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TASK 5.1

$$u(e,t) \longrightarrow \boxed{g(t)} \longrightarrow y(e,t)$$

$$S_{uy} = \frac{S_1}{(1-j\omega b)(1+j\omega T_1)} \quad ; \quad S_{yy} = \frac{S_1}{1+\omega^2 T_1^2}$$

(a) Transfer function, $G(j\omega)$ of the linear system.

$$G^*(j\omega) = \frac{S_{yy}}{S_{uy}} = \frac{\dfrac{\cancel{S_1}}{1+\omega^2 T_1^2}}{\dfrac{\cancel{S_1}}{(1-j\omega b)(1+j\omega T_1)}} = \cancel{\frac{\dfrac{S_1}{1+\omega^2 T_1^2}}{(1-j\omega b)(1+j\omega T_1)}}$$

$$= \frac{(1-j\omega b)(1+j\omega T_1)}{1+\omega^2 T_1^2} = \frac{(1-j\omega b)(1+j\omega T_1)}{(1+j\omega T_1)(1-j\omega T_1)} = \frac{1-j\omega b}{1-j\omega T_1}$$

$$\Rightarrow G(j\omega) = \frac{1+j\omega b}{1+j\omega T_1}$$

(b) Is the system described by $G(j\omega)$ a causal system?
Begin by identifying if the pole exist:

$$\Rightarrow 1+j\omega T_1 = 0$$

$$\omega = \frac{-1}{jT_1} = \frac{j}{T_1}$$

$\therefore$ No, T_1 must be greater than zero for the system to be causal.

(c) Calculate the auto correlation function $S_{uu}(\tau)$ of the input signal $u(e,t)$.

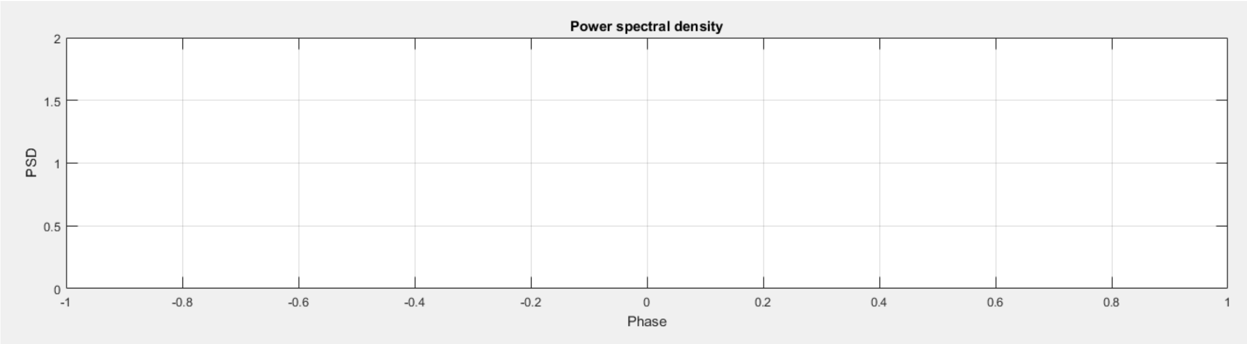
$S_{uu}(\tau) = ?$

$$S_{uu}(\tau) = \frac{S_{uy}}{G(j\omega)} = \frac{\dfrac{S_1}{(1-j\omega b)(1+j\omega T_1)}}{\dfrac{1+j\omega b}{1+j\omega T_1}} = \frac{S_1}{(1-j\omega b)(1+j\omega b)} = \frac{S_1}{1+\omega^2 b^2}$$

$$\Rightarrow S_{uu}(\tau) = F^{-1}[S_{uu}] = F^{-1}\left[\frac{S_1}{1+\omega^2 b^2}\right] = F^{-1}\left(\frac{S_1/b}{b\left(\frac{1}{b^2}+\omega^2\right)}\right)$$

$$= S_{uu}(\tau) = \frac{S_1}{2b} e^{-\left|\frac{\tau}{b}\right|}$$

TASK 5.2




```
clear all;
close all;

N  = 1024; % Number of Samples
Fs = 100; % Sampling Frequency in Hz
T = 1; % period
w = 0 : pi/T : 6.*pi./T % phase angle
y = (2./((w.^2).*T)).*((1-(2.*cos(w.*T))).^2)

% Power Spectral Density using Wiener Khintchine Theorem with Hamming window
Rxxdft = abs(fftshift(fft(y)));
freq = -Fs/2:Fs/length(y):Fs/2-(Fs/length(y));

subplot(2,1,1);
plot(freq, Rxxdft,'b'),title('Power spectral density'),axis([-1 1 0
2]),xlabel('Phase'),ylabel('PSD');
grid on;
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TASK 5.3

$x(\zeta, t) =$, mean $= 0$, standard deviation $= 1$

$S_{xx}(\tau) = a e^{-\alpha|\tau|} + b$

Given random process, $y(\zeta, t)$:

$$y(\zeta, t) = \begin{cases} 0, & \text{for } t \leq t_0 \\ \int_{t_0}^{t} x(\zeta, \lambda) d\lambda & \text{for } t > t_0 \end{cases}$$

(a) Determine the constants a and b.

$$m_x^{(1)}(t) = \sqrt{\lim_{\tau \to \infty} a e^{-\alpha|\tau|} + b} = 0$$

$$\Rightarrow \sqrt{0 + b} = 0$$

$$\therefore b = 0$$

$$\sigma^2 = m_x^{(2)}(t) - [m_x^{(1)}(t)]^2 = 1$$

$$= m_x^{(2)}(t) - 0 = 1$$

$$= \lim_{\tau \to 0} a e^{-(\tau)} + b = 1$$

$\Rightarrow a + b = 1$, $\therefore a = 1$ since $b = 0$ above.

(b) Determine the cross-correlation function

$$S_{xy}(t_1, t_2) = E\{x(\zeta, t_1)\, y(\zeta, t_2)\} = ?$$

$$= E\left\{x(\zeta, t_1) \int_0^{t_2} x(\zeta, t_2) du\right\} = \int_{t_0}^{t_2} S_{xx}(t - u) du = \int_{t_0}^{t_2} e^{-\alpha(t_1 - u)} du$$

Case 1: $t_0 \leq t_1 \leq t_2$

$$S_{xy}(t_1, t_2) = \int_{t_0}^{t_1} e^{-\alpha(t_1 - u)} du + \int_{t_0}^{t_1} e^{-\alpha(u - t_1)} du = \frac{1}{\alpha}\left[e^{-\alpha(t_1 - u)}\right]_{t_0}^{t_1} - \frac{1}{\alpha}\left[e^{-\alpha(u - t_1)}\right]_{t_1}$$

$$S_{xy}(t_1, t_2) = \frac{1}{\alpha}\left[1 - e^{-\alpha(t_1 - t_0)} - e^{-\alpha(t_2 - t_1)}\right] \quad \text{— (i)}$$

Case 2: $t_0 \leq t_2 \leq t_1$

$$S_{xy}(t_1, t_2) = \int_0^{t_2} e^{-\alpha(t - u)} du = \frac{1}{\alpha}\left[e^{-\alpha(t_1 - u)}\Big|_{t_0}^{t_2}\right]$$

$$S_{xy}(t_1, t_2) = \frac{1}{\alpha}\left[e^{-\alpha(t_1 - t_2)} - e^{-\alpha(t_1 - t_0)}\right] \quad \text{— (ii)}$$

Case 3: $t_1 \leq t_0 \leq t_2$

$$S_{xy}(t_1, t_2) = \int_{t_0}^{t_2} e^{-\alpha(u - t_1)} du = \frac{1}{\alpha}\left[e^{-\alpha(u - t_1)}\right]_{t_0}^{t_2}$$

$$S_{xy}(t_1, t_2) = \frac{1}{\alpha}\left[e^{-\alpha(t_0 - t_1)} - e^{-\alpha(t_2 - t_1)}\right] \quad \text{— (iii)}$$

From ~~Eqn~~ (i), (ii), & (iii)

$$S_{xy}(t_1, t_2) = \begin{cases} \frac{1}{\alpha} e^{-\alpha(t_2 - t_1)} - e^{-\alpha(t_2 - t_0)} & \text{for } t_1 \leq t_0 \leq t_2 \\ \frac{1}{\alpha} (2 - e^{-\alpha(t_1 - t_0)} - e^{-\alpha(t_2 - t_0)} & \text{for } t_0 \leq t_1 \leq t_2 \\ 0, & \text{elsewhere} \end{cases}$$

(c) The process $y(\zeta, t)$ is a non-stationary process.