Eigenvalue decomposition

* Algebraic multiplicity = number of numbers an eigenvalue repeats
* Geometric multiplicity = dimension of null space of A – lambda \* I
* i.e. number of vectors for an eigenvalue

Diagonalization

* If A is diagonalizable then for P^-1 \* A \* P = LAMBDA columns of P have to be eigenvectors of A and diagonal matrix LAMBDA is eigenvalues of A
* A can only be diagonalized iff its eigenvectors are linearly independent
* If A is diagonalizable then algebraic multiplicity = geometric multiplicity

Schur’s Triangularization

* **Definition:** Every **square** matrix A can be triangularized by a **unitary** matrix
* A = U\*T\*U^H , where U is unitary – i.e. U^H\*U = U\*U^H = I
* T is **upper triangular** matrix
* Unitary matrix is orthogonal matrix with unit norm columns and full rank
* U and T are not necessarily unique, but the diagonals of T are

Rayleigh Quotient

* Is maximized when q is eigenvector corresponding to largest eigenvalue
* December 3rd notes
* Is minimized when q is smallest eigenvector

SVD

* ANY matrix can be singular value decomposed – A = U \* SIGMA \* V^H
* U and V are unitary
* Rank of A is denoted by r
* Can be rewritten then as: A = U\_r \* SIGMA\_rr \* V\_r^H
* 4 Fundamental Subspaces
* Cols of U\_r = basis for R(A)
* Cols of V\_(n-r) = basis for N(A)
* Cols of V\_r = basis for R(A^H)
* Cols of U(n-r) = basis for N(A^H)
* Singular values always exist – eigenvalues only exist for square matrices
* **PSD Matrix**
* A PSD matric (A) can always be factored as A = B^HB where B is not unique. If A = U \* LAMDA \* U^H, then one choice of B is U\*LAMDA^1/2
* SVD of pseudoinverse: A^+ = V \* SIGMA^+ \* U^H
* Columns of U\_r = basis for R(A)
* Columns of V\_(n-r) = basis for N(A)
* Columns of V\_r = basis for R(A^H)
* Columns of U\_(n-r) = basis for N(A^H)

PCA

* If B ( = [b1 b2 b3 … bk] ) is set of othonormal basis vectors for dimension k subspace then BB^T is orthogonal projection matrix onto subspace of dim k
* Then BB^Tx is projection of x onto subspace of dim k
* Minimize projection error: min ||x-BB^Tx||^2 (frobenius/Euclidean norm squared)
* Simplified to max Tr[ x^TBB^Tx ]= max [ B^Txx^TB] where xx^T is variance of x
* Solution is rayleigh’s quotient – since x^Tx = 1--- so answer is eigenvector 1 cause lambda 1 is largest eigenvalue

Norms

* The stuff in discussion 9 generally
* Frobenius Norm
* || X + Y ||^2 = Trace[ (X+Y)^T (X+Y) ] = ||X||^2 + ||Y||^2
* L2 Norm
* ||X||^2 = X^T\*X

Projection Matrix

* Projection of y onto A is: y\_hat = A (A^H\*A)^-1 \* A^H \* y
* If columns of A are orthogonal then A^H\*A = I so y\_hat = AA^Hy
* A T Ax = A T b

QR Factorization and Gram-Schmidt

* Last slides of summary slides 2, may want to add some notes?

Pseudoinverse

1. (*AA*+ need not be the general identity matrix, but it maps all column vectors of *A* to themselves);
2. {\displaystyle A^{+}AA^{+}=A^{+}}   (*A*+ is a [weak inverse](https://en.wikipedia.org/wiki/Weak_inverse) for the multiplicative [semigroup](https://en.wikipedia.org/wiki/Semigroup));
3. {\displaystyle (AA^{+})^{\*}=AA^{+}}   (*AA*+ is [Hermitian](https://en.wikipedia.org/wiki/Hermitian_matrix));
4. {\displaystyle (A^{+}A)^{\*}=A^{+}A}   (*A*+*A* is also Hermitian).

Helpful Stuff

* Trace(A) = sum of eigenvalues
* Det(A) = product of eigenvalues
* A\_perp is N(A^T)
* rank(AB)=rank(B)−dimN (A)∩R(B).