Question 1

A tissue measures 300 mm × 260 mm. There are 100 tissues in a box.
Find the total area of tissue in the box in m².

$0 \cdot 3 \times 0 \cdot 26 = 0 \cdot 078 \text{ m}^2$

$0 \cdot 078 \times 100 = 7 \cdot 8 \text{ m}^2$

$300 \times 260 = 78\,000$
$78\,000 \times 100 = 7\,800\,000$
$= 7 \cdot 8 \text{ m}^3$

Question 2

(a) A container in the shape of a cylinder has a capacity of 50 litres. The height of the cylinder is 0·7 m.
Find the length of the diameter of the cylinder.
Give your answer correct to the nearest whole number.

$\pi r^2 h = V$

$\pi \times r^2 \times 70 = 50\,000$

$r^2 = \dfrac{50\,000}{\pi \times 70}$

$r^2 = 227 \cdot 36$

$r = 15$

Diamater $= 30$ cm

(b) A rectangular tank has a length of 0·6 m, a width of 0·35 m and its height measures 15 cm.
Find the capacity of the rectangular tank.

$60 \times 35 \times 15 = 31500 \text{ cm}^3$ 　　or 31·5 L

(c) The rectangular tank is full of water. This water is then poured into the cylindrical container in **(a)** above. Find the depth of water in the cylinder.
Give your answer correct to one decimal place.

$$\pi \times 15 \times 15 \times h = 31500$$

$$h = \frac{31500}{\pi \times 15 \times 15}$$

$$h = 44{\cdot}6 \text{ cm}$$

$$\frac{31{\cdot}5}{50} \times 100 = 63\%$$

63% of the height of 70 cm is 44·1cm

Question 3

An ornament is carved from a rectangular block of wood which has a square base and a height of 24 cm. The ornament consists of two identical spheres and two identical cubes as illustrated in the diagram. The diameter of each sphere is equal to the length of the side of each cube. The ornament has the same width as the original block.

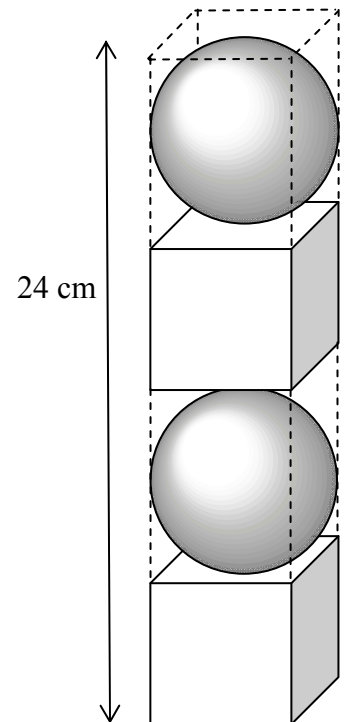**(a)** Find the length of a side of one of the cubes.

$$\frac{24}{4} = 6 \text{ cm}$$

24 cm

(b) Find the volume of the ornament.

$$2(6 \times 6 \times 6) + 2\left(\frac{4}{3}\pi \times 3^3\right)$$

$$= 432 + 226$$

$$= 658 \text{ cm}^3$$

(c) In making the ornament, what percentage of the original block of wood is carved away?

$$24 \times 6 \times 6 - 658{\cdot}2 = 864 - 658{\cdot}2 = 205{\cdot}8$$

$$\frac{205{\cdot}8}{864} \times \frac{100}{1} = 23{\cdot}82\%$$

$6 \times 6 \times 6 = 216$
$4/3 \times \pi \times 3^3 = 113{\cdot}097$
$216 - 113{\cdot}097 = 102{\cdot}9$
$102{\cdot}9 \times 2 = 205{\cdot}8$
$$\frac{205{\cdot}8}{864} \times 100 = 23{\cdot}82\%$$

Question 4

A soup tin in the form of a cylinder has a diameter of 7 cm and a height of 10 cm. The cylinder is constructed from pieces of metal cut from a thin sheet measuring 23 cm by 18 cm.

(a) Which one of the four diagrams A, B, C or D could represent the sheet of metal from which the cylinder has been cut?

A
23 cm

18 cm

B
23 cm

18 cm

C
23 cm

18 cm

D
23 cm

18 cm
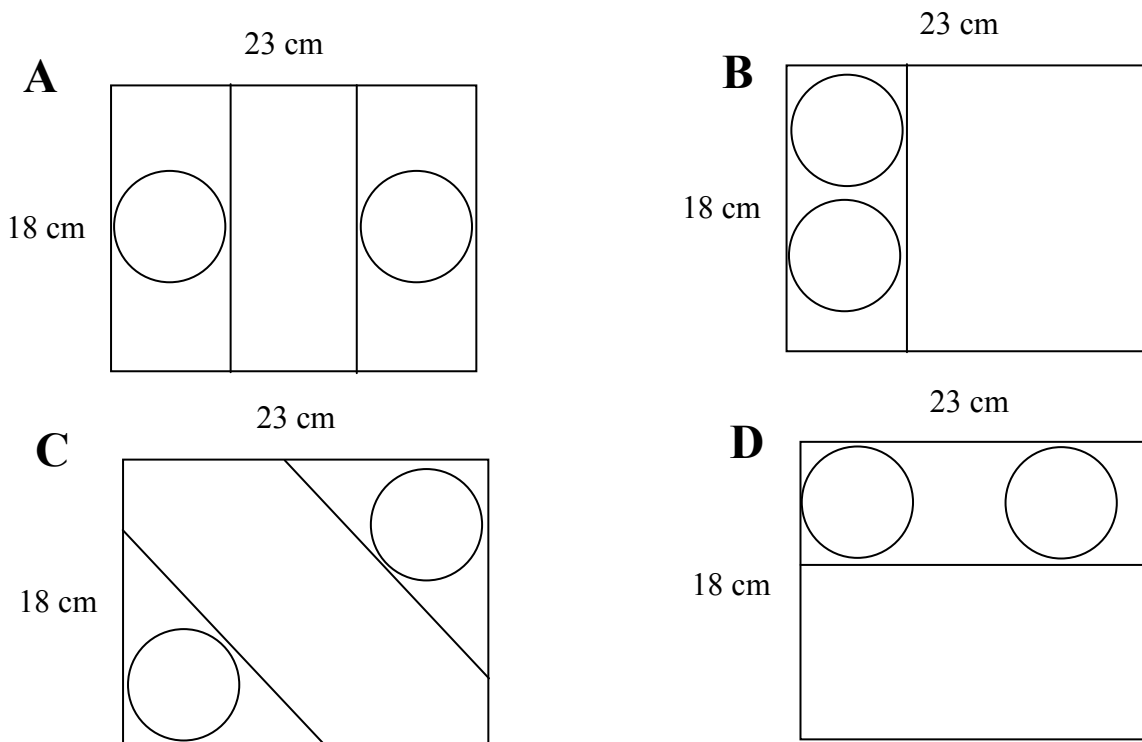
Answer = Diagram ___D___.   Give a reason for your choice.

$2\pi r = 2 \times \pi \times 3{\cdot}5 = 22$ cm
Need a piece 10 cm ×22 to make this cylinder. Only D has this.

(b) Find the area of metal which remains after the pieces have been cut out.

Area of sheet: $18 \times 23 = 414$

Surface area of cylinder: $10 \times 22 + 2(\pi \times 3{\cdot}5^2) = 220 + 77 = 297$

Metal remaining: $414 - 297 = 117$ cm^2

(c) Find the capacity of the soup tin.

$V = \pi \times 3{\cdot}5^2 \times 10 = 384{\cdot}85$ cm^3

**Question 5**   **(Suggested maximum time: 5 minutes)**

A, B, C, D and E represent the probabilities of certain events occurring.

(a) Write the probability of each of the events listed into the table below.

Event		Probability
A club is selected in a random draw from a pack of playing cards	A	$\frac{1}{4}$
A tossed fair coin shows a tail on landing	B	$\frac{1}{2}$ OR evens OR 50/50
The sun will rise in the east tomorrow	C	1 OR certain
May will follow directly after June	D	0 OR impossible
A randomly selected person was born on a Thursday	E	$\frac{1}{7}$

(b) Place each of the letters A, B, C, D and E at its correct position on the probability scale below.

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├──┼────┼────────┼──────────────────┤
0                                    1
     E      A        B
D                                    C
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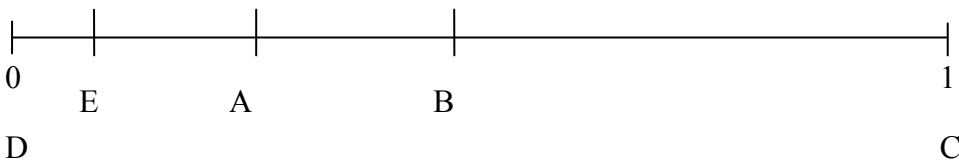
Question 6

The ages of the Academy Award winners for best male actor and best female actor (at the time they won the award) from 1992 to 2011 are as follows:

Male actor   54 52 37 38 32 45 60 46 40 36 47 29 43 37 38 45 50 48 60 50

Female actor   42 29 33 36 45 49 39 26 25 33 35 35 28 30 29 61 32 33 45 29

(a) Represent the data on a back-to-back stem-and-leaf diagram.

Male actors								Female actors								
					9	2	5	6	8	9	9	9				
8	8	7	7	6	2	3	0	2	3	3	3	5	5	6	9	
8	7	6	5	5	3	0	4	2	5	5	9					
			4	2	0	0	5									
				0	0	6	1									

Key: 2|5 is 25 years old

(b) State one similarity and one difference that can be observed between the ages of the male and female winners.

Same shape of distribution
No one over 61
No one under 24
The range is similar in both

Outlier in female winners
No female in her 50s
The female are younger

(c) Mary says "The female winners were younger than the male winners."
Investigate this statement in relation to:

(i) The mean age of the male winners and mean age of the female winners.

Male
Sum = 887
Mean = 44·35

Female
Sum =714
Mean = 35·7

Mean age of women is lower so statement is true for mean age

(ii) The median age of the male winners and the median age of the female winners.

> Male
> (45+45)/2
> Median = 45

> Female
> (33+33)/2
> Median = 33

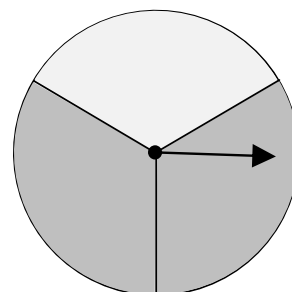> Median age of women is lower so statement is true for median age

(d) Find the interquartile ranges of the ages of the male winners and of the female winners.

Male

> $50 - 37 \cdot 5$
> $12 \cdot 5$

Female

> $40 \cdot 5 - 29$
> $11 \cdot 5$

Question 7 (25 marks)

A fair circular spinner consists of three equal sectors. Two are coloured blue and one is coloured red.
The spinner is spun and a fair coin is tossed.

(a) What is the probability of the spinner landing on a blue sector?

> $\dfrac{2}{3}$ or $\dfrac{240}{360}$

(b) Find the probability of getting a head and a red.

> $\dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$

(c) Find the probability of getting a tail and a blue.

> $\dfrac{1}{2} \times \dfrac{2}{3} = \dfrac{1}{3}$

Question 8 **(15 marks)**

(a) What is the probability of getting a 1 when a fair die is tossed?

$$\frac{1}{6}$$

A fair die is tossed 500 times.
The results are partially recorded in the table below.

Number on die	1	2	3	4	5	6
Frequency	70	82	*86*	90	91	81
Relative Frequency	*·14*	*·16*	*·17*	*·18*	*·18*	*·16*

(b) Calculate the number of times a 3 appeared. Write your answer in the table above.

500 – (70+82+90+91+81)
86

(c) Calculate the relative frequency of each outcome and write it into the table above.
Give your answers correct to 2 decimal places.

$$\frac{70}{500} = 0 \cdot 14 \qquad \frac{90}{500} = 0 \cdot 18$$

$$\frac{82}{500} = 0 \cdot 16 \qquad \frac{91}{500} = 0 \cdot 18$$

$$\frac{86}{500} = 0 \cdot 17 \qquad \frac{81}{500} = 0 \cdot 16$$

(d) Give a possible reason for the difference in value between the relative frequency for 1 in the table and your answer to part **(a)**.
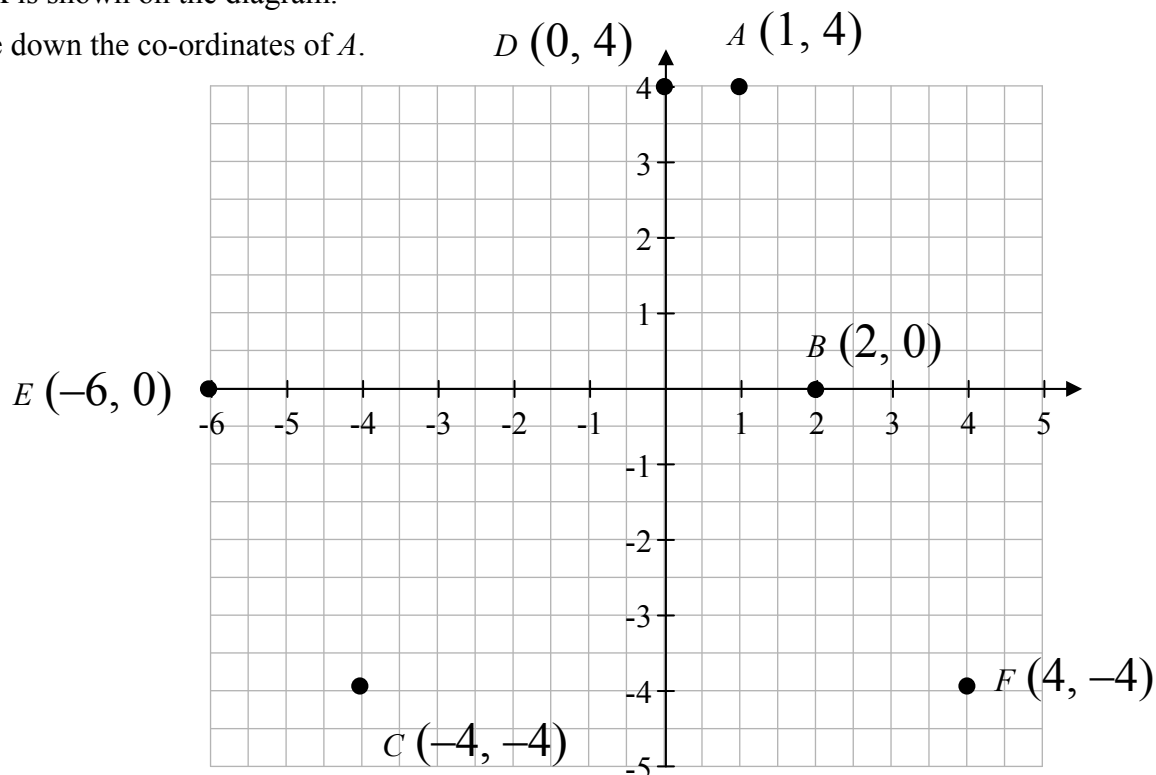
0·166 V 0·14
Experimental error

Question 9   **(20 marks)**

The point A is shown on the diagram.

(a) Write down the co-ordinates of A.

$D\,(0, 4)$ $A\,(1, 4)$

$E\,(-6, 0)$

$B\,(2, 0)$

$F\,(4, -4)$

$C\,(-4, -4)$

(b) Plot the following points on the diagram above.

B	C	D	E	F
(2, 0)	(− 4, − 4)	(0, 4)	(− 6, 0)	(4, − 4)

(c) Calculate the midpoint of [DF].

$$\left(\frac{0+4}{2}, \frac{4-4}{2}\right) = (2, 0)$$

(d) Find the slope of BF.

$$\frac{-4-0}{4-2} = \frac{-4}{2} - 2$$

(e) Write down the equation of the line BF in the form $y = mx + c$.

$$y = -2x + 4$$

$$y - 0 = -2(x - 2)$$
$$y = -2x + 4$$

(f) Find the slope of the line *CE*.

$$\frac{0-(-4)}{-6-(-4)} = \frac{4}{-2} = -2$$

(g) Write the equation of the line *CE* in the form of $ax + by + c = 0$.

$$y - 0 = -2(x+6)$$
$$y = -2x - 12$$
$$2x + y + 12 = 0$$

(h) What is the ratio of the area of the triangle *BCE* to the area of the triangle *BCF?*

Area of $\triangle BCE$

$$\frac{1}{2}(8)(4)$$
16

Area of $\triangle BCF$

$$\frac{1}{2}(8)(4)$$
16

Ratio:　　1:1 or 1/1

(i) State whether the two triangles in part **(h)** above are congruent.
Give a reason for your answer.

Answer:　　yes

Reason:　　*CFBE* is a parallelogram and *CB* is a diagonal which divides the parallelogram into two congruent triangles.

Or

SSS or SAS or ASA argument

Question 10 (50 marks)

The table below gives the equations of six lines.

Line 1 $y = 3x - 6$

Line 2 $y = 3x + 12$

Line 3 $y = 5x + 20$

Line 4 $y = x - 7$

Line 5 $y = -2x + 4$

Line 6 $y = 4x - 16$

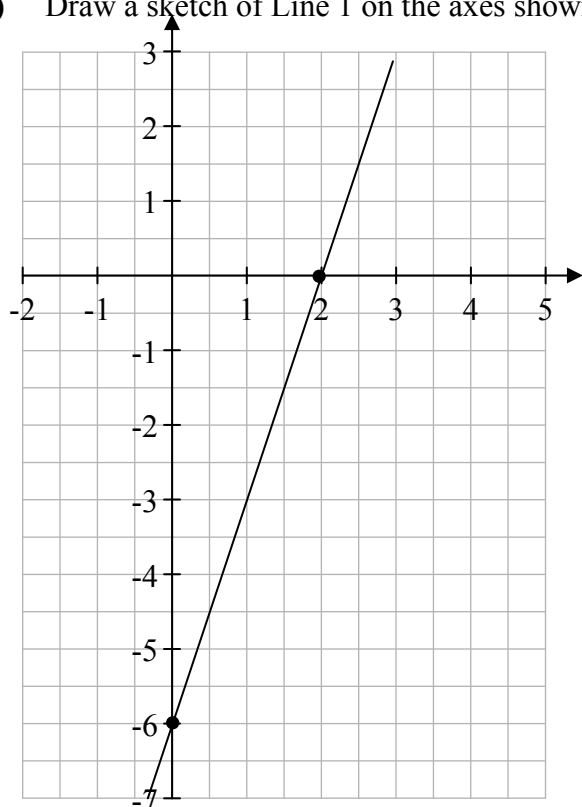**(a)** Which line has the greatest slope? Give a reason for your answer.

Line 3 OR $y = 5x + 20$
5 is the biggest number in front of x for any of the lines

(b) Which lines are parallel? Give a reason for your answer.

Line 1   and   Line 2
$y = 3x - 6$ and $y = 3x + 12$

They have the same slope (3)

(c)   Draw a sketch of Line 1 on the axes shown.

$$y = 3x - 6$$
$$x = 0, y = -6$$
$$y = 0, x = 2$$

$$(2,0),(0,-6)$$

(d)   The diagram below represents one of the given lines. Which line does it represent?
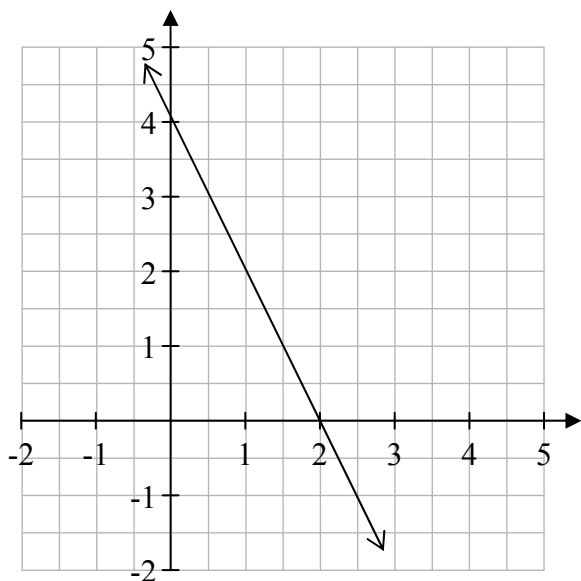
$$\text{slope} = \frac{0-4}{2-0} = -2$$
$$y\text{-intercept} = 4$$
$$\text{equation} \qquad y = -2x + 4$$

Answer = Line _____5 $(y = -2x + 4)$

(e) The table shows some values of x and y for the equation of one of the lines.
Which equation do they satisfy?

x	y
7	12
9	20
10	24

$$y = 4x - 16$$
$$y = 4(7) - 16 = 12$$
$$y = 4(9) - 16 = 20$$
$$y = 4(10) - 16 = 24$$

$$m = \frac{20 - 12}{9 - 7} = 4$$
$$y - 12 = 4(x - 7)$$
$$y = 4x - 16$$

Answer = Line _____6_____

(f) There is one value of x which will give the same value of y for Line 4 as it will for Line 6. Find, using algebra, this value of x and the corresponding value of y.

$$y = x - 7$$
$$y = 4x - 16$$
$$0 = 3x - 9$$
$$x = 3$$
$$y = 3 - 7 = -4$$

$$(3, -4)$$
Ans: $x = 3$

(g) Verify your answer to **(f)** above.
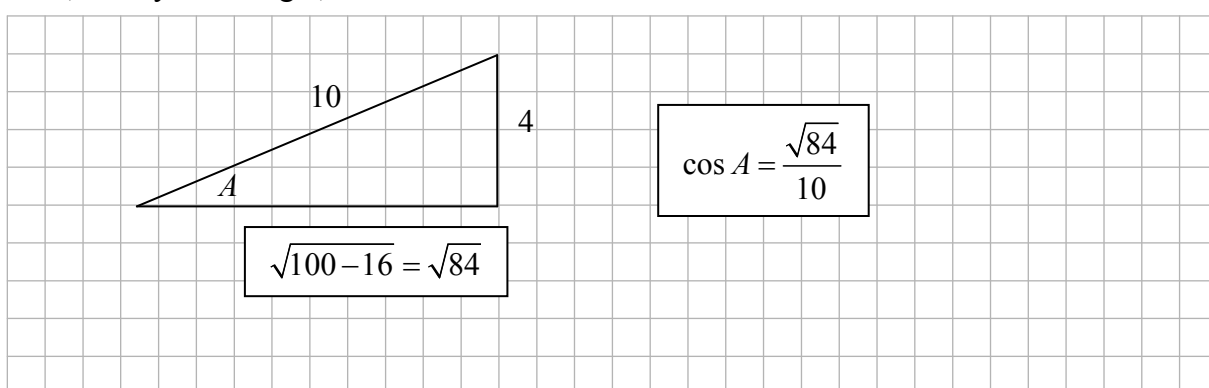
Line 4
$$y = (3) - 7 = -4$$
Line 6
$$y = 4(3) - 16 = -4$$

Question 11 (9 marks)

**(a)** Construct a right-angled triangle containing an angle A such that $\sin A = 0\cdot 4$.

10

4

A
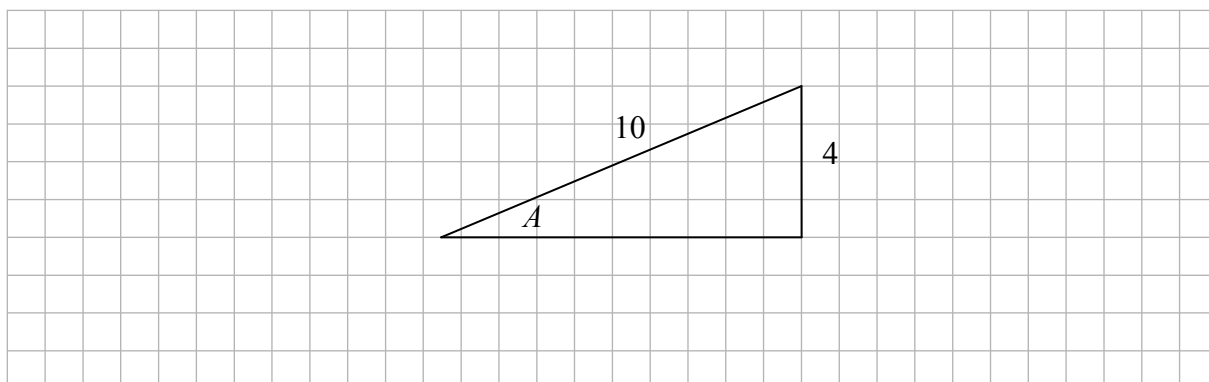
(b) Find, from your triangle, $\cos A$ in surd form.

10

4

A

$\sqrt{100-16} = \sqrt{84}$

$\cos A = \dfrac{\sqrt{84}}{10}$

Question 12 (30 marks)

A homeowner wishes to replace the three identical steps leading to her front door with a ramp. Each step is 10 cm high and 35 cm long. Find the length of the ramp. Give your answer correct to one decimal place.

$\leftarrow$ 35 cm $\rightarrow$

10 cm
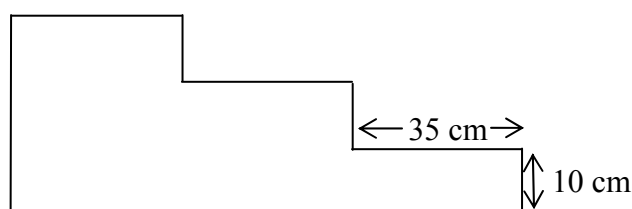
$\text{Ramp} = \sqrt{30^2 + 105^2}$

$= 109\cdot 2$ cm

OR

30 cm

ramp

105 cm

$H^2 = 35^2 + 10^2$

$H = \sqrt{1225 + 100}$

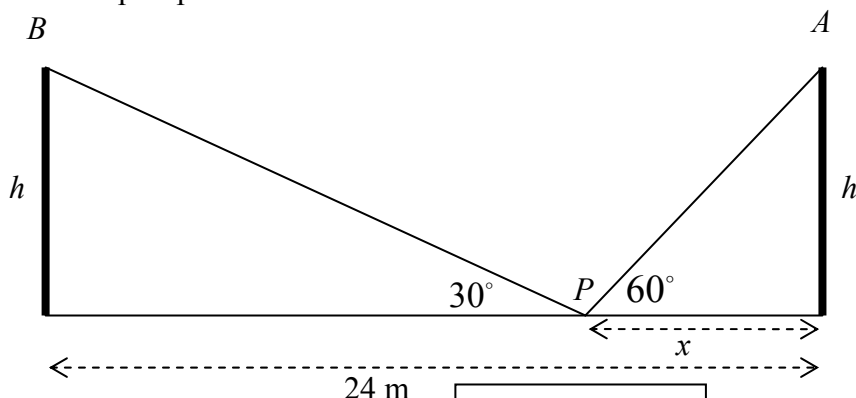$H = \sqrt{1325}$

$Ramp = 3 \times H$

$Ramp = 3 \times \sqrt{1325}$

$Ramp = 109.2 cm$
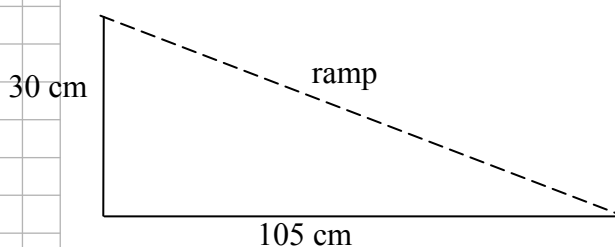
Question 13 (20 marks)

Two vertical poles A and B, each of height h, are standing on opposite sides of a level road. They are 24 m apart. The point P, on the road directly between the two poles, is a distance x from pole A. The angle of elevation from P to the top of pole A is 60°.

(a) Write h in terms of x.

$$\tan 60° = \frac{h}{x}$$
$$\sqrt{3} = \frac{h}{x}$$
$$h = \sqrt{3}\, x$$

$$\tan 60° = \frac{h}{x}$$
$$1{\cdot}732 = \frac{h}{x}$$
$$h = 1{\cdot}732\, x$$

$$\tan 30° = \frac{h}{24 - x}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{24 - x}$$
$$h = \frac{24 - x}{\sqrt{3}}$$

(b) From P the angle of elevation to the top of pole B is 30°. Find h, the height of the two poles.

$$\tan 30° = \frac{h}{24 - x}$$
$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{24 - x}$$
$$3x = 24 - x$$
$$4x = 24$$
$$x = 6$$
$$h = 6\sqrt{3} \text{ m}$$

$$\tan 30° = \frac{h}{24 - x}$$
$$\frac{1}{1{\cdot}732} = \frac{1{\cdot}732x}{24 - x}$$
$$3x = 24 - x$$
$$4x = 24$$
$$x = 6$$
$$h = 6\sqrt{3} \text{ m} \quad \text{or } 6(1{\cdot}732)\text{m} = 10{\cdot}39 \text{ m}$$

Question 14 (16 marks)

Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Given: A circle with centre O, with points A, B and C on the circle

To Prove: $|\angle BOC| = 2|\angle BAC|$

Construction: Join A to O and extend to R

Proof: In the triangle AOB

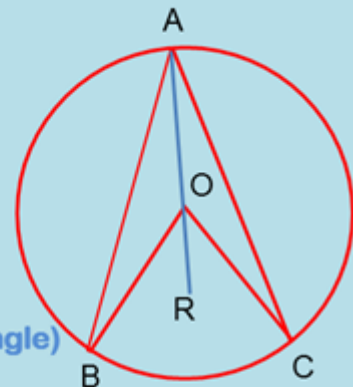$|AO| = |OB|$ Radii

$\Rightarrow |\angle OBA| = |\angle OAB|$ Theorem 2 (isosceles △)

$|\angle BOR| = |\angle OBA| + |\angle OAB|$ Theorem 6 (exterior angle)

∴ $|\angle BOR| = |\angle OAB| + |\angle OAB|$

∴ $|\angle BOR| = 2|\angle OAB|$

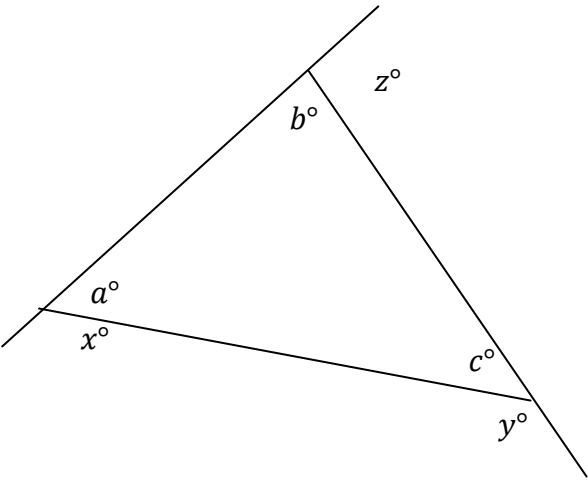Similarly $|\angle ROC| = 2|\angle OAC|$

∴ $|\angle BOC| = 2|\angle BAC|$

Question 15 (16 marks)
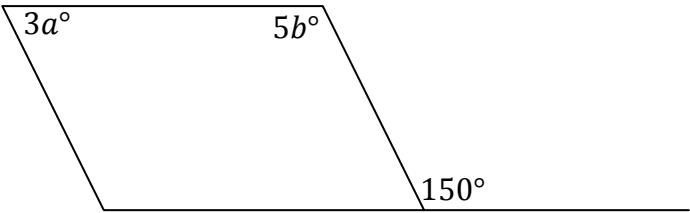
(a) Prove that $x + y + z = 360$.

$x = b + c$ (external angle) $a + b + c = 180$ (Triangle)

$y = a + b$ (external angle) $x + y + z = 360$

$z = a + c$ (external angle)

$x + y + z = 2(a + b + c)$

(b) The diagram below shows a parallelogram and one exterior angle. Find the value of a and the value of b.

$5b = 150 (alt)$

$b = 30$

$3a + 5b = 180$

$3a = 30$

$a = 10$