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Petroleum production optimization – A static or dynamic problem?

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ABSTRACT

This paper considers the upstream oil and gas domain, or more precisely the daily production optimization problem in which production engineers aim to utilize the production systems as efficiently as possible by for instance maximizing the revenue stream. This is done by adjusting control inputs like choke valves, artificial lift parameters and routing of well streams. It is well known that the daily production optimization problem is well suited for mathematical optimization. The contribution of this paper is a discussion on appropriate formulations, in particular the use of static models vs. dynamic models. We argue that many important problems can indeed be solved by repetitive use of static models while some problems, in particular related to shale gas systems, require dynamic models to capture key process characteristics. The reason for this is how reservoir dynamics interacts with the dynamics of the production system.

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1. Introduction

Petroleum assets are sizable amounts of hydrocarbons which are trapped in appropriate underground geological structures. After discovering and deciding on developing an asset wells are drilled and production starts. Typically the production lifetime of an asset contains three stages; the ramp-up, plateau and decline phases. During the ramp-up phase new wells are drilled and completed while the production rate steadily increases. During the plateau phase production stays fairly constant. New wells may, however, still be drilled while others are abandoned. Dry shale-gas fields require a higher drilling frequency than conventional petroleum assets to maintain the plateau phase due to the early decline in productivity of these wells. Typically reservoir fluid composition changes with more and more less valuable products like water entering the production stream. Further, fluids may be injected to maintain reservoir pressure. During the decline phase production can no longer be maintained, thus production rates drop.

Asset production planning is performed on different horizons from a life-cycle perspective to daily production planning, thus decisions may be described by a control hierarchy as shown in Fig. 1. The uppermost level includes life-cycle related decisions such as selecting an investment strategy, appropriate technologies and an operations model.

The second highest level, level 2 in Fig. 1, typically refers to decision horizons of one to five years, even though this may vary significantly, and includes choices on production strategies. This includes drilling schedules, the location and completion design of new wells, injection rates and fluids, and target production rates to mention a few. Additionally for shale-gas fields is the decision of which wells to refracture to stimulate existing wells (Cafaro et al., 2016). Level 2 decisions are usually supported by simulator studies using high fidelity reservoir models.

Level 3 takes us to the operations domain since the planning horizon ranges from a few hours to a week, thus from a process systems perspective this is equivalent to real-time optimization (RTO). We denote this by daily production optimization (DPO). There are two important contrasts between level 2 and level 3 formulations. First, the shorter time horizon most often allows for the use simple reservoir models on level 3, and, second it is critical to include the production network in DPO formulations. The reason is that DPO production bottlenecks are normally found both in the reservoir as well as in the network.

The lowest level in Fig. 1 includes an automatic control system, which normally is implemented in proprietary control systems. This includes control loops for flow, pressure and level control functions to mention some of its functionality. Due to lower profit margins and land-based operations with easier access, the control and automation system is often simpler for shale-gas wells compared to offshore oil and gas installations. It may also be noted that there will always exist a separate safety system, which is automatically activated in emergency situations.

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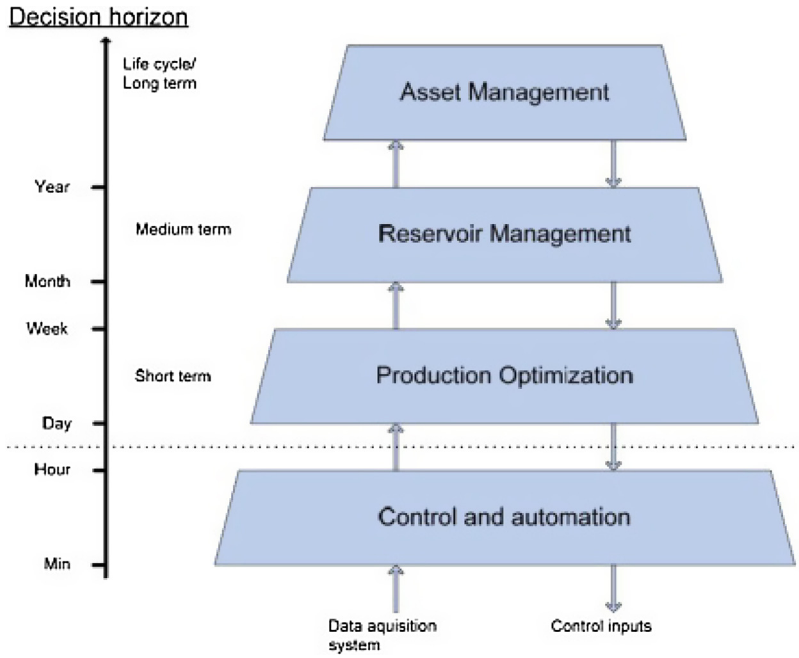

Fig. 1. A multilevel control hierarchy.

There is a clear business case for mathematical optimization in DPO in the sense that decision support systems based on this methodology report production increases in the range of 1–4% (Stenhouse et al., 2010; Teixeira et al., 2013). These improvements are more pronounced for fields in the late plateau and decline phases than earlier since the DPO bottleneck structure tends to become more complicated with time, e.g. due to increased water and gas production, and reduced reservoir pressure.

This paper contributes to *DPO formulations* by discussing static and dynamic formulations. We define DPO formulations with static models as *static DPOs*, and DPO formulations with one or more dynamic models and a control horizon as *dynamic DPOs*. The paper continues with relevant references before an efficient formulation of the DPO problem, which is new and captures a broad class of static and dynamic DPO problems, is presented. Subsequently two case studies, one based on a dynamic formulation and the other on a static formulation, are presented as a prelude to a discussion section. Towards the end some conclusions are presented.

1.1. Mathematical programming in daily production optimization

There is a broad class of technologies for DPO to increase production or reduce down-time. This includes virtual flow metering systems, flow assurance systems, and more recently condition and performance monitoring systems. These are real-time monitoring systems that have in common that they attempt to increase the operational awareness of the operator. In the subsequent survey we focus on methods for daily production optimization using the framework of mathematical programming.

The dynamics of conventional reservoirs are usually slow. Thus, in the DPO context reservoir conditions around wells change slowly meaning that pressures and fluids change only marginally from one day to another. There are numerous references on the use of a strategy in which a static formulation is re-optimized according to changing conditions, for instance a couple of times a day. To the authors' knowledge Kosmidis et al. (2004, 2005) were the first to formulate the static DPO problem as a mixed integer nonlinear programming (MINLP) problem in which discrete variables allude to routing decisions in a flow network. Further, this paper applies a piecewise linearization approach to approximate the original MINLP model with a mixed integer linear programming (MILP) model to take advantage of the high efficiency of state-of-the-art MILP-solvers. Some other contributions along these lines are Misener et al. (2009) and Codas and Camponogara (2012). Piecewise linear approximation scales badly for nonseparable functions, thus different multidimensional piecewise models for DPO were compared in Silva and Camponogara (2014) indicating that SOS2 models and MILP models with a logarithmic number of binary variables were preferable choices. Other surrogate models have also been successfully applied, one example being B-spline models (Grimstad et al., 2016). In this case an MINLP formulation was retained, however, with cubic B-spline nonlinearities only. This property can be exploited to improve MINLP solver efficiency. The latter reference also includes energy balances to account for temperature sensitive flow velocity constraints in gas dominated flow.

DPO may include many wells and a large flow network. In these cases it is particularly important to exploit structure to reduce runtime since it may be exceedingly long. Flow networks can often be partitioned into smaller parts with only a few linking constraints. Gunnerud and Foss (2010) analyse a large problem based on a Statoil field where Lagrange relaxation and Dantzig-Wolfe decomposition is applied with significant reduction in runtime compared to a formulation without decomposition. In addition SOS2 models were used as proxy models to reduce runtime compared to more comprehensive models.

Uncertainty is a severe DPO challenge, in particular model errors in the well models. This was first handled explicitly in Bieker et al. (2007). In a later paper (Elgsæter et al., 2010) uncertainty is accounted for by iteratively changing the setpoints until the capacity constraints, which should be active, are utilized to their limits. This approach assumes that there exists measurements to monitor capacities. It may be noted there are very few papers on uncertainty in DPO. This is quite different from the reservoir management domain where there exists numerous such publications.

There is limited work on dynamic DPO, i.e. using dynamic models. A recent paper (Codas et al., 2016) explores integration of low level control with nonlinear model predictive control for DPO. An even more ambitious approach is taken in Krishnamoorthy

Table 1
Utility sets.

Set	Description
$\mathbf{N}$	Set of nodes $i \in \mathbf{N}$.
$\mathbf{E}$	Set of edges $e = (i, j) \in \mathbf{E}$, with $i, j \in \mathbf{N}$.
$\mathbf{K}$	Timesteps $\mathbf{K} = \{0, 1, \ldots, N\}$. $\mathbf{K}^- = \mathbf{K} \setminus N$.
$\mathbf{E}_i^{in}$	Edges entering node i, i.e. $\mathbf{E}_i^{in} = \{e : e = (j, i) \in \mathbf{E}\}$.
$\mathbf{E}_i^{out}$	Edges leaving node i, i.e. $\mathbf{E}_i^{out} = \{e : e = (i, j) \in \mathbf{E}\}$.
$\mathbf{E}^{snk}$	Edges entering a sink node, i.e. $\mathbf{E}^{snk} = \{e : e = (i, j),\ \mathbf{E}_j^{out} = \emptyset\}$.
$\mathbf{E}^d$	Set of discrete edges, i.e. $\mathbf{E}^d \subset \mathbf{E}$.
$\mathbf{N}^d$	Nodes with discrete leaving edges, i.e. $\mathbf{N}^d = \{i : i \in \mathbf{N}, \mathbf{E}_i^{out} \subset \mathbf{E}^d\} \subset \mathbf{N}$.
$\mathbf{R}$	Set of phases – oil, gas and water {oil, gas, wat}.

et al. (2016) where dynamic DPO is combined with uncertainty by using a scenario-based optimization strategy. These papers include dynamics in pipelines. However, reservoir dynamics are not included. Specific applications that require dynamic DPO will be discussed later, one example is the case study on shale-gas. Corresponding references will then also be included.

2. A general DPO formulation

Consider the petroleum production system illustrated in Fig. 2, in which the produced fluid flows through well bores, manifolds, and flowlines, to finally enter the separators. At the separators the fluid phases, typically including oil, gas and water, are separated. It may be noted that the process diagram for the processing section in practice is far more complicated than shown in Fig. 2. Oil and/or gas are exported separately through transmission pipelines. One important feature of the production system is that each well flow can be routed to one of the flowlines by configuring the on/off valves in the manifold.

Upstream of the wells lies the reservoir. Thus, the reservoir defines the inflow boundary conditions on the production system. The downstream boundary conditions are given by the (nearly) constant pressure in the separators, which is maintained by regulatory control. In this paper we will mainly be concerned with production systems where the upstream and downstream boundaries are placed in the reservoir and the inlet separator, respectively.

We can now formulate a fairly general network optimization problem. The topology of the network is represented by a directed graph $G = (\mathbf{N}, \mathbf{E})$, with nodes $\mathbf{N}$ and edges $\mathbf{E}$ (Ahuja et al., 1993). In the sequel we adopt the notation in Grimstad et al. (2016). There are three mutually exclusive sets of nodes, $\mathbf{N}$, which all represent a junction: source nodes ($\mathbf{N}^{src}$), sink nodes ($\mathbf{N}^{snk}$) and intermediate nodes ($\mathbf{N}^{int}$), the latter representing junctions in the graph. An edge $\mathbf{E}$ connects two nodes and represents a pipe segment such as a well bore or a flowline, a valve, or an active element like a pump. A subset of edges, $\mathbf{E}^d$, represents chokes and on/off valves. These edges have two states: either open or closed. Thus, discrete edges are used to route the flow through the network by restricting the flow through the valve. It is advantageous to define certain utility sets, and certain requirements need to be placed on the graph structure, cf. Grimstad et al. (2016). Some utility sets are defined in Table 1 in order to compactify notation.

Alternative formulations like compositional models instead of three phases is sometimes necessary, especially for condensate reservoirs. The variables of the problem formulation are listed in Table 2. Note that the flow rates $\{q_{rek}\}$ are given as mass flow rates or as volumetric flow rates at standard conditions. For brevity, the phase flow rates on an edge $e \in \mathbf{E}$ at time $k \in \mathbf{K}$ are collectively denoted $\mathbf{q}_{ek}$, that is, with an oil, gas, and water phase, $\mathbf{q}_{ek} = [q_{\text{oil},ek}, q_{\text{gas},ek}, q_{\text{wat},ek}]^T$.

To efficiently represent edges $\mathbf{E}$ of the directed graph approach for modeling petroleum production networks, we introduce Boolean variables Y_{ek} and formulated the DPO problem using generalized disjunctive programming (GDP) (Raman and Grossmann, 1994; Grossmann and Trespalacios, 2013). GDP modeling enables flexibility in formulation of optimization problems with logical structures and conditions. Moreover, starting with a GDP model rather than a weak, ad-hoc mixed-integer formulation enables the possibility of posing and evaluating various mixed integer reformulations of the GDP, or solving the GDP with specialized logic-based methods (Lee and Grossmann, 2000). Consequently, to capture the connections between the logical part and the constraints of the DPO problem, we formulate the following GDP model $\mathbf{P}$:

$$\text{maximize} \quad z = \sum_{r \in \mathbf{R}} \sum_{e \in \mathbf{E}^{snk}} \sum_{k \in \mathbf{K}} g_{rk}(q_{rek}) \tag{1}$$

subject to

$$\sum_{e \in \mathbf{E}_i^{in}} q_{rek} = \sum_{e \in \mathbf{E}_i^{out}} q_{rek}, \quad \forall r \in \mathbf{R}, i \in \mathbf{N}^{int}, k \in \mathbf{K} \tag{2}$$

$$\zeta_{rik}(\mathbf{q}_{e,k+1}, \mathbf{q}_{ek}, p_{ik}) = 0, \ \forall r \in \mathbf{R}, \forall e \in \mathbf{E}_i^{out}, i \in \mathbf{N}^{src}, k \in \mathbf{K}^- \tag{3}$$

$$\mathbf{q}_{e,0} = \text{given}, \ \forall e \in \mathbf{E}_i^{out}, i \in \mathbf{N}^{src} \tag{4}$$

$$p_{ik} = \text{const.}, \ \forall i \in \mathbf{N}^{snk}, k \in \mathbf{K} \tag{5}$$

$$p_{ik} - p_{jk} = f_e(\mathbf{q}_{ek}, p_{ik}), \ \forall e \in \mathbf{E} \backslash \mathbf{E}^d, k \in \mathbf{K} \tag{6}$$

$$\begin{bmatrix} Y_{ek} \\ p_{ik} - p_{jk} = f_e(\mathbf{q}_{ek}, p_{ik}) \end{bmatrix} \vee \begin{bmatrix} \neg Y_{ek} \\ \mathbf{q}_{ek} = 0 \end{bmatrix}, \ e \in \mathbf{E}^d, k \in \mathbf{K} \tag{7}$$

$$\left(\underset{e \in \mathbf{E}_i^{out}}{\veebar} Y_{ek} \right) \vee \left(\bigwedge_{e \in \mathbf{E}_i^{out}} \neg Y_{ek} \right), \ \forall i \in \mathbf{N}^d, k \in \mathbf{K} \tag{8}$$

$$Y_{ek} \in \{\text{True}, \text{False}\}, \ \forall e \in \mathbf{E}^d, k \in \mathbf{K} \tag{9}$$

$$\sum_{e \in \mathbf{E}^{snk}} q_{rek} \leq C_{rk}, \ \forall r \in \mathbf{R}, k \in \mathbf{K} \tag{10}$$

The objective function (1) is a sum of univariate, possibly nonlinear functions of flow rate. This general form allows for the inclusion of various cost and penalty terms. A particularly common objective in operational settings is the total oil production, which may be represented by $g_{rk}(q_{rek}) = q_{rek}$, for r = oil.

The constraints in (2) are the mass balances for each phase, for all the internal nodes in the system. The upstream boundary conditions are covered by constraints (3), with initial conditions in (4), while the downstream boundary conditions are given by the (piecewise) constant separator pressure in (5). Downstream conditions may vary. However, a constant separator pressure is quite common and is adopted here.

Eqs. (6) and (7) define the momentum balance, or pressure drop, across pipe segments. These are modelled as a function of inlet flow rates and inlet pressure. For a discrete edge, the momentum balance is only enforced when the edge is open; otherwise, when the edge is closed, the flow rate is required to be zero according to (7).

A node with discrete leaving edges may route the flow to *one or zero* of these edges. This logic is captured by the routing constraints in (8). Notice that these constraints do not allow for flow splitting, which is an uncommon configuration in production systems.[1] The final constraints in (10) are processing capacity constraints, typically included for the gas and water phase. Note that (9) defines the Boolean variable Y_{ek}.

[1] The extension of the formulation to allow flow splitting is not considered here as it raises the complexity of the formulation without contributing to our discussion.

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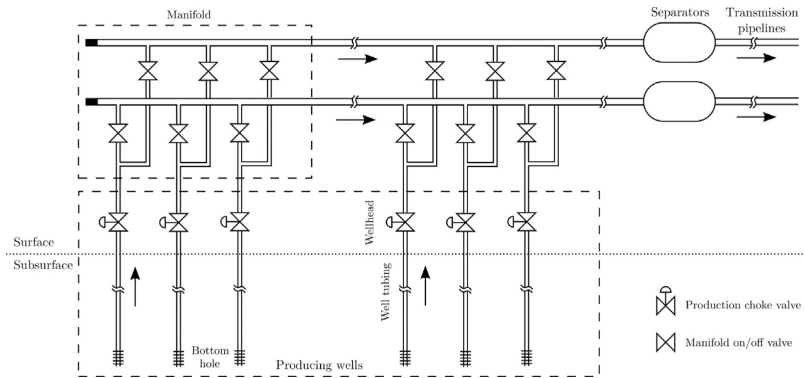

Fig. 2. A petroleum production system.

Table 2
Variables.

Variable	Description
p_{ik}	Pressure in node $i \in \mathbf{N}$ at time $k \in \mathbf{K}$.
q_{rek}	Flow rate of phase $r \in \mathbf{R}$ on edge $e \in \mathbf{E}$ at time $k \in \mathbf{K}$.
Y_{ek}	Boolean variable associated with edge $e \in \mathbf{E}^d$ at time $k \in \mathbf{K}$. The edge may be open (Y_{ek} = True) or closed (Y_{ek} = False).

2.1. Discussion

Upstream conditions are of particular interest to this study and will thus be elaborated upon. Problem **P** defines a dynamic optimization problem on a prediction horizon **K** where the dynamics enter through the reservoir model (3), where a natural approach is to embed this into a receding horizon optimization strategy. This implies that **P** is solved repetitively at each timestep, and that only the first control move is actually implemented. In situations where the reservoir dynamics can be neglected a quasi dynamic approach may be used in which **P** is simplified by limiting the prediction horizon to one time step and substituting (3) with a static model, i.e.,

$$\zeta_{rik}(\mathbf{q}_{ek}, p_{ik}) = 0, \ \forall r \in \mathbf{R}, \forall e \in \mathbf{E}_i^{\text{out}}, i \in \mathbf{N}^{\text{src}}, k = 0 \tag{11}$$

Further, (4) needs to be omitted in this case. Note that the static version of **P** is well known. Kosmidis et al. (2004) was the first to formulate a MINLP for the well oil rate allocation problem.

The static upstream inflow condition, which describes the mass flowrate from the reservoir into the well, may be linear or nonlinear depending on the actual well and reservoir characteristics near the well. Common models are linear productivity index models and nonlinear Vogel curves. We do not put any requirements on the properties of ζ_{rik}. We note, however, that it is common to assume continuity, but not necessarily differentiability. For cases where ζ_{rik} is non-differentiable it may be advantageous to approximate it by a piecewise defined surrogate model to obtain a formulation better suited for optimization.

There is a vast literature on dynamic reservoir models, also related to optimization, see e.g. Jansen et al. (2008) and references therein. Reservoir model (3) describes the well known class of black oil models, which include oil, water and gas phases. Such models are typically based on a spatial discretization of a PDE model (Aziz and Settari, 1979), and may for instance be solved using a Newton iteration scheme. It may be noted that there exists more complex models, like compositional models as well as simpler, surrogate models. Streamline models is an example of the latter.

The GDP formulation **P** generalizes many of the DPO problems cited in the references in Section 1.1, capturing a variety of DPO problems both with or without dynamics and with simple and complex network topologies. The problem formulation also enables several alternative solution approaches. Pertaining to a GDP model, the most common solution approach is to reformulate **P** as a (nonconvex) mixed-integer nonlinear programming (MINLP), while specialized logic-based algorithms may also be considered (e.g. Lee and Grossmann, 2000). Yet, since Eqs. (3), (6) and (7) often are embedded in one simulator or several independent simulators for each well and pipeline, it is usually necessary to apply a derivative-free method since gradient information is seldom available. On the other hand, by acknowledging the fact that each nonlinear model, (3), (6) and (7), only has a few (local) inputs and that all integer variables appear linearly, it makes sense to replace each well and pipeline simulator with a piecewise linear surrogate model. This transforms the MINLP problem into a MILP problem with all the benefits that come from such a formulation as alluded to in Section 1.1.

3. Case studies

We now present two quite different DPO problems. The first one is based on a shale-gas scenario while the second one refers to an offshore oil field.

3.1. Scheduling shale-gas wells with demand-side response

This case study, adopted from Knudsen et al. (2014), considers a small field with dry, mature shale-gas wells. Such wells have an unique ability to quickly recover from loss of production due to a well shut-in, thereby enabling operators to accommodate varying demands during periods of low demands, without incurring revenue losses due to depreciation (Knudsen et al., 2014). Together with their land-based nature, this property provides a potential to use such fields as a proxy or substitute for underground gas storage (Knudsen and Foss, 2017). As illustrated in Fig. 3, this enables bypassing of conventional, third-party underground gas storage, thereby increasing gas energy efficiency from source to end-user, in addition to reducing supply-related greenhouse gas emissions (Knudsen, 2016; Knudsen and Foss, 2017). In terms of DPO for shale-gas wells, this production strategy couples the gas demand side tightly to the upstream production, as well as integrating electric power generation into shale-gas well-scheduling.

In the case study, we consider 20 wells, all mature, mid-life wells producing at low rates, assumed to be located in proximity to a natural gas power plant (NGPP). An electric utility company

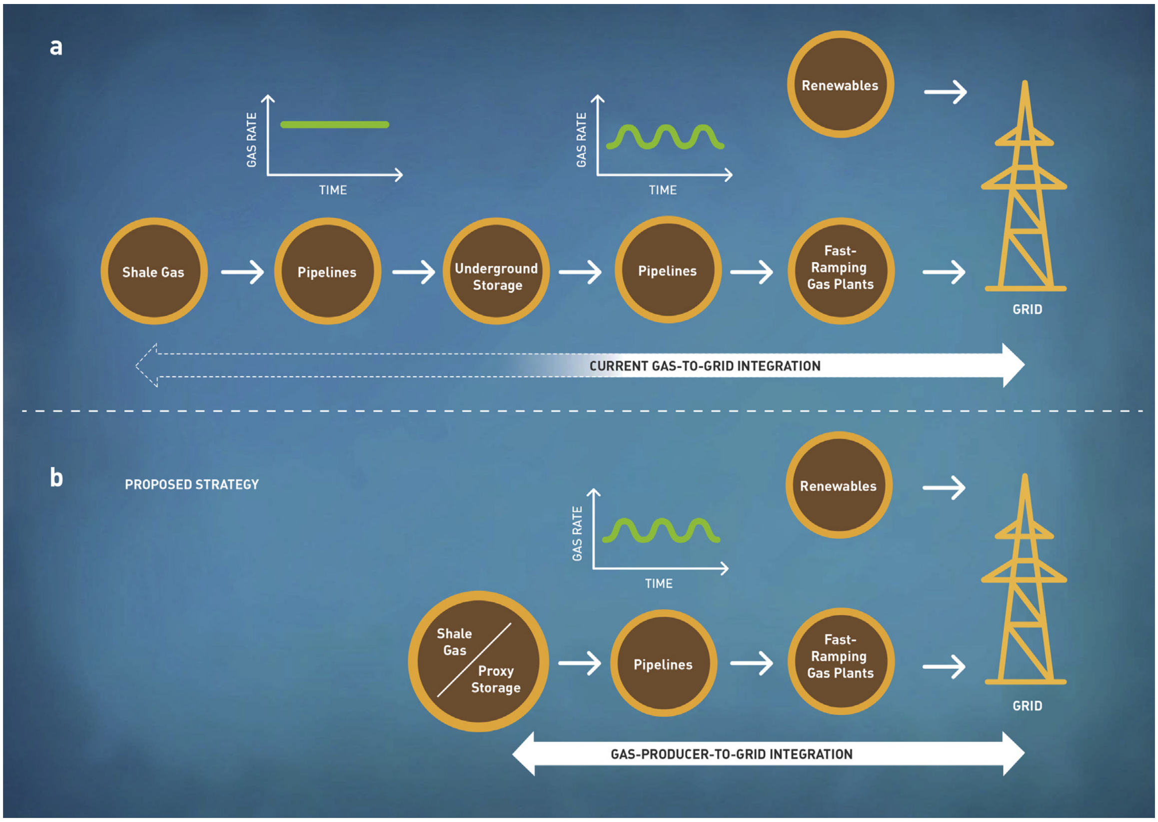

Fig. 3. Illustration of the coupling of shut-in based shale-gas production with varying electric power production, as a means of demand-based gas production and reduced use of conventional underground gas storage. From Knudsen and Foss (2017).

(EUC) operates the NGPP, using the power plant as a ramp-up source together with generation from intermittent renewables. We assume that wells produce gas onto a transmission pipeline with sufficient capacity, and also that the demand predictions from the NGPP is given sufficiently early to compensate for a short pipeline transmission time. Hence, we omit the delay caused by pipeline transmission in the optimization model.

The shut-in ability of dry, mature shale-gas wells is enabled by the characteristic shale-fracture system. This system, generated through hydraulic fracturing stimulation, ensures fast formation pressure build-up during well shut-ins, and a subsequent peak in production recovering the loss of production during shut-in. The dynamics of the pressure build-up, however, ranges from the time-scale of hours in and close to the fracture network, and on the time-scale of years further into the low-permeable shale matrix blocks. This composite dynamics, with fast near-well dynamics and steady-state like dynamics elsewhere in the reservoir, necessitates the use of a dynamic upstream model. To this end, we apply a linear dynamic shale-well proxy model (Knudsen et al., 2014) for $\zeta_{rik}(\cdot)$ in (3), which we tune using prediction-error filtering to fit the proxy model in the frequency range necessary to capture the dominating dynamics during recurrent shut-in operations. The validity of this model is hence constrained to hourly shut-in operations, yet the model may also be fitted to transients on other time scales by altering the filter cut-off frequencies, see Knudsen and Foss (2015), Ljung (1999). For other proxy-modeling techniques for shale-gas wells, see Wilson and Durlofsky (2013), Cafaro et al. (2016) and Ghassemzadeh and Charkhi (2016).

The resulting scheduling model includes an embedded disjunction in place of (8), allowing for on/off and pipeline routing decisions for each well. To solve **P**, we apply a combined big-M and convex hull reformulation of the embedded disjunction, see e.g. Grossmann and Trespalacios (2013), and approximate the nonlinearities with SOS2 approximations. The resulting MILP consists 429 binary variables, 9786 continuous variables and 12469 constraints. As the problem has a block structure with the wells coupled only by the compressor capacity and demand constraints, we implement a Lagrangian relaxation scheme to decrease the computation time. The scheme includes a proximal bundle method to solve the Lagrangian dual and a combined projection-fixing heuristic to recover primal feasibility. For details on the GDP model and computational scheme, we refer the reader to Knudsen et al. (2014).

We assume that the EUC operates a set of intermittent renewable generation sources with hourly varying generation. The EUC uses the NGPP as ramping source in order to compensate for the variability in renewable generation and meet the power demand. Furthermore, we assume that the EUC uses weather forecasts to predict some hours ahead its gas demand necessary to balance its electric-power generation. Upon receiving this gas demand, the optimization-based DPO needs to to meet (or track) the varying gas demands by scheduling well shut-ins. Every third hour, the shale-gas operator receives an updated gas demand d^{EUC} for the next 24 h from the EUC.

In order to meet the varying gas demands, the shale-gas operator must re-optimize its well schedule each time he receives an updated demand curve. To this end, we implement **P** on a receding horizon, re-optimizing the well schedule and operating pressures each time a new demand curve is received. We implement the receding horizon scheme using a prediction horizon of three days and re-optimization of the well schedule each 3 h. Beyond the first 24 h of the prediction horizon, we impose the predicted demand to be tracked as the *average* $\bar{d}^{\mathrm{EUC}}$ of the current EUC demand, as a simple means of taking into account the uncertainty of the future demand. The shale-gas operator commits to supplying the

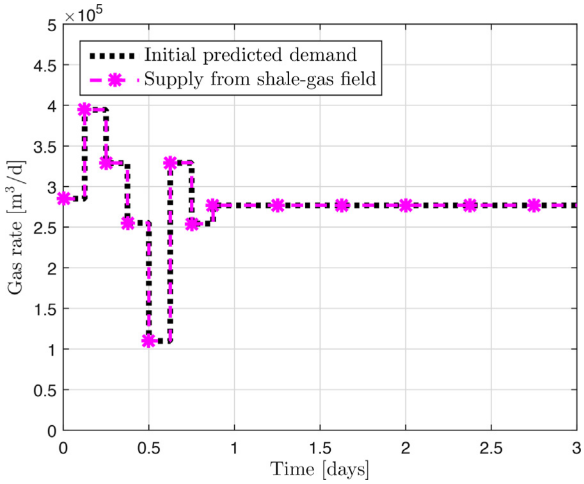

Fig. 4. Open-loop gas rates given initial demand from electric utility company.

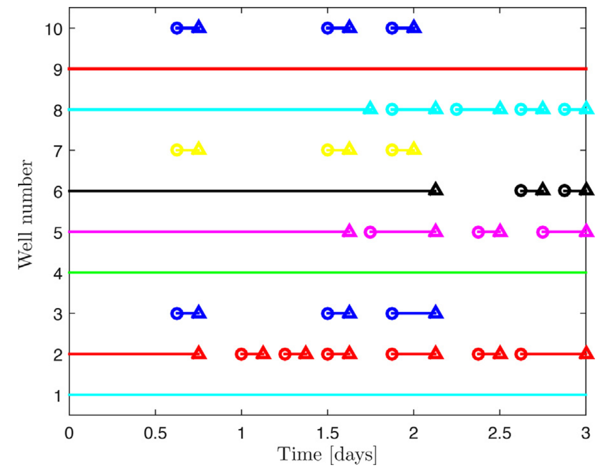

Fig. 6. Shut-ins and start-ups for the first 10 wells of the shale-gas case. The circle ○ symbolizes a start-up of the well, while a △ symbolizes a shut-in.

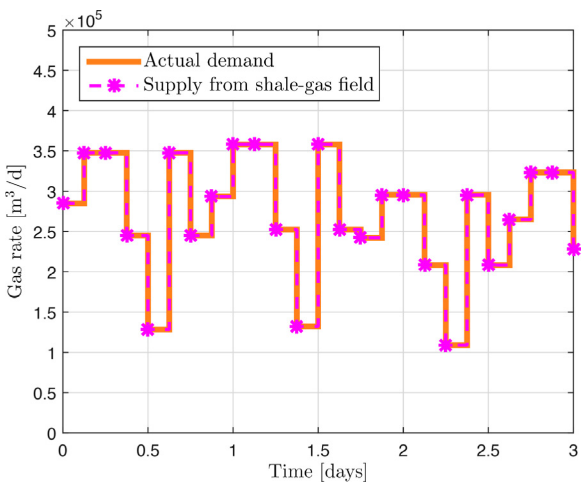

Fig. 5. Gas rates using receding horizon optimization.

demanded rate, incurring high penalties if the operator fails to do so. Compared to Knudsen et al. (2014), we assume that the shale-gas operator supplies all its gas produced to the EUC, i.e. no surplus gas is sold on the spot market.

In Fig. 4, we show the gas rates from the open-loop response computed at the first sampling instant. The total well-supply meets the initially provided gas demand exactly. At the next sampling time, 3 h later, the gas demand from the EUC changes due to variability in renewable generation. To prevent penalties, the operator updates the well schedule by incorporating the updated demand curve. As shown in Fig. 5, by optimizing over a receding horizon, the operator is able to exactly meet the varying gas demand by properly shutting in its wells and adjusting wellhead pressures. The resulting well-schedule is highly dynamic, as shown by the shut-in pattern in Fig. 6. Some wells are frequently shut in and re-opened in order to build up pressure and thereby store gas for the predicted future demand. Meanwhile, three of the wells produce continuously as a result of higher formation pressure support, seen by the solid lines in Fig. 6. Solving the DPO **P** in the receding horizon scheme requires on average 5 min for each iteration.

3.2. DPO in conventional wells

In this case we consider a production system with 13 wells and 7 flowlines, two risers included. All of the wells produce a mix of oil, gas, and water. A part of the daily production optimization problem facing the operator, is to decide which of the wells to produce from and which to shut in. Furthermore, four of the wells may be routed to one of the two risers, labeled Riser A and B. The two risers produce to a platform on which the fluids are processed for export. The platform also gathers the production from another oil field. Thus, the capacity of the processing facility must be shared between the two oil fields, and during periods of maintenance and test campaigns the sharing ratio may vary.

A problem formulation for the production system described above is derived from **P**. The resulting formulation has 310 constraints and 220 variables, 17 binary variables included. The formulation has a total of $2^{17} = 131{,}072$ possible network configurations. The routing constraints cut the number of valid network configurations to $2^9 \cdot 3^4 = 41{,}472$: the routing constraints enforce wells with routing capabilities to produce to zero or one flowline, giving three possible well configurations per well.

The inflow performance relationship of the wells are modeled as cubic B-splines $\zeta_{rik}(\mathbf{q}_{ek}, p_{ik})$ fitted to well test data. The pressure drop over the network flowlines are modeled as cubic B-splines $f_e(\mathbf{q}_{ek}, p_{ik})$ fitted to data generated by a multiphase flow simulator. Cubic B-splines ensures C^2 continuity (continuous second-order derivatives); this a desired property in continuous optimization that may accelerate the solution process. With the nonlinearities represented with B-splines, the case in question can be solved to global optimality within 1–3 min on a laptop computer. These surrogate models were also used by Grimstad et al. (2016) to model a similar production network.

An operational window of three days is considered, in which the gas processing capacity on the platform available for the production system is lowered by 20%, approximately from 8.48 to 6.78 MSm^3/day, to accommodate increased production from the other oil field. The production system must produce with a limited gas processing capacity for one day, before production may return to a normal level. The available capacity changes in step-wise manner and the control must be adjusted accordingly. Optimal control is achieved by performing re-optimizations every time the gas processing capacity changes; assuming that the production system operates in a steady state between the steps.

The result of the sequential optimization is shown in Fig. 7. The optimal well routings, yielding the highest oil rate, are illustrated in Fig. 8. When the gas processing capacity is reduced by 20%, Well 7 is shut-in and Wells 4–7 flips routing combination. Notice that, with these changes in control, the reduction in oil production is kept below 10%, even though the gas production is reduced by 20%. Throughout this scenario, the production of gas lies on the active gas

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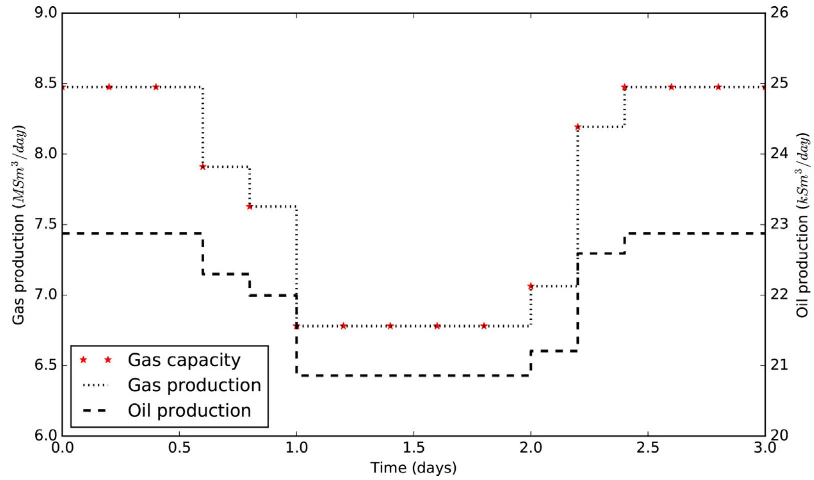

Fig. 7. Oil and gas production with changing gas processing capacity.

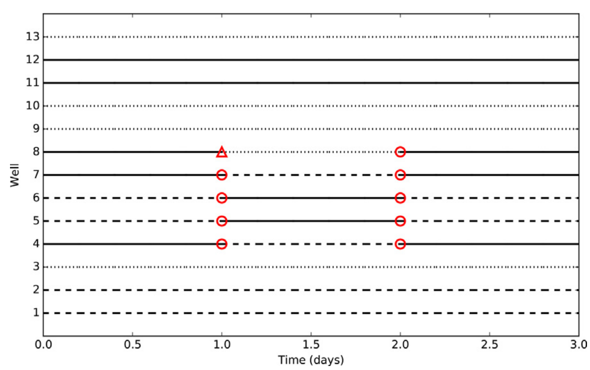

Fig. 8. Optimal routing of wells as the gas processing capacity changes. A dotted line means that the well is shut-in; a dashed or solid line indicates that the well is routed to Riser A or B, respectively. A red circle ○ symbolizes a start-up or re-routing of a well, while a red △ symbolizes a shut-in. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

capacity constraint. When the gas processing capacity is increased back to 8.48 MSm^3/day, the controls return to their initial state.

4. Discussion

The cases above support both static and dynamic formulations. As alluded to earlier a large class of DPO problems can be treated by a static formulation by acknowledging the time scale separation between reservoir dynamics and the prediction horizon for DPO, thus the use of repetitive optimization on a static model suffices. This is supported by Fig. 9, which shows the power spectrum for the total oil rate, i.e. the export oil rate, from an offshore oil platform on the Norwegian continental shelf. Frequencies corresponding to dynamics in the 2–12 h range are shaded with a grey background. Noting that this is a log–log plot there are significantly less variations in this frequency range than below and above, except for frequencies above 2×10^{-3} Hz. Thus, arguably including dynamic models to improve control in this frequency range is unnecessary since disturbances are limited. For completeness, the frequency range above 2×10^{-3} Hz corresponds to dynamics faster than 10 min. The power spectrum shown in Fig. 9 is quite typical for offshore operations. However, there are cases where reservoir dynamics are important on a daily production horizon such as in shale-gas cases. Referring back to the power spectrum this implies that dynamics in the shaded area will be more pronounced in the shale-gas case than in Fig. 9. This is supported by the on/off well pattern in Fig. 6 where 50% of the wells switch on and off a couple of times a day.

The clear majority of cases uses a static DPO formulation for reasons discussed already. However, there is another reason for choosing the simpler option, static models, namely model accuracy. Model accuracy poses a particular challenge for the well models. Flow rate estimates from individual wells are inaccurate since multiphase flow meters are few and far between. Further, model calibration is performed quite infrequently since they rely on data from well tests.

A dynamic formulation may be necessary because of demand side dynamics, a situation which is encountered in the shale-gas example discussed earlier. This case includes dynamics both in the upstream reservoir and downstream demand side. Another situation is if fast dynamics appear only on the demand side. This latter situation, with only demand side dynamics, is covered by $\mathbf{P}$ by optimizing on a suitable horizon $\mathbf{K}$ with a static reservoir model (3) while discarding (4). As alluded to in Fig. 3 one may expect more pronounced NGPP demand side dynamics in future electric grids with renewable electricity production. This is supported by Lee et al. (2012) who study the complementaries between NGPP and renewable power production feeding into a common electric grid. They stress the need for NGPP to provide the flexibility required due to variable renewable generation.

Oil rims are reservoirs where a thin oil rim, typically 10–25 m, is sandwiched between a large gas cap above and water beneath. One well known case is the Troll oil field operated by Statoil. Oil rims can only be exploited economically through horizontal wells which are placed just above the water-oil-contact. The reservoir dynamics are fast. In particular the gas-oil-contact may move downwards towards the well, in a matter of hours, in the event of increased choke opening. This can expose the well to free gas, a situation which limits oil production severely due to gas processing limitations, as modelled in (10). Nennie et al. (2009) compares alternative dynamic control strategies for production optimization of a thin oil rim and concludes that dynamic strategies enables increased production. Similar results are obtained in Sagatun (2010) and Hasan et al. (2013). To conclude, DPO in oil rims may call for a dynamic reservoir model.

The focus up until now has been on the need for dynamics related to reservoir behavior and the demand side, while the wells and network uses static models since $\mathbf{P}$ does not allow dynamic models for these parts. This is a valid assumption except for certain

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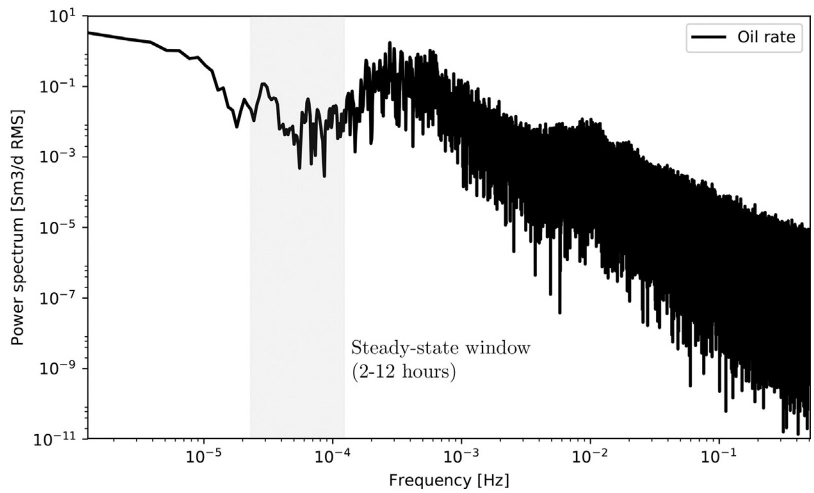

Fig. 9. Power spectrum of oil rate measured at topside separator.

extended networks that are closely integrated with downstream processing, LNG plants being one example. In Foss and Halvorsen (2009), where the Statoil Snøhvit subsea field was considered, the pipeline between 9 wells and the LNG plant is 140 km. The network dynamics are then in the range of 8–10 h and thus in this case the use of a dynamic formulation was considered and actually applied in a DPO study. To limit complexity a dynamic surrogate model for the pipeline was used.

DPO applications, which are normally treated by a static formulation, may in some cases benefit from a dynamic formulation. For short DPO horizons, less than a few hours, a dynamic formulation may be in order, in particular in situations where the link to lower level control is particularly important. This link is reinforced in situations with severe slug flow since the slug cycling time typically lies in the range from 20 min to 1 h. As mentioned earlier, Codas et al. (2016) explores the integration between low level control and dynamic DPO. Improved results are reported, however, at the expense of a significant complexity increase. Another situation where a dynamic description is needed is production optimization that includes well startup. This, however, adds significant complexity because of the need for more complex well models than the ones for instance used in the dynamic DPO study in Codas et al. (2016). The reason is that the well models need to be accurate in a wide range of operating conditions, not only during normal operation.

5. Conclusions

A static optimization formulation suffices in most relevant DPO cases. Important exceptions are shale gas wells and thin oil rims.

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