Stochastic Signals and Systems

Exercises

1 Task 4.1

Given is the following signal

$$x(t) = A_1 sin(2\pi f_1 t) + A_2 cos(2\pi f_2 t)$$

where, frequencies f_1 and f_2 are 5 Hz and 21 Hz respectively and amplitudes A_1 and A_2 are 1 and 0.4 respectively.

a) Find the power spectral density of the signal $x(t)$ using MATLAB. What is the effect of changing number of samples (e.g. 1024, 2018) and sampling frequency (e.g. 1000 Hz, 200 Hz) on power spectral density?

b) What happens when sampling frequency becomes less than Nyquist sampling frequency, (e.g. 30 Hz)? Plot the result in MATLAB. Explain: In this case, is there any relation between Nyquist sampling frequency, maximum signal frequency and the frequency component present in the power spectral density?

2 Task 4.2

Given is the following random process with random noise added to it

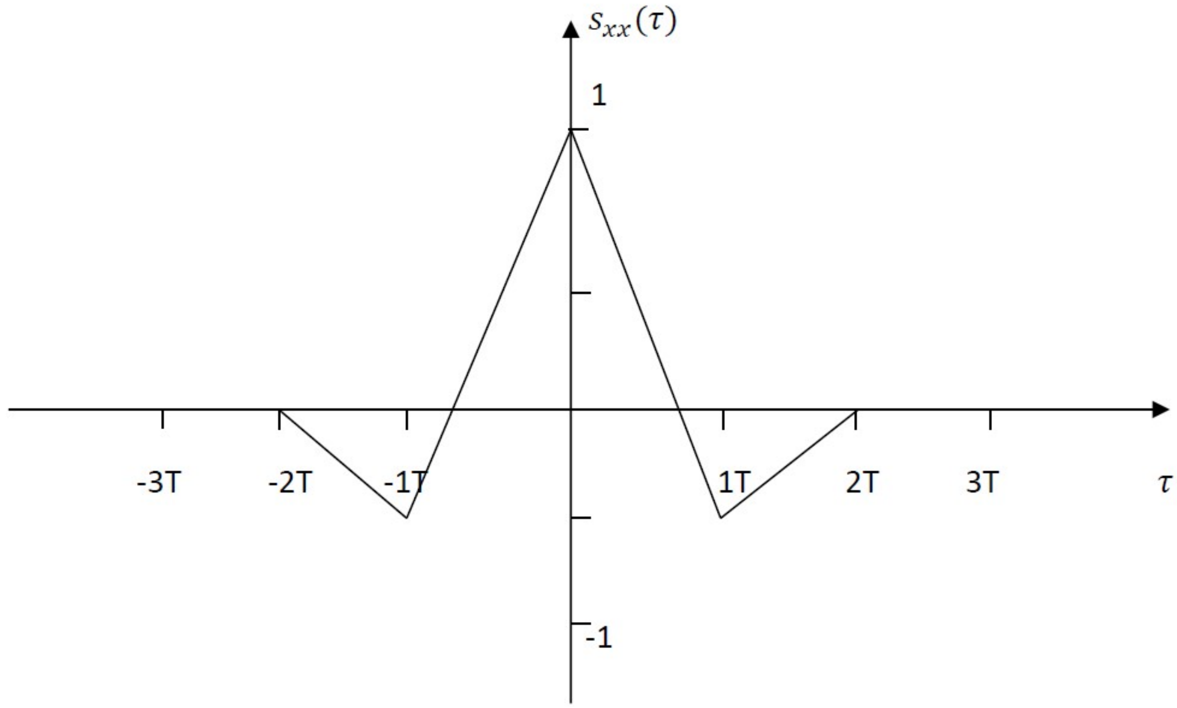
$$x(\zeta, t) = \sin(2\pi f t) + \alpha \cdot n(\zeta, t)$$

where, the frequency f is 20 Hz, α is 3.0 and $n(\zeta, t)$ is random noise.The random process is sampled at a sampling frequency of 100 Hz and the number of samples is 1024. Write MATLAB programs.
a) Plot the signal.
b) Calculate the ACF.
c) Apply the Fourier transform to the ACF using a Hamming window (window length 2048) and plot the result both with linear scale and logarithmic scale.
d) Apply the Fourier transform to the ACF using a rectangular window (window length

2048) and plot the result both with linear scale and logarithmic scale.
e) Explain the results of c) and d).

3 Task 4.3

This is the autocorrelation function $s_{xx}(\tau)$ of the stationary random process $x(\zeta, t)$:



Calculate and sketch the power spectral density $S_{xx}(\omega)$.