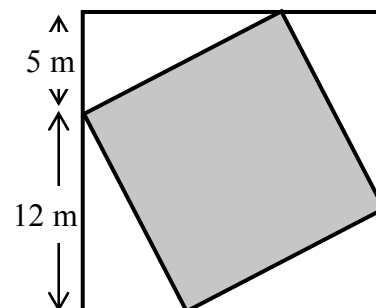**Question 1**   **20 marks**

Liam's garden is in the shape of a square. It has four equal
right-angled triangular lawns and a smaller square patio in
the centre, as shown.

5 m

12 m

(a) Find the length of the hypotenuse of one of the right angled
triangular lawns.

$$h^2 = a^2 + b^2$$

Length $= \sqrt{12^2 + 5^2}$ $=$ 13 m

(b) Find the area of one of the triangular lawns.

Area $= \dfrac{1}{2}(12)(5)$ $=$ 30 m^2

(c) Find the area of the square patio in the middle.

Area of patio $=$ 13^2 $=$ 169 m^2

or

Area of patio $=$ $17^2 - (4 \times 30)$ $=$ 169 m^2

(d) The patio is to be paved with rectangular flagstones of length 80cm and width 50cm.
Calculate the number of flagstones Liam needs to buy to cover the patio, allowing an extra
20% for waste.

Area of flagstone $=$ $0 \cdot 8 \times 0 \cdot 5$ $=$ $0 \cdot 4$ m^2

Number of flagstones $=$ $\dfrac{169}{0.4}$ $=$ $422 \cdot 5$ (or 423)

Extra 20% $=$ $422 \cdot 5 \times 0 \cdot 2 = 84 \cdot 5$ **or** 120% $=$ $422 \cdot 5 \times 1 \cdot 2$
Total number of flagstones $=$ 507

or

Extra 20% $=$ $423 \times 0 \cdot 2 = 84 \cdot 6$ **or** 120% $=$ $423 \times 1 \cdot 2$
Total number of flagstones $=$ $507 \cdot 6$

or

Area to cover $=$ $169 \times 1 \cdot 2$ $=$ $202 \cdot 8$

Total number of flagstones $=$ $\dfrac{202 \cdot 8}{0 \cdot 4} = 507$

[28]

Question 2   **30 marks**

The ages of the 30 people who took part in an aerobics class are as follows:

18	24	32	37	9	13	22	41	51	49
15	42	37	58	48	53	27	54	42	24
33	48	56	17	61	37	63	45	20	39

The ages of the 30 people who took part in a swimming class are as follows:

16	22	29	7	36	45	12	38	52	13
33	41	24	35	51	8	47	22	14	24
42	62	15	24	23	31	53	36	48	18

(a) Represent this data on a back-to-back stem-and-leaf diagram.

					Aerobics class					Swimming class				
					9	0	7	8						
		8	7	5	3	1	2	3	4	5	6	8		
	7	4	4	2	0	2	2	2	3	4	4	4	9	
9	7	7	7	3	2	3	1	3	5	6	6	8		
9	8	8	5	2	2	1	4	1	2	5	7	8		
	8	6	4	3	1	5	1	2	3					
				3	1	6	2							
							Key: 1 | 5 means 15							

(b) Use your diagram to identify the median in each case.

Aerobic median: $\dfrac{37+39}{2} = 38$

Swimming median: $\dfrac{29+31}{2} = 30$

(c) What other measure of central tendency could have been used when examining this data?

Mean or Mode

**(d)** Based on the data make one observation about the ages of the two groups.

An older age group take Aerobics class
or
A younger age group take Swimming class
or
Similar

[29]

Question 3   **15 marks**

The arrows represent the different routes that a skier can take when skiing down a mountain.
The circles on the diagram represent different points on the routes.

(a)   When leaving any particular point on the mountain a skier is equally likely to choose any of
the available routes from that point. Fill in the boxes in the diagram which represent the
probability that the skier will take that route.
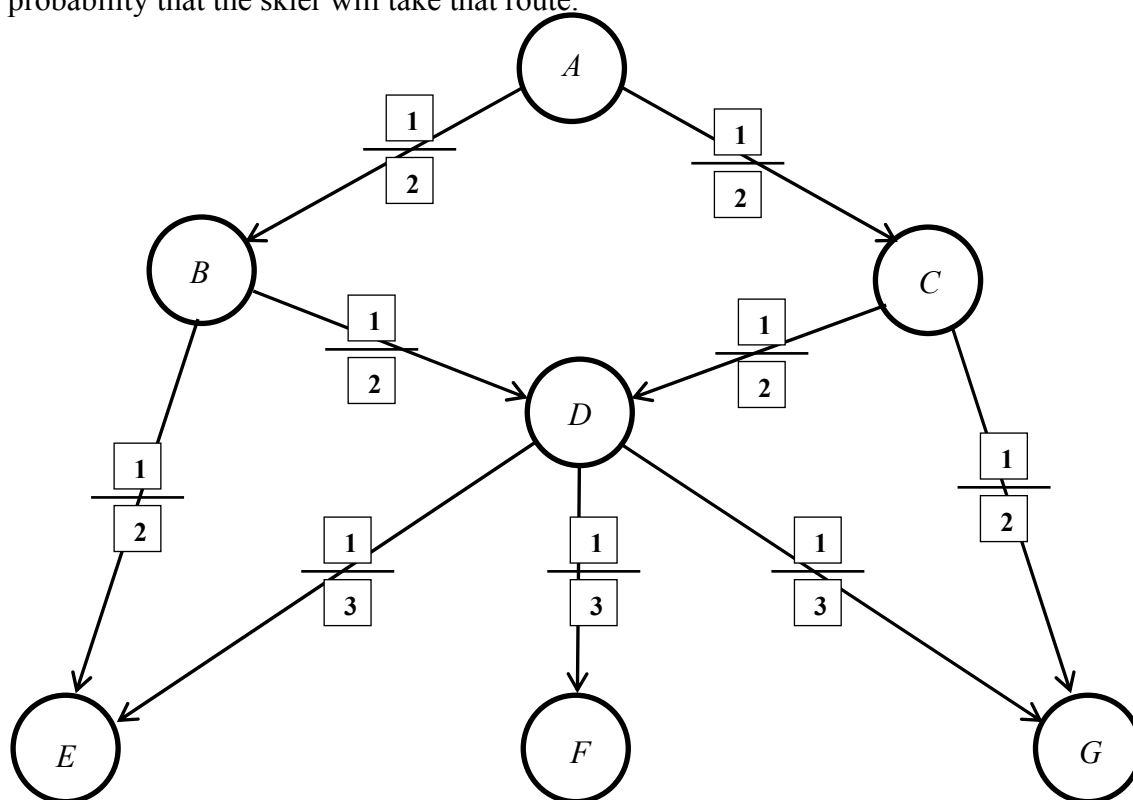
(b)   **(i)**   If the skier starts at point A, in how many different ways can the skier reach the point E?

1. A → B → E	
2. A → B → D → E	} 3 ways
3. A → C → D → E	

(ii)   If the skier starts at point A, find the probability that the skier will reach the point E.

1. $\dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

2. $\dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{12}$ } Probability $= \dfrac{1}{4} + \dfrac{1}{12} + \dfrac{1}{12}$

3. $\dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{12}$ $= \dfrac{5}{12}$

[30]

Question 4   **10 marks**

A football strip consists of a shirt, shorts and socks.
Aspen United has two shirts, blue and green, from which to select. They also can select from three different colours of shorts and five different colours of socks, including red in each case.

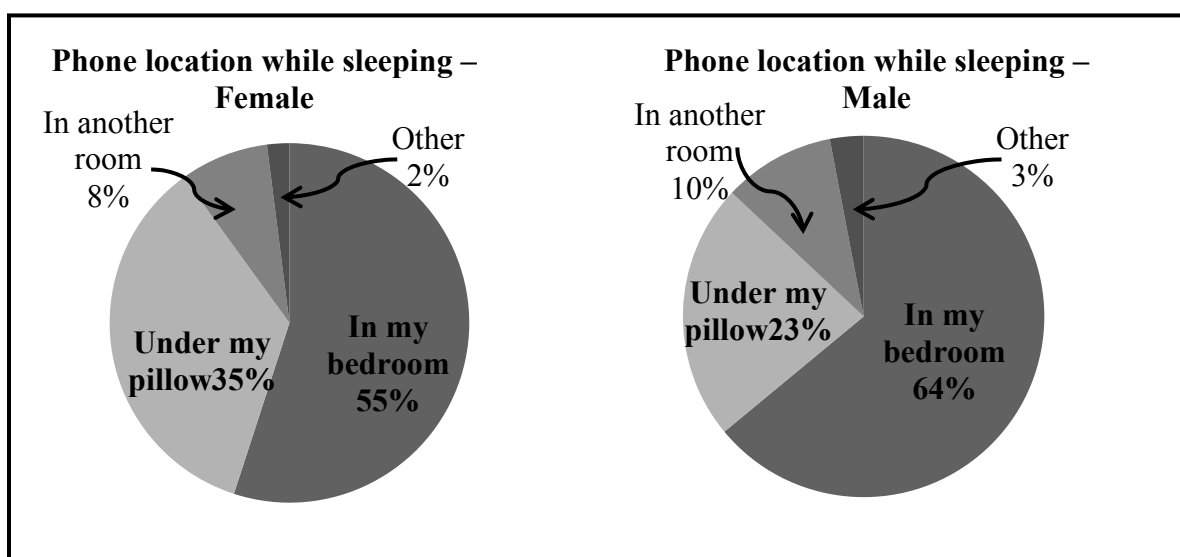**(a)** Calculate how many different strips Aspen United can have.

Different strips: $2 \times 3 \times 5 \quad = \quad 30$

(b) Willow Celtic plays in an all red strip. When Aspen United plays Willow Celtic, Aspen United are not allowed to use their red shorts or their red socks. Calculate how many different strips Aspen United can have when they play Willow Celtic.

Different strips: $2 \times 2 \times 4 \quad = \quad 16$

Question 5   **25 marks**

In total 7150 second level school students from 216 schools completed the 2011/2012 phase 11*CensusAtSchool* questionnaire. The questionnaire contained a question relating to where students keep their mobile phones while sleeping.

Phone location while sleeping – Female

In another room 8%
Other 2%
Under my pillow 35%
In my bedroom 55%

Phone location while sleeping – Male

In another room 10%
Other 3%
Under my pillow 23%
In my bedroom 64%

(a) Given that this question was answered by 4171 girls and 2979 boys, calculate how many female students kept their mobile phones under their pillows.

Girls – phone under pillow   =   35% of 4171

     = $4171 \times 0{\cdot}35$

     = $1459{\cdot}85$

     =   1460   (or 1459·85 or 1459)

(b) Calculate the overall percentage of students who kept their mobile phones under their pillows.

Total number of students   =   7150

Boys – phone under pillow   =   23% of 2979

     = $685{\cdot}17$

     =   685   (or 685·17 or 686)

Total   = $1460 + 685$ =   2145   (or 2145·02)

Percentage   = $\dfrac{2145}{7150} \times 100$ (or $\dfrac{2145{\cdot}02}{7150} \times 100$)

     =   30%   (or 30·0002%).

(c) A new pie chart is to be drawn showing the mobile phone location for all students. Calculate the measure of the angle that would represent the students who kept their mobile phones under their pillows.

Angle   =   30% of 360°
        = $360 \times 0{\cdot}3$
        =   108°   (or 108·00072)°

[32]

Question 6   **30 marks**

The salaries, in €, of the different employees working in a call centre are listed below.

22000	16500	38000	26500	15000	21000	15500	46000
42000	9500	32000	27000	33000	36000	24000	37000
65000	37000	24500	23500	28000	52000	33000	25000
23000	16500	35000	25000	33000	20000	19500	16000

(a) Use this data to complete the grouped frequency table below.

Salary (€1000)	0 – 10	10 –20	20 –30	30–40	40 –50	50 –60	60 –70
No. of Employees	1	6	12	9	2	1	1

[Note: 10–20 means €10000 or more but less than €20000, etc.]

(b) Using mid-interval values find the mean salary of the employees.

The mid - interval values are 5000, 15000, 25000, 35000, 45000, 55000, 65000

$$\text{Mean} =$$

$$\frac{(5000 \times 1) + (15\,000 \times 6) + (25\,000 \times 12) + (35\,000 \times 9) + (45\,000 \times 2) + (55\,000 \times 1) + (65\,000 \times 1)}{32}$$

$$= \frac{5000 + 90000 + 300000 + 315000 + 90000 + 55000 + 65000}{32}$$

$$= \frac{920000}{32}$$

$$= €28{,}750$$

(c) **(i)** Outline another method which could have been used to calculate the mean salary.

Add up all the individual salaries and divide by 32.

(ii) Which method is more accurate? Explain your answer.

Answer	Adding up individual salaries and dividing by 32

Reason	This gives the actual mean as estimates (mid-intervals) are not used.

[33]

Question 7   **20 marks**

In a survey, 54 people were asked which political party they had voted for in the last three elections.
The results are as follows:

      30 had voted for the Conservatives
      22 had voted for the Liberals
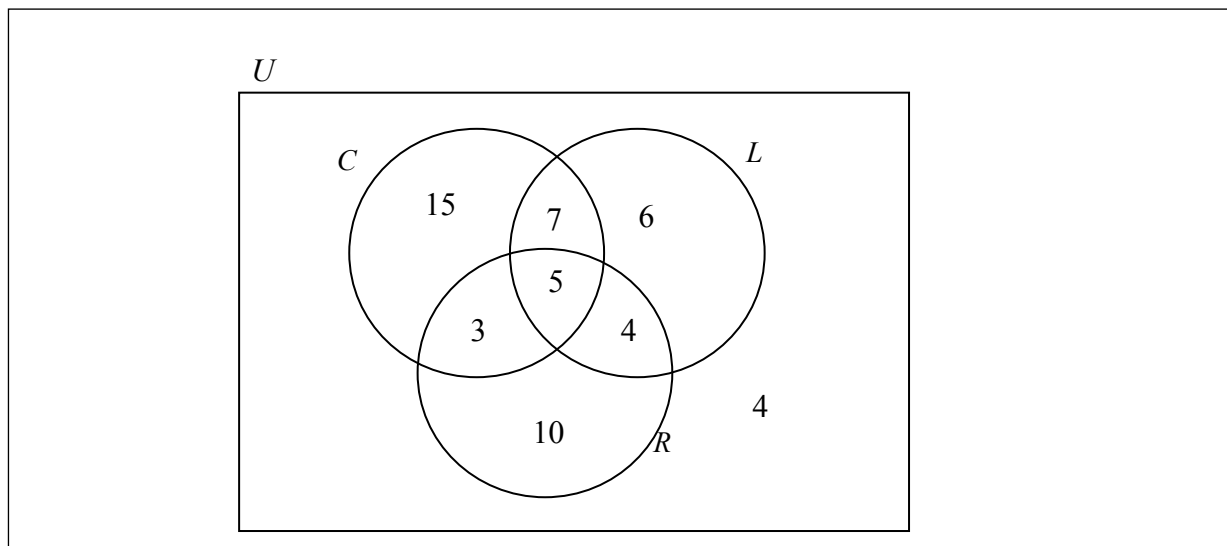   22 had voted for the Republicans

      12 had voted for the Conservatives and for the Liberals
      9 had voted for the Liberals and for the Republicans
      8 had voted for the Conservatives and for the Republicans

         5 had voted for all three parties.

(a)   Represent the information in a Venn diagram.

(b)   If one person is chosen at random, what is the probability that the person chosen did not vote
in any of the three elections?

> Probability person did not vote $\quad=\quad \dfrac{4}{54}\quad$ or $\quad\dfrac{2}{27}$

(c)   If one person is chosen at random, what is the probability that the person chosen voted for at
least two different parties?

> Probability person voted for at least two parties $\quad=\quad \dfrac{3+5+7+4}{54}=\dfrac{19}{54}$

(d)   If one person is chosen at random, what is the probability that the person chosen voted for the
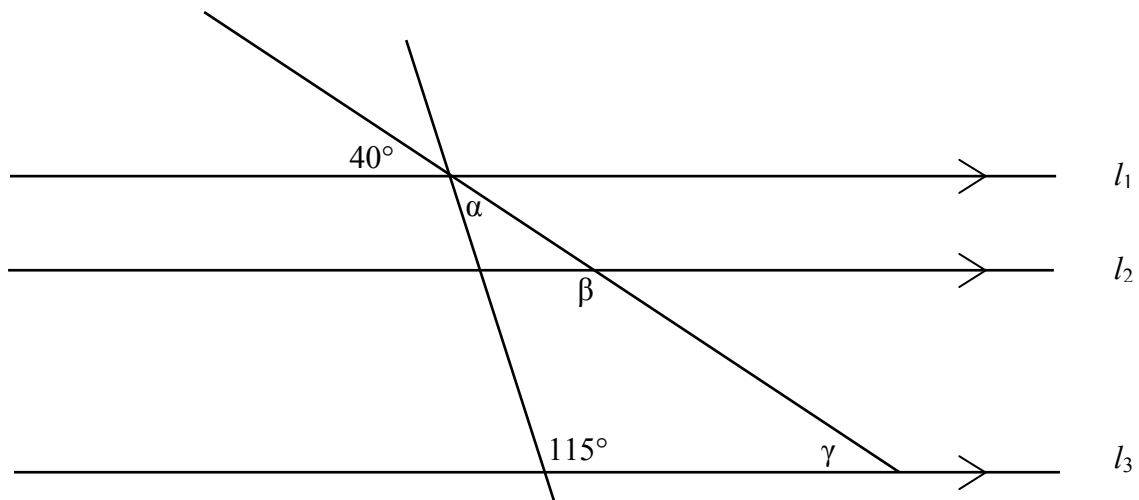same party in all three elections?

> Probability person voted for the same party $\quad=\quad \dfrac{15+6+10}{54}=\dfrac{31}{54}$

[34]

Question 8   **10 marks**

Construct the bisector of the ∠ABC below, using only a compass and straight edge.
Show all construction work.
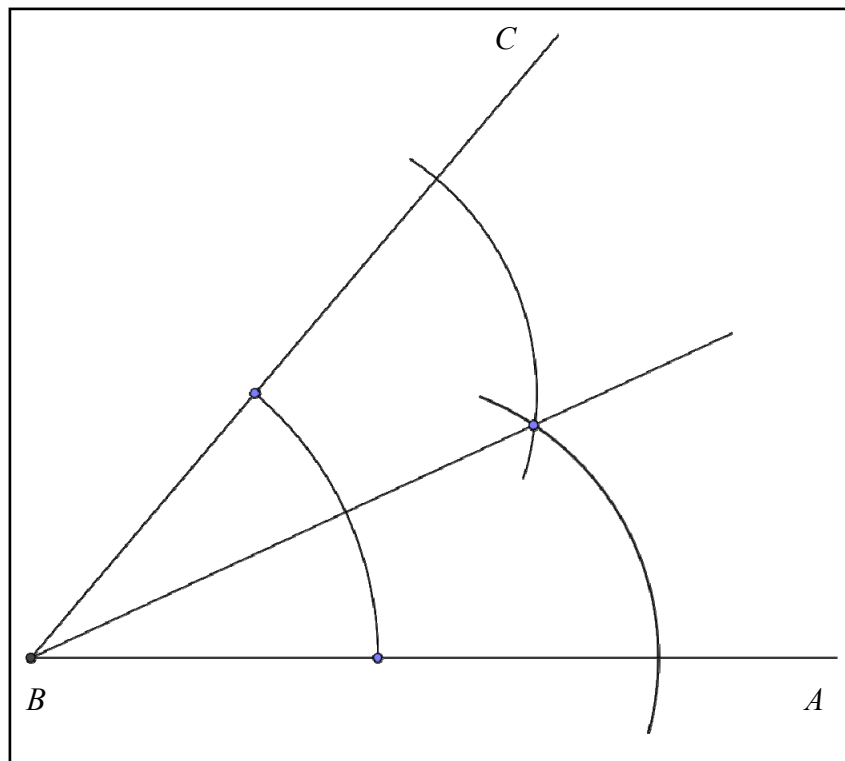
Question 9   **15 marks**

If l_1, l_2 and l_3 are parallel lines, find the measure of the angles α, β and γ.

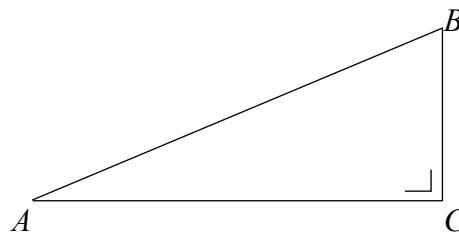|				
α	=	180 − (115 + 40)	=	25°
β	=	180 − 40	=	140°
γ	=	40°		

[35]

Question 10   **20 marks**

In the triangle ABC, $|AB| = 2$ and $|BC| = 1$.

(a) Find $|AC|$, giving your answer in surd form.

$$h^2 = a^2 + b^2$$
$$2^2 = |AC|^2 + 1^2$$
$$\Rightarrow |AC| \quad = \quad \sqrt{2^2 - 1^2} \quad = \quad \sqrt{3}$$

(b) Write $\cos \angle BAC$ and hence find $|\angle BAC|$.

$$\cos \angle BAC \quad = \quad \frac{\sqrt{3}}{2}$$
$$|\angle BAC| \quad = \quad 30°$$

(c) Sketch a right angled isosceles triangle in which the equal sides are 1 unit each and use it to write $\cos 45°$ in surd form.

$$\text{Hypotenuse} \quad = \quad \sqrt{1^2 + 1^2} \quad = \quad \sqrt{2}$$

$$\cos 45° \quad = \quad \frac{1}{\sqrt{2}}$$
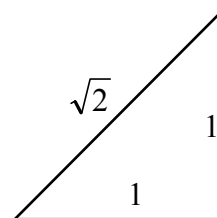
(d) Show that $\cos 75° \neq \cos 45° + \cos 30°$.

$$\cos 75° \quad = \quad \frac{\sqrt{6} - \sqrt{2}}{4} \quad = \quad 0{\cdot}2588$$

$$\cos 45° + \cos 30° \quad = \quad \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \quad = \quad 0{\cdot}7071 + 0{\cdot}8660 = 1{\cdot}5731$$

$$(0{\cdot}2588 \neq 1{\cdot}5731)$$

Question 11   **10 marks**

The two triangles shown are similar. Find the value of x.

$$\frac{x}{6} \quad = \quad \frac{25}{15}$$

$$\Rightarrow \quad x \quad = \quad 10$$

25

x

6

15

[36]
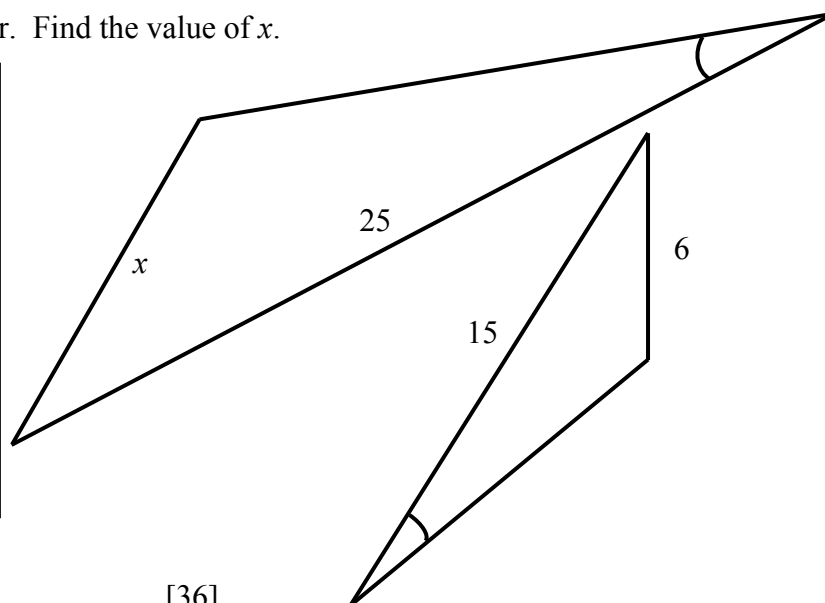
Question 12   **45 marks**
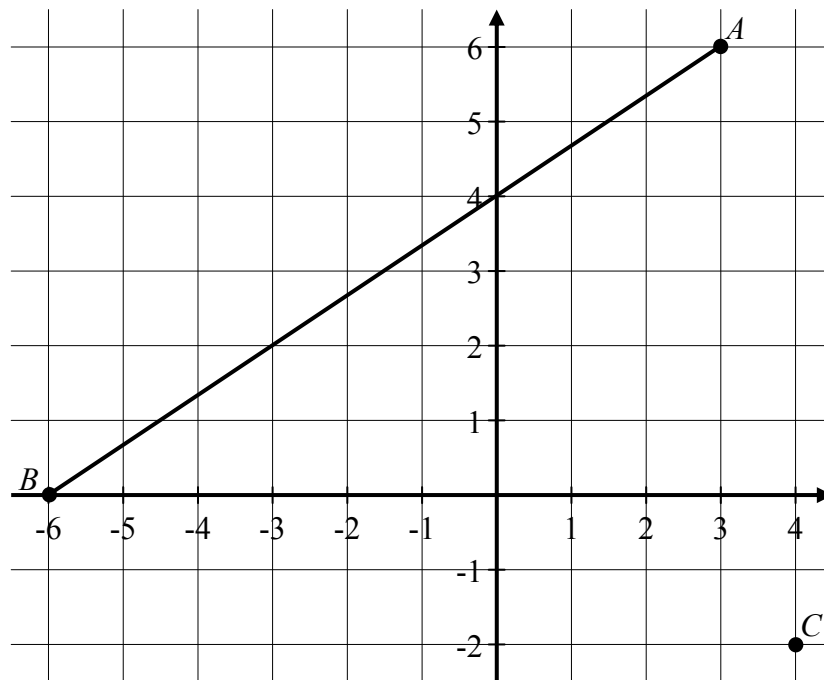
(a) Write the coordinates of *A*, *B* and *C*.

$A(\ 3\quad,\quad 6\ )\qquad B(-6\quad,\quad 0)\qquad C(\ 4\ ,-2\ )$

(b) Find the co-ordinates of *D*, the mid-point of [*AB*].

$$D\quad=\quad \left(\frac{3-6}{2},\frac{6+0}{2}\right)\quad=\quad \left(-\frac{3}{2},3\right)$$

(c) Find the equation of the line *AB*.

$$\text{Slope } AB\quad=\quad \frac{0-6}{-6-3}\quad=\quad \frac{2}{3}$$

Equation *AB*: $\qquad y-0=\dfrac{2}{3}(x+6)\qquad$ **or** $\qquad y-6=\dfrac{2}{3}(x-3)$

or

$$y=\frac{2}{3}x+4$$

$$2x-3y+12\ =\ 0$$

[37]

(d) Find the equation of the line through C, perpendicular to AB.

Perpendicular slope $= -\dfrac{3}{2}$

Line through C: $\quad y + 2 = -\dfrac{3}{2}(x-4)$

$\quad\quad\quad\quad\quad 3x + 2y - 8 = 0$

or

The line is of the form $3x + 2y + c = 0$

$(4, -2)$: $3(4) + 2(-2) + c = 0 \Rightarrow c = -8$

$\quad\quad\quad\quad\quad 3x + 2y - 8 = 0$

(e) Let E be the point where this perpendicular line through C intersects AB.
Calculate the coordinates of the point E.

E the point of intersection of two lines $\quad\quad 2x - 3y + 12 = 0$ (i)

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 3x + 2y - 8 = 0$ (ii)

$2\times$(i) $4x - 6y = -24$ $\quad$ **or** $\quad$ $y = \dfrac{2x+12}{3}$

$+3\times$(ii) $9x + 6y = 24$ $\quad\quad\quad\quad \Rightarrow 3x + 2\left(\dfrac{2x+12}{3}\right) - 8 = 0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \Rightarrow 9x + 4x + 24 - 24 = 0$

$\quad\quad\quad\quad \Rightarrow \quad x = 0 \quad\quad$ and $\quad\quad y = 4$

(f) Which is the shorter distance, $|CD|$ or $|CE|$? Find this distance.

$|CD| = \sqrt{\left(4+\dfrac{3}{2}\right)^2 + (-2-3)^2} = \sqrt{55.25} \quad$ or $\quad 7{\cdot}433$

$|CE| = \sqrt{(4-0)^2 + (-2-4)^2} = \sqrt{52} \quad$ or $\quad 7{\cdot}211$
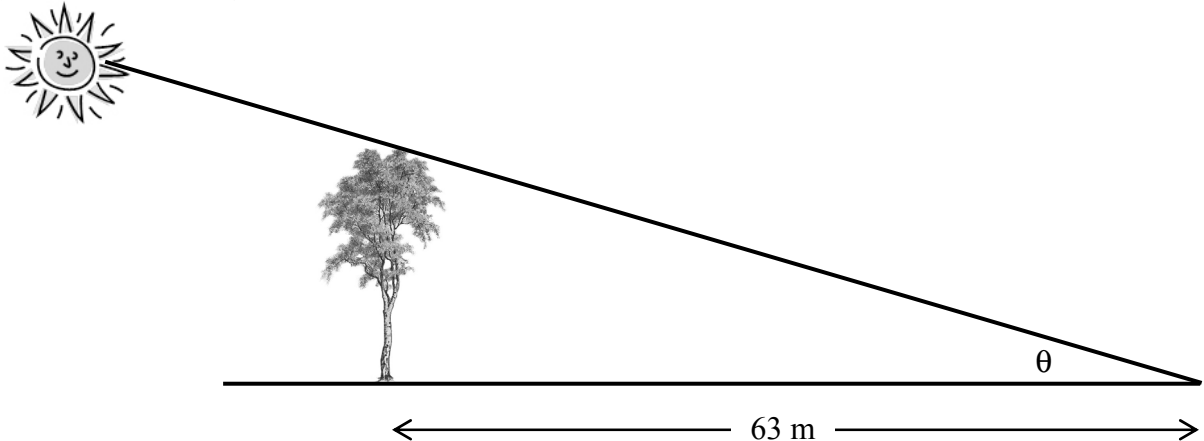
$|CE|$ is the shorter distance

or

$|CE|$ (is the perpendicular distance and therefore is the shorter distance.)

$|CE| = \sqrt{(4-0)^2 + (-2-4)^2} \quad = \quad \sqrt{52} \quad$ or $\quad 7{\cdot}211$

[38]

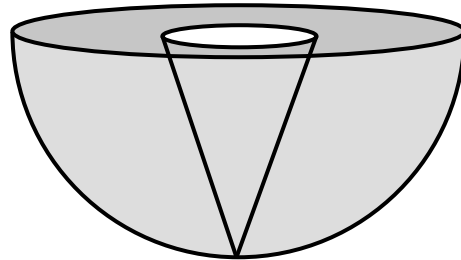**Question 13**   **10 marks**

A tree 32 m high casts a shadow 63m long. Calculate θ, the angle of elevation of the sun.
Give your answer in degrees and minutes (correct to the nearest minute).

$$
\begin{aligned}
\tan\theta &= \frac{32}{63} & \textbf{or} \quad \tan\alpha &= \frac{63}{32} \\
\Rightarrow \theta &= 26{\cdot}9277 & \Rightarrow \alpha &= 63{\cdot}0723 \\
\Rightarrow \theta &= 26°55'39{\cdot}64" & \Rightarrow \theta &= 90 - 63{\cdot}0723 \\
&= 26°56' & &= 26{\cdot}9277 \\
& & &= 26°55'39{\cdot}64" \\
& & &= 26°56'
\end{aligned}
$$

Question 14   **20 marks**

A solid metal hemisphere has a radius of 12 cm.

(a) Calculate the volume of the hemisphere. Give your answer in terms of π.

$$
\begin{aligned}
\text{Volume of hemisphere} &= \frac{2}{3}\pi r^3 \\
&= \frac{2}{3} \times \pi \times 12^3 \\
&= 1152\pi \ \text{cm}^3
\end{aligned}
$$

[39]

(b)   A solid cone of radius 4 cm and height 12 cm is cut from the hemisphere.
Calculate the volume of the cone. Give your answer in terms of π.

$$\begin{aligned}
\text{Volume of cone} \quad &= \quad \frac{1}{3}\pi r^2 h \\
&= \quad \frac{1}{3} \times \pi \times 4^2 \times 12 \\
&= \quad 64\pi \text{ cm}^3
\end{aligned}$$

(c)   The remaining metal in the hemisphere is melted down and recast into cones of the same
dimensions as the cone above. How many cones can be formed from the remaining metal?

$$\begin{aligned}
\text{Remaining metal} \quad &= \quad 1152\pi - 64\pi \\
&= \quad 1088\pi \\
\text{Volume of cone} \quad &= \quad 64\pi \\
\text{Number of cones} \quad &= \quad \frac{1088\pi}{64\pi} \\
&= \quad 17
\end{aligned}$$

or

$$\begin{aligned}
\text{Number of cones} \quad &= \quad \frac{1152\pi}{64\pi} - 1 \\
&= \quad 18 - 1 \\
&= \quad 17
\end{aligned}$$

[40]

Question 15   **20 marks**

The dimensions of two solid cylinders are shown in the diagrams below.
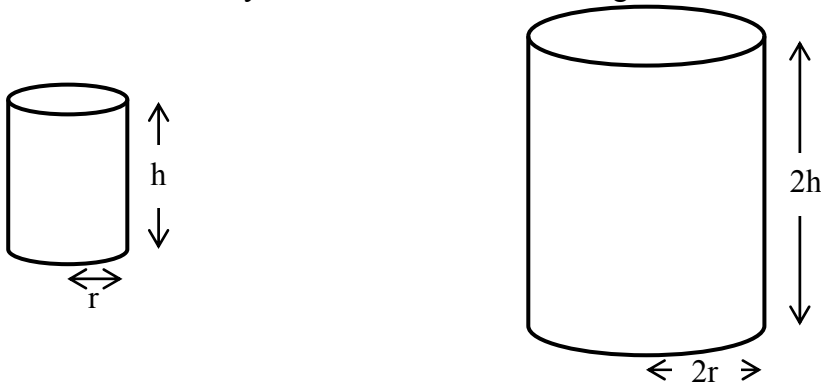
(a) Calculate the ratio of the curved surface area of the smaller cylinder to the curved surface area of the larger cylinder.

Curved surface area of a cylinder	=	$2\pi rh$
Curved surface area of small cylinder	=	$2 \times \pi \times r \times h$
	=	$2\pi rh$
Curved surface area of large cylinder	=	$2 \times \pi \times (2r) \times (2h)$
	=	$8\pi rh$
Ratio	=	$2\pi rh : 8\pi rh$
	=	$1 \cdot 4$

(b) Calculate the ratio of the volume of the smaller cylinder to the volume of the larger cylinder.

Volume of a cylinder	=	$\pi r^2 h$
Volume of small cylinder	=	$\pi \times r^2 \times h$
	=	$\pi r^2 h$
Volume of large cylinder	=	$\pi \times (2r)^2 \times (2h)$
	=	$8\pi r^2 h$
Ratio	=	$\pi r^2 h : 8\pi r^2 h$
	=	$1 : 8$

[41]