

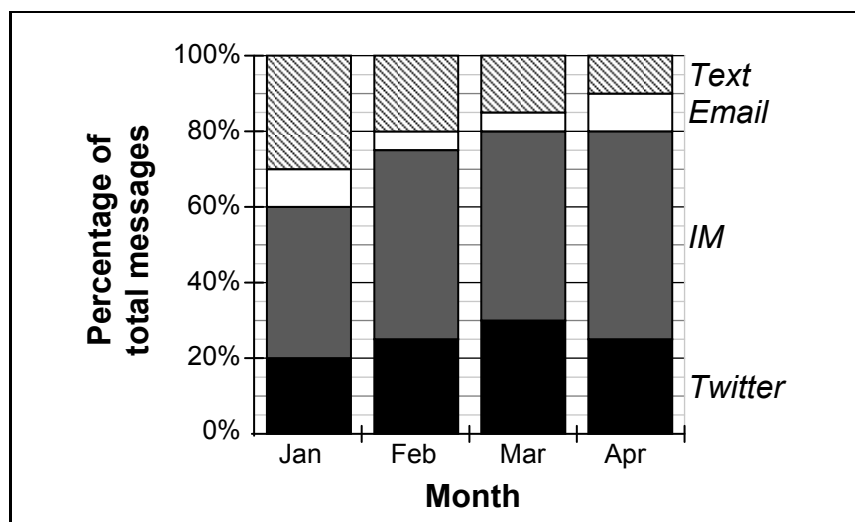
Question 1

15 Marks

The students in a class recorded how many messages they sent using different forms of messaging (*Text*, *Email*, *IM*, and *Twitter*) over four months.

Diagram 1 shows the percentage of messages sent using each form of messaging in each of the four months.

- (a) Using **Diagram 1**, complete the table below to show the percentage of messages sent using *Email* in each of the four months.

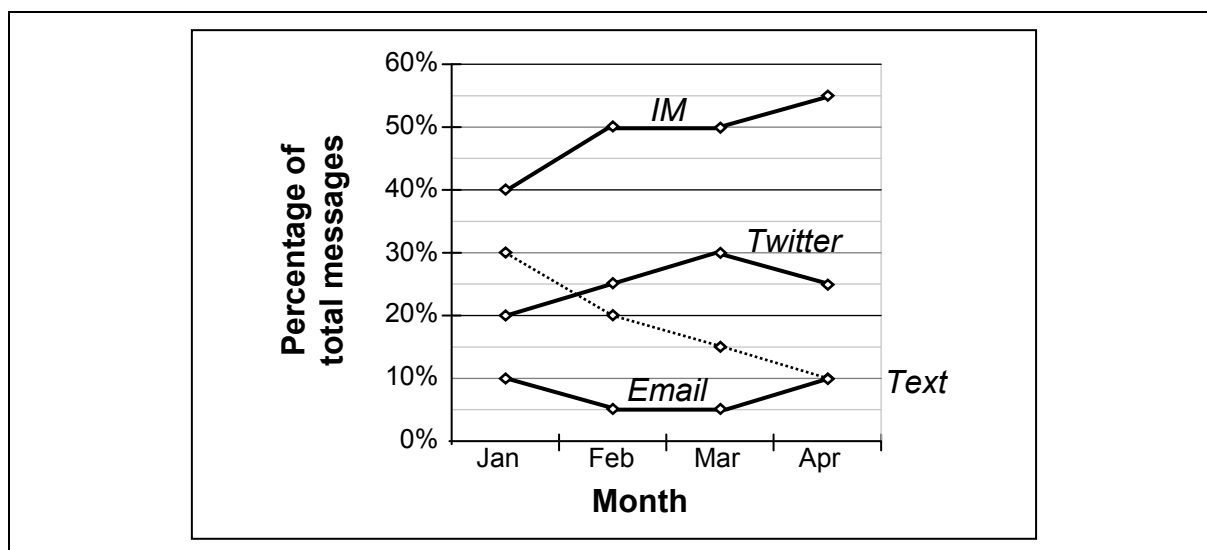


Month	Jan	Feb	Mar	Apr
Percentage of messages sent by <i>Email</i>	10%	5%	5%	10%

Diagram 2 shows the trend graph for *Text* over the four months.

- (b) Complete **Diagram 2** to show the trend graphs for *Email*, *IM*, and *Twitter* over the four months, using the data in **Diagram 1**.

Label each trend graph clearly.



Question 2

20 Marks

Below is a menu from a restaurant.

A 3-course dinner is made up of one *Starter*, one *Main Course*, and one *Dessert*.

<i>Starter</i>	<i>Main Course</i>	<i>Dessert</i>
<ul style="list-style-type: none"> • Soup • Garlic Bread • Onion Rings • Chowder 	<ul style="list-style-type: none"> • Pizza • Spaghetti • Steak • Lamb • Salmon 	<ul style="list-style-type: none"> • Cheesecake • Chocolate Cake • Ice-cream

- (a) Calculate the number of different 3-course dinners that can be ordered from this menu.

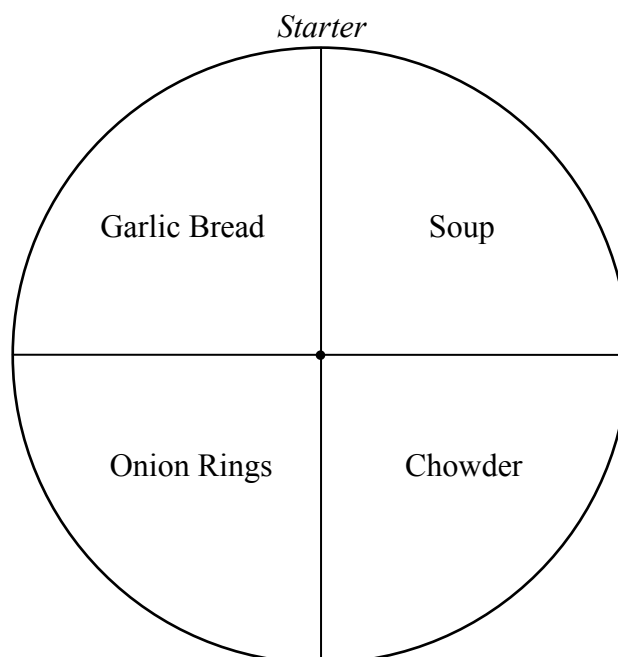
$$\text{Number of different 3-course dinners} = 4 \times 5 \times 3 = 60.$$

In the restaurant there is a circular poster for each course, where the options are represented by sectors of equal area.

The poster for the *Starter* is shown in the diagram on the right.

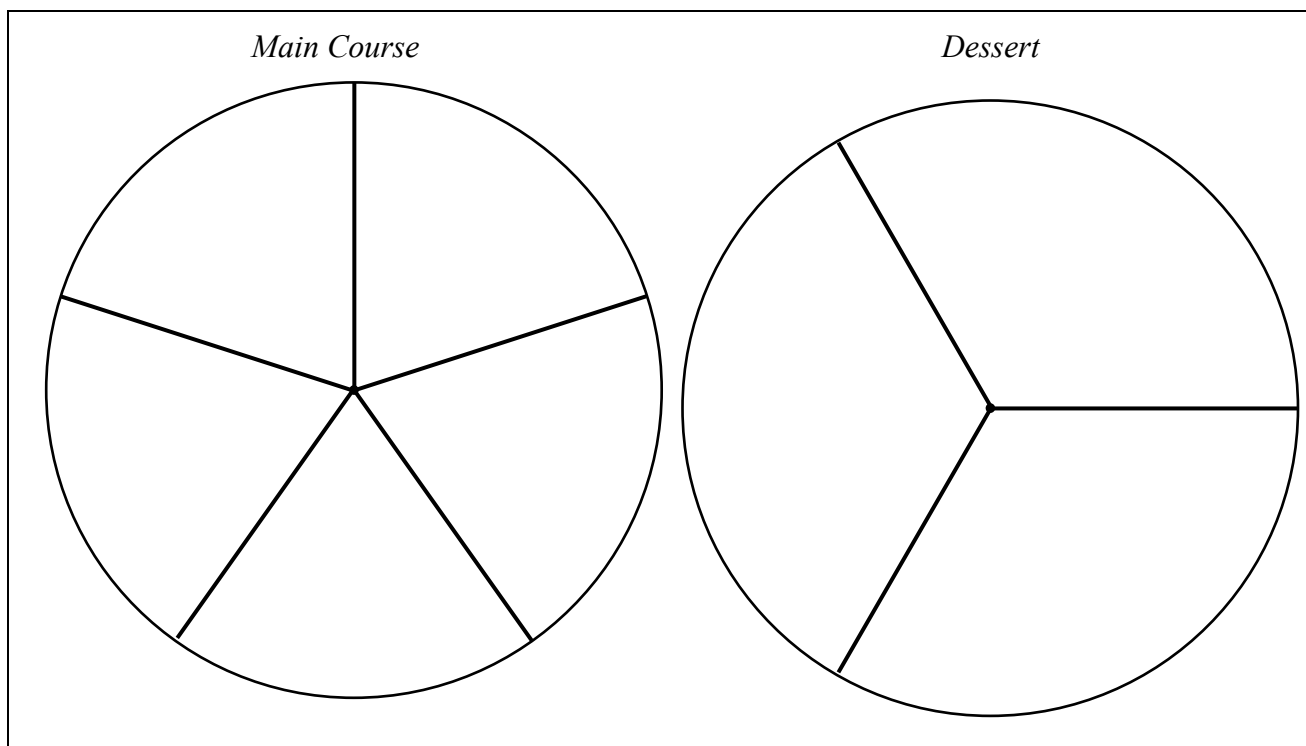
- (b) **Calculate** the angle of each sector in the *Main Course* poster and each sector in the *Dessert* poster.

Construct these sectors in the diagrams on the next page.



$$\text{Main Course Angle} = 360 \div 5 = 72^\circ.$$

$$\text{Dessert Angle} = 360 \div 3 = 120^\circ.$$



- (c) The owner of the restaurant wants to increase the number of different 3-course dinners that can be ordered.

She will add **either** one *Starter*, **or** one *Main Course*, **or** one *Dessert* to the menu.

Which should she add to make the number of different 3-course dinners that can be ordered as **large** as possible? Justify your answer fully.

Answer:	Dessert
Justification:	Extra Starter \Rightarrow $5 \times 5 \times 3 = 75$ dinners
	Extra Main Course \Rightarrow $4 \times 6 \times 3 = 72$ dinners
	Extra Dessert \Rightarrow $4 \times 5 \times 4 = 80$ dinners

Question 3

20 Marks

Eithne is going to survey post-primary Geography teachers in Ireland.

- (a) Some of the questions in the survey are shown in the table below.
Put a tick (✓) in the correct box to show what type of data each question would give.

Question	Numerical Continuous	Numerical Discrete	Categorical Nominal	Categorical Ordinal
How many Geography classes do you teach each week?		✓		
How much do you like teaching Geography? A lot A little Not at all <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>				✓
What subjects (other than Geography) do you teach?			✓	

Eithne is going to send her survey to some of the post-primary schools in Ireland.

- (b) Describe how Eithne could select a **Simple Random Sample** from all the post-primary schools in Ireland.

<p>Get a list of all of the post-primary schools in Ireland. [Step 1]</p> <p>Randomly select a number of them, e.g. using random number generator. [Step 2]</p>

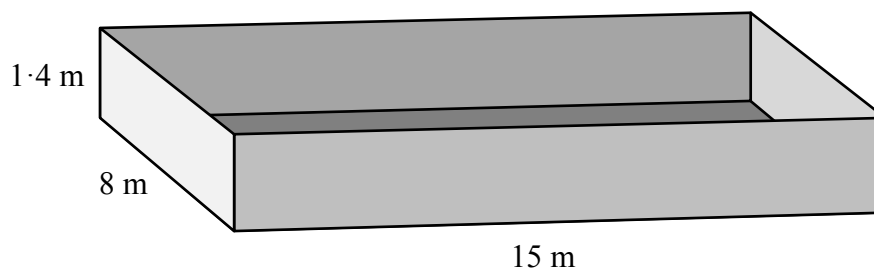
Eithne is considering sending her survey by email.

- (c) State **one advantage** and **one disadvantage** of using email to collect data.

<p>Advantage: Quick / Convenient / Cheap / <i>etc.</i></p>	<p>Disadvantage: Not everyone has email / May go to Spam / Faulty computer / <i>etc.</i></p>
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Question 4**30 Marks**

A swimming pool is 15 m long, 8 m wide, and 1.4 m deep, as shown in the diagram.



Harry says: “The area of the bottom of the swimming pool is $8 \times 15 = 120 \text{ cm}^2$.”

(a) Explain what is **wrong** with Harry’s answer.

The unit is cm^2 – it should be m^2 .

Harry will use $20 \text{ cm} \times 20 \text{ cm}$ tiles to cover the **inside** of the pool.

(b) Find the **minimum** number of tiles that Harry will need.

Area of bottom	$= 8 \times 15$	$= 120 \text{ m}^2$	or	$1,200,000 \text{ cm}^2$
Area of front & back sides	$= 2 \times 15 \times 1.4$	$= 42 \text{ m}^2$	or	$420,000 \text{ cm}^2$
Area of left & right sides	$= 2 \times 8 \times 1.4$	$= 22.4 \text{ m}^2$	or	$224,000 \text{ cm}^2$
Total Area		$= 184.4 \text{ m}^2$	or	$1,844,000 \text{ cm}^2$
Total Area of 1 tile	0.2×0.2	$= 0.04 \text{ m}^2$	or	400 cm^2
Number of tiles	$= 184.4 \div 0.04$ or $1,844,000 \div 400 = 4610$			

The surface of the water in the swimming pool is 10 cm below the top of the pool.

(c) Find the volume of water in the swimming pool.

$$\text{Volume} = 15 \times 8 \times 1.3 = 156 \text{ m}^3$$

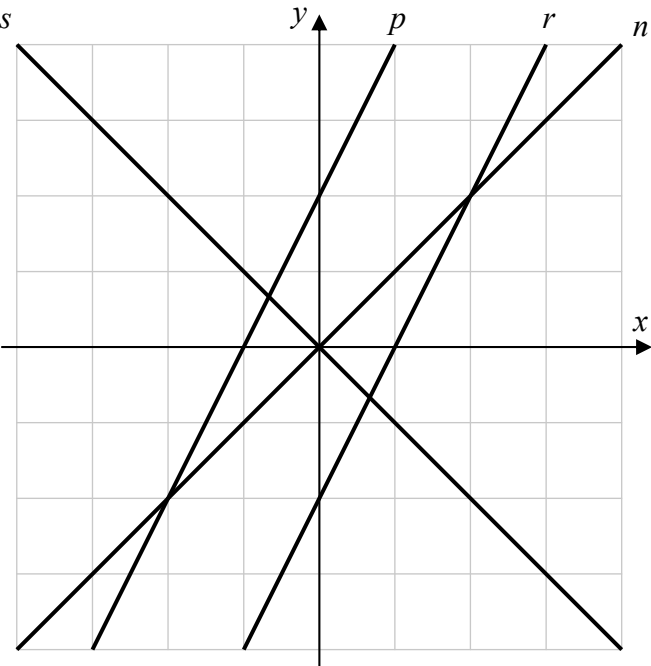
OR

$$\text{Volume} = 1500 \times 800 \times 130 = 156,000,000 \text{ cm}^3$$

Question 5

25 Marks

The co-ordinate diagram below shows the lines n , p , r , and s .
The table shows the equation of each line.



Equation	Line
$y = 2x - 4$	r
$y = x$	n
$y = -x$	s
$y = 2x + 4$	p

- (a) Write the letters n , p , r , and s into the table to match each line to its equation.

[See table above]

Complete the following sentences. Write one of the letters n , p , r , or s in each box.

- (b) You can use a **translation** to map the line p or r onto the line r or p .
- (c) You can use an **axial symmetry** in the y -axis to map the line n or s onto the line s or n .
- (d) The line s or n is mapped onto itself under **central symmetry** in the point $(0, 0)$.

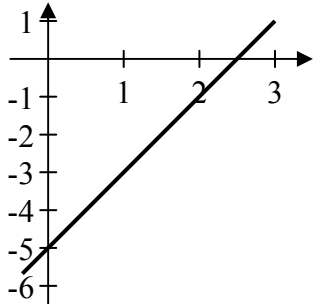
Question 6**20 Marks**

The equation of the line l is $5 + y - 2x = 0$.

- (a) Find the co-ordinates of the points where l cuts the axes.

l cuts the x -axis at $(2.5, 0)$	l cuts the y -axis at $(0, -5)$
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- (b) Find the **slope** of the line l .

$y = 2x - 5 \quad \Rightarrow \quad \text{slope} = 2$
OR
$m = \frac{-5 - 0}{0 - 2.5} = 2$
OR
<div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> $\begin{aligned} \text{slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{5}{2.5} \\ &= 2 \end{aligned}$ </div> </div>

The line j goes through the point $(11, 6)$ and is **perpendicular** to the line l .

- (c) (i) Write down the **slope** of the line j .

Slope of $j = -\frac{1}{2}$.

- (ii) Find the **equation** of the line j .

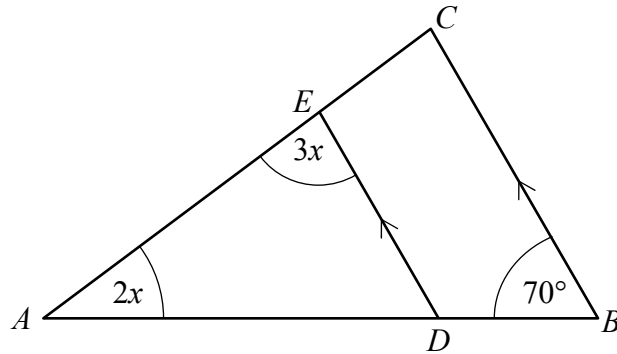
Equation of j :	$y - 6 = -\frac{1}{2}(x - 11)$
	$\Rightarrow x + 2y - 23 = 0.$

Question 7**15 Marks**

The diagram shows the triangle ABC .

DE is parallel to BC .

The sizes of some of the angles are shown.



- (a) Find the value of x .

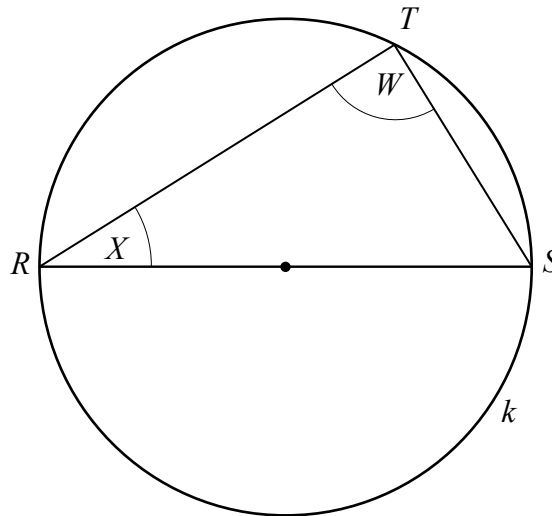
$2x + 3x + 70^\circ$	$=$	180°
$\Rightarrow 5x$	$=$	110°
$\Rightarrow x$	$=$	22°

- (b) Given that $|AE| = 100$, $|AC| = 130$, and $|DE| = 74$, find the value of $|BC|$.

$\frac{ BC }{130}$	$=$	$\frac{74}{100}$
$\Rightarrow 100 BC $	$=$	9620
$\Rightarrow BC $	$=$	96.2

Question 8**15 Marks**

The diagram shows the triangle RST inscribed in the circle k .
The line segment $[RS]$ is a **diameter** of the circle.



Gavin says: “The size of the angle W **must** be 90° .”

- (a) State one result on your course (a theorem or a corollary) that shows that Gavin is correct.

The angle at the centre of a circle is twice the angle at the circumference standing on the same arc [Theorem 19].

OR

Each angle in a semi-circle is a right angle [Corollary 3].

$|ST| = 10$ and $|RS| = 30$.

- (b) Using this information, and trigonometry, find the size of $\angle X$.
Give your answer in degrees, correct to one decimal place.

$$\begin{aligned} \sin X &= \frac{10}{30} \\ \Rightarrow X &= \sin^{-1}\left(\frac{10}{30}\right) \\ &= 19.47... \\ &= 19.5^\circ \text{ (1 decimal place)} \end{aligned}$$

Question 9**30 Marks**

A class of 25 students was surveyed to find out how many *WhatsApp* messages they each sent in a particular week. The results are shown in the table below.

Number of messages	0 – 30	30 – 50	50 – 70	70 – 100	100 – 160
Number of students	1	2	10	7	5

Note: 30 – 50 means at least 30 but less than 50, etc.

- (a) A student is picked at random from the class.
Find the probability that this student sent 50 or more messages.

$$\begin{aligned}
 \text{50 or more messages:} & \quad 10 + 7 + 5 = 22 \\
 \text{Probability of } \geq 50 & \quad = \frac{22}{25} \text{ or } 0.88.
 \end{aligned}$$

- (b) A student is picked at random from those who sent 50 or more messages.
Find the probability that this student sent 50 – 70 messages.

$$\begin{aligned}
 \text{50 or more messages:} & \quad 10 + 7 + 5 = 22 \\
 \text{Probability of } 50 - 70 & \quad = \frac{10}{22} \text{ or } \frac{5}{11} \text{ or } 0.4545\dots
 \end{aligned}$$

- (c) Using mid-interval values, estimate the **mean** number of messages sent per student.

$$\begin{aligned}
 \text{Mean} & \quad = \frac{(1)(15) + (2)(40) + (10)(60) + (7)(85) + (5)(130)}{1 + 2 + 10 + 7 + 5} \\
 & \quad = \frac{15 + 80 + 600 + 595 + 650}{25} \\
 & \quad = \frac{1940}{25} = \frac{388}{5} \text{ or } 77.6.
 \end{aligned}$$

The students also found the **total** number of *WhatsApp* messages they sent in this particular week.

- (d) Use the data in the table to find the **smallest** value that this total could be.

$$\begin{aligned}
 & (0 \times 1) + (30 \times 2) + (50 \times 10) + (70 \times 7) + (100 \times 5) \\
 = & \quad 1550.
 \end{aligned}$$

Question 10**15 Marks**

There are 10 students in a class. All 10 of them sat a test.

The table below shows the **mean** mark, the **median** mark, and the **range** of the marks on the test.

	Results on the test	Answers to part (b)
Mean mark	25.1	27.1
Median mark	24	26
Range of the marks	14	14

32 was the **highest mark** got by a student on the test.

- (a) Use the range to find the **lowest mark** got by a student on the test.

$$\begin{aligned}\text{Lowest mark} &= 32 - 14 \\ &= 18.\end{aligned}$$

An external examiner suggested that 2 be added onto each student's mark.

- (b) Find what the **mean**, the **median**, and the **range** would be in this case.
Fill your answers into the table above.

[See table above]

Bob says: "Whenever the median of a list of numbers is 24, then at least one of the numbers in the list **must be 24**."

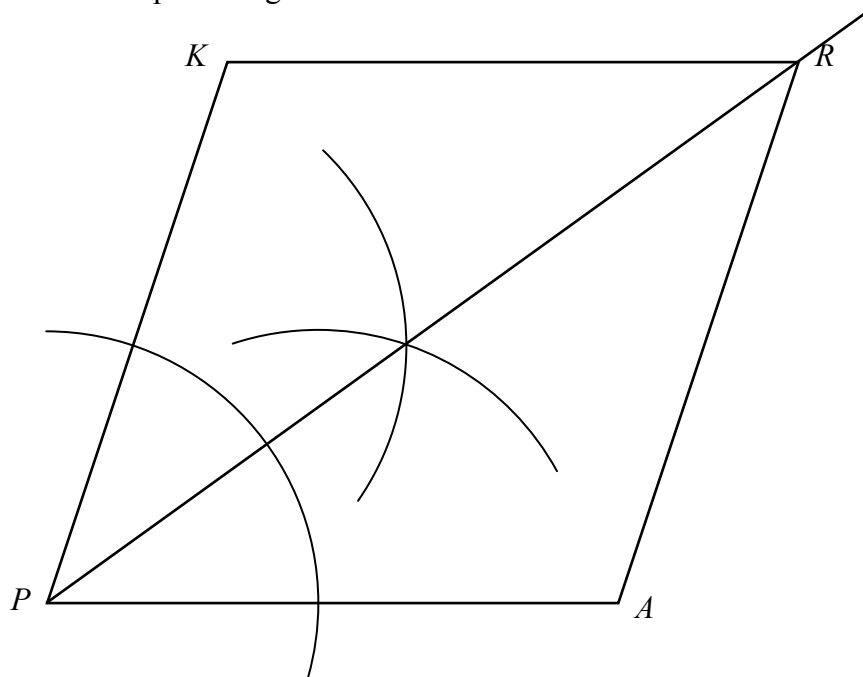
- (c) Give an example to show that Bob is **not** correct.

$$\begin{aligned}\text{List} &= 7, 23, 25, 96. \\ \text{Median} &= 24.\end{aligned}$$

Question 11

20 Marks

The diagram below shows the parallelogram $PARK$.



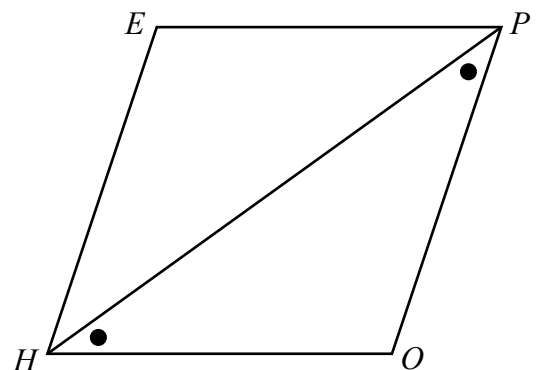
- (a) **Construct the bisector of $\angle KPA$** on the diagram above, using only a compass and straight edge. Show your construction lines clearly.

[See diagram above]

The diagram on the right shows the parallelogram $HOPE$.
 $|\angle PHO| = |\angle OPH|$, as shown.

- (b) **Prove** that all four sides of the parallelogram are equal in length.

Give a reason for each of the statements that you make in your proof.



$ HO = EP $ & $ HE = OP $	Opposite sides of parallelogram	[Step 1]
$ HO = OP $	Isosceles triangle PHO	[Step 2]
$\Rightarrow HE = OP = HO = EP $		[Step 3]

OR

$$|\angle PHO| = |\angle HPE| \dots\dots\dots \text{Alternate angles}$$

$$|\angle OPH| = |\angle EHP| \dots\dots\dots \text{Alternate angles}$$

$$|HP| = |HP| \dots\dots\dots \text{Common side}$$

$$\therefore \triangle HOP \equiv \triangle PEH \dots\dots\dots \text{A.S.A}$$

$$\therefore |OP| = |HE| \text{ and } |HO| = |EP| \dots\dots \text{Corresponding sides} \quad [\text{Step 1}]$$

$$|HO| = |OP| \dots\dots\dots \text{Isosceles triangle } PHO \quad [\text{Step 2}]$$

$$\Rightarrow |HE| = |OP| = |HO| = |EP| \quad [\text{Step 3}]$$

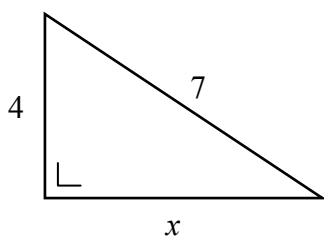
Question 12

20 Marks

- (a) The triangle PQR has sides of length 8, 11, and y .
Write down **one** value of y for which $\triangle PQR$ is an **isosceles** triangle.

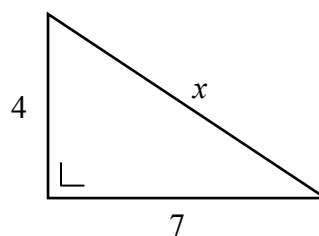
$$y = \boxed{8 \text{ or } 11}$$

- (b) The triangle STU has sides of length 4, 7, and x .
Find the **two** values of x for which $\triangle STU$ is a **right-angled** triangle.
Give each answer in surd form.



$$4^2 + x^2 = 7^2$$

$$\Rightarrow x = \sqrt{33}$$



$$4^2 + 7^2 = x^2$$

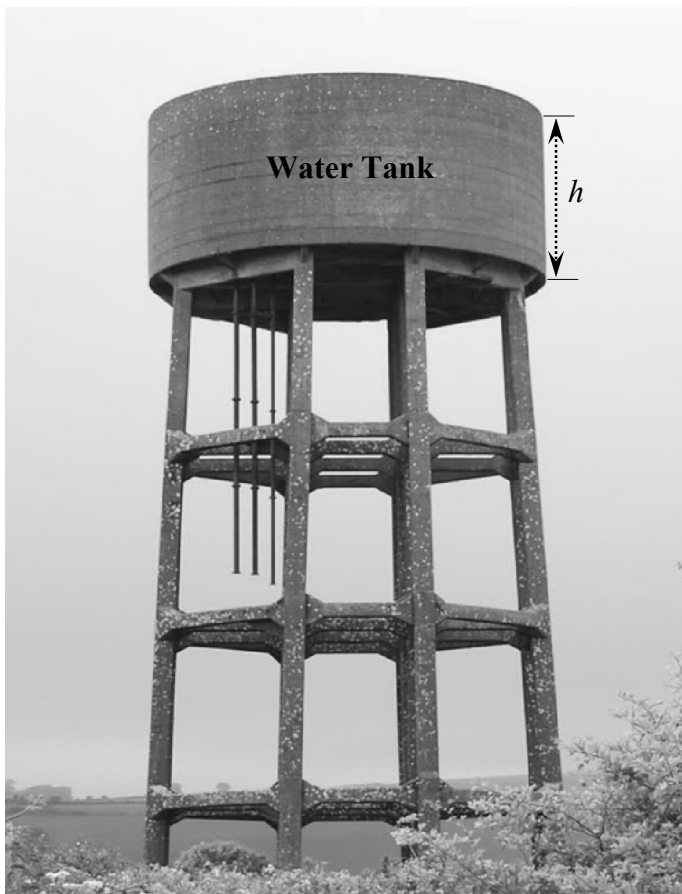
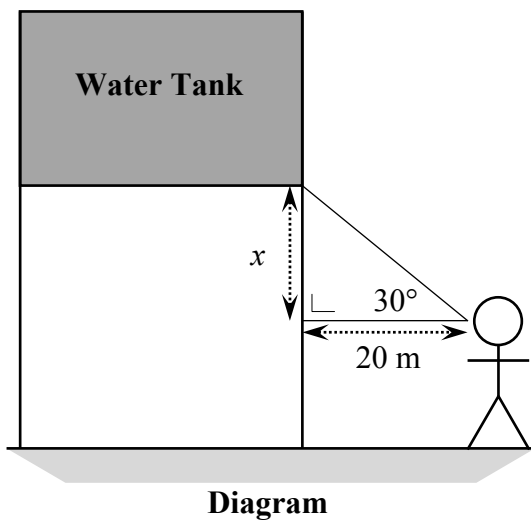
$$\Rightarrow x = \sqrt{65}$$

Question 13

35 Marks

Miriam is trying to find the volume of the water tank shown in the photograph on the right.

She takes some measurements and draws a diagram. Part of her diagram is shown below.



Source: www.watertowersofireland.com. Altered.

- (a) Using the diagram, find the value of x . Give your answer in metres, correct to two decimal places.

$$\frac{x}{20} = \tan 30^\circ \quad \Rightarrow \quad x = 11.547 = 11.55 \text{ m (2 decimal places)}$$

- (b) The angle of elevation to the bottom of the water tank is 30° , as shown in the diagram. The angle of elevation to the top of the water tank is 38° . Find the distance marked h on the photograph. Give your answer correct to one decimal place.

	$\frac{x+h}{20} = \tan 38^\circ$ $\Rightarrow x + h = 20 \tan 38^\circ = 15.626$ $\Rightarrow h = 15.63 - 11.55$ $= 4.08 = 4.1 \text{ m (1 decimal place)}$
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- (c) Hugh is also trying to find the volume of the water tank.
He estimates that the height, h , is 4.5 m.

By taking **measurements** from the photograph and performing **calculations**, use Hugh's value of h to estimate the volume of the water tank as accurately as you can.

Give your answer correct to the nearest m^3 .

State clearly what shape you are taking the water tank to be.

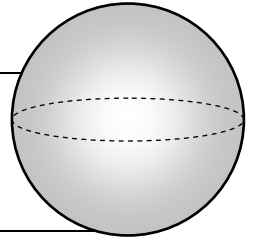
<i>Shape of water tank:</i> <div style="display: flex; justify-content: space-between; align-items: center;"> Cylinder [Step 1] </div>	
<i>Measurements from photograph (label each measurement):</i> <div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: center;"> Diameter = 5.2 cm Height = 2.2 cm </div> <div style="text-align: center;"> [\Rightarrow Radius = 2.6 cm] [Step 2] </div> </div>	
<i>Calculations:</i> <div style="display: flex; justify-content: space-between; align-items: center;"> $\frac{\text{Actual radius}}{2.6} = \frac{4.5}{2.2} \Rightarrow \text{Actual radius} = 5.318... \text{ m}$ [Step 3] </div> <div style="display: flex; justify-content: space-between; align-items: center;"> $\begin{aligned} \text{Volume of cylinder} &= \pi \times r^2 \times h \\ &= \pi \times (5.318)^2 \times 4.5 \\ &= 399.81... \end{aligned}$ <div style="text-align: right;"> [Step 4] [Step 5] </div> </div>	
<i>Volume of water tank, in m^3:</i> <div style="text-align: center;"> 400 m^3 (nearest m^3) </div>	

Question 14**20 Marks**

A small sphere has a radius of 1.5 cm.

- (a) Find the **volume** of the small sphere. Give your answer in cm^3 , in terms of π .

$$\text{Volume of small sphere} = \frac{4}{3} \times \pi \times r^3 = \frac{9}{2} \pi \text{ cm}^3$$



The volume of a large sphere is three times the volume of the small sphere.

- (b) Find the **radius** of the large sphere.

Give your answer in cm, in the form $\frac{a \sqrt[3]{a}}{b}$, where $a, b \in \mathbb{N}$.

$$\text{Radius of large sphere} = R$$

$$\text{Volume of large sphere} = \frac{4}{3} \times \pi \times R^3 = 3 \times \left(\frac{9}{2} \pi \right) = \frac{27}{2} \pi$$

$$\Rightarrow R^3 = \frac{27 \times 3}{2 \times 4} = \frac{81}{8}$$

$$\Rightarrow R = \sqrt[3]{\frac{81}{8}} = \frac{3\sqrt[3]{3}}{2} \text{ cm}$$