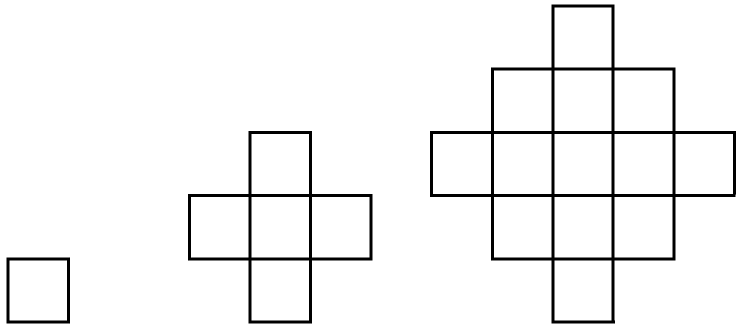
Question 8 **(Suggested maximum time: 20 minutes)**

The first three stages of a pattern are shown below.

Each stage of the pattern is made up of small squares.
Each small square has an area of one square unit.



Stage 1 **Stage 2** **Stage 3**

(a) Draw the next two stages of the pattern.

(b) The perimeter of Stage 1 of the pattern is 4 units.
The perimeter of Stage 2 of the pattern is 12 units.

Find a general formula for the **perimeter** of Stage n of the pattern, where $n \in \mathbb{N}$.

(c) Find a general formula for the **area** of Stage n of the pattern, where $n \in \mathbb{N}$.

(d) What kind of sequence (linear, quadratic, exponential, or none of these) do the **areas** follow? Justify your answer.

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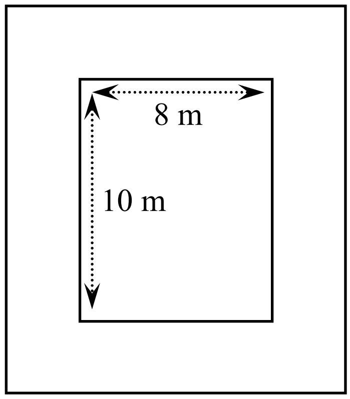
Question 9 **(Suggested maximum time: 20 minutes)**

A plot consists of a rectangular garden measuring 8 m by 10 m, surrounded by a path of constant width, as shown in the diagram.
The total area of the plot (garden and path) is 143 m^2.



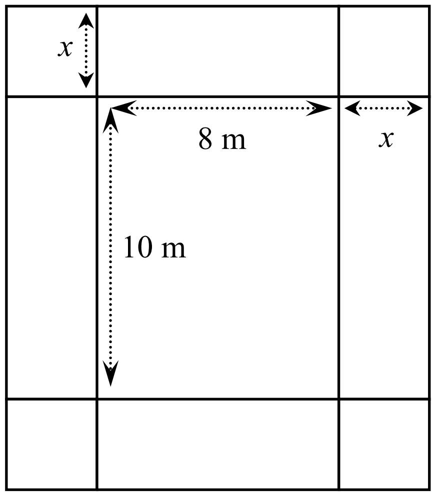
Three students, Kevin, Elaine, and Tony, have been given the problem of trying to find the width of the path. Each of them is using a different method, but all of them are using x to represent the width of the path.

Kevin divides the path into eight pieces. He writes down the area of each piece in terms of x. He then forms an equation by setting the area of the path plus the area of the garden equal to the total area of the plot.

(a) Write, in terms of x, the area of each section into Kevin's diagram below.

(b) Write down and simplify the equation that Kevin should get. Give your answer in the form $ax^2 + bx + c = 0$.

Equation:



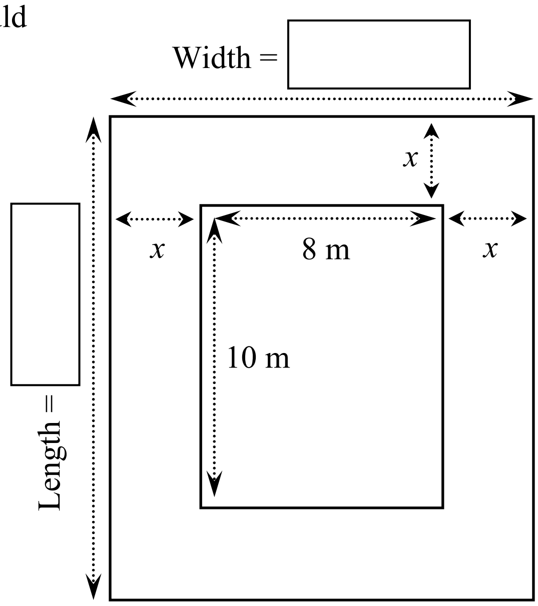
Kevin's Diagram

Elaine writes down the length and width of the plot in terms of x.
She multiplies these and sets the answer equal to the total area of the plot.

(c) Write, in terms of x, the length and the width of the plot in the spaces on Elaine's diagram.

(d) Write down and simplify the equation that Elaine should get. Give your answer in the form $ax^2 + bx + c = 0$.

Equation:



Elaine's Diagram

(e) Solve an equation to find the width of the path.

(f) Tony does not answer the problem by solving an equation. Instead, he does it by trying out different values for x. Show some calculations that Tony might have used to solve the problem.

(g) Which of the three methods do you think is best? Give a reason for your answer.

Answer:

Reason:

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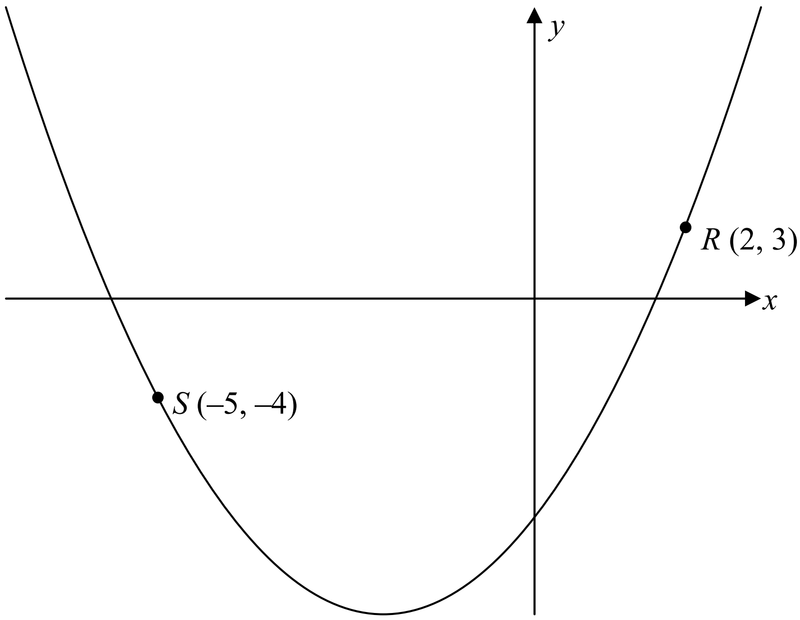
Question 10 **(Suggested maximum time: 20 minutes)**

Part of the graph of the function $y = x^2 + ax + b$, where $a, b \in \mathbb{Z}$, is shown below.



The points $R(2, 3)$ and $S(-5, -4)$ are on the curve.

(a) Use the given points to form two equations in a and b.

(b) Solve your equations to find the value of a and the value of b.

(c) Write down the co-ordinates of the point where the curve crosses the y-axis.

(d) By solving an equation, find the points where the curve crosses the x-axis.
Give each answer correct to one decimal place.

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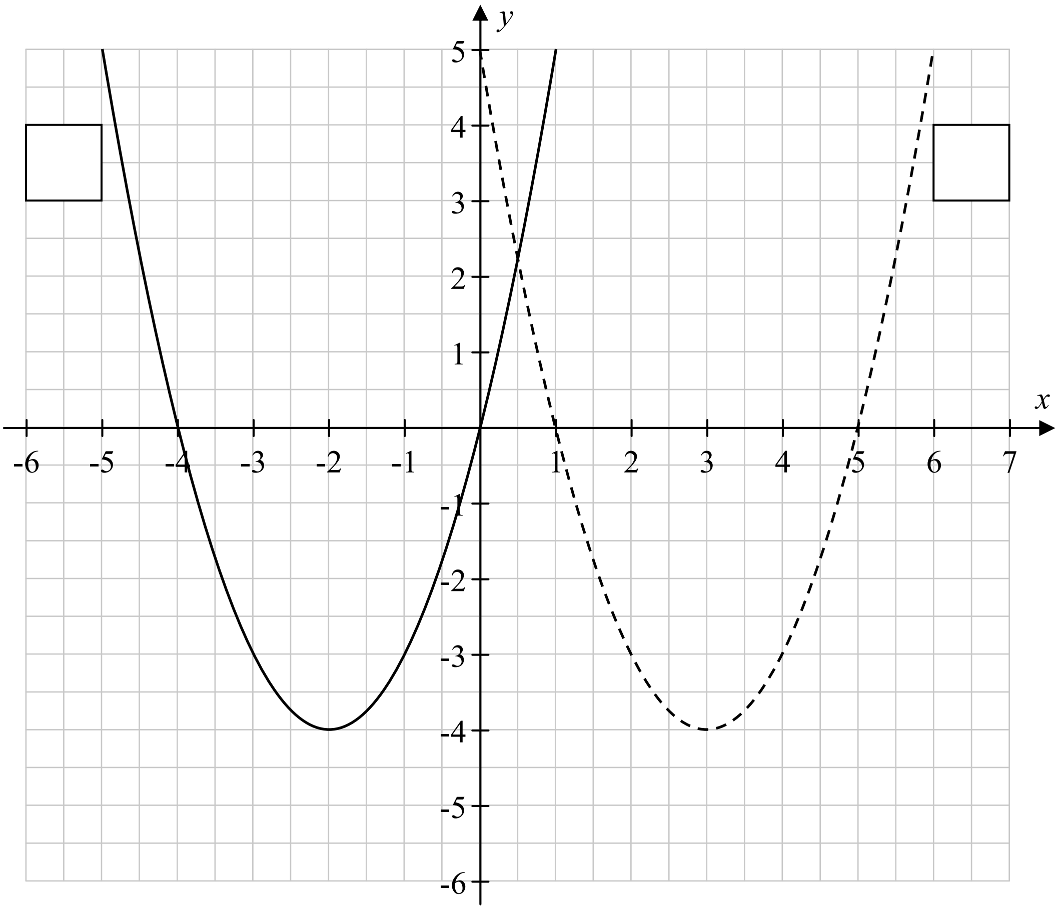
Question 11 **(Suggested maximum time: 15 minutes)**

The graphs of two functions, f and g, are shown on the co-ordinate grid below.
The functions are:

$$f : x \mapsto (x+2)^2 - 4$$
$$g : x \mapsto (x-3)^2 - 4.$$



(a) Match the graphs to the functions by writing f or g beside the corresponding graph on the grid.

(b) Write down the roots of f and the roots of g.

Roots of f:

Roots of g:

(c) Sketch the graph of $h : x \mapsto (x-1)^2 - 4$ on the co-ordinate grid above, where $x \in \mathbb{R}$.

(d) p is a natural number, such that $(x-p)^2 - 2 = x^2 - 10x + 23$.

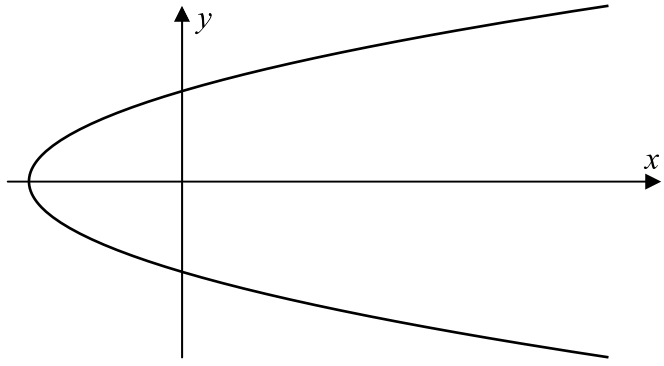
Find the value of p.

(e) Write down the equation of the axis of symmetry of the graph of the function:

$$k(x) = x^2 - 10x + 23.$$

Question 12 **(Suggested maximum time: 5 minutes)**

Give a reason why the graph below does **not** represent a function of x.



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