Dark Soliton Analysis in Highly Nonlinear Fiber with Optical and Raman Gain

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Abstract: The evolution of dark solitons in highly nonlinear fiber with optical and Raman gain is analytically and numerically studied. Starting from Nonlinear Schrodinger's Equation, we derive new equation for dark solitons embraced in Raman shockwaves.

1. Introduction

The optical pulses experience nonlinear effects and group velocity dispersion (GVD) while propagating through nonlinear optical fibers. When the pulse broadening and compression caused by nonlinearity and dispersion is perfectly balanced, the solitons are generated. The solitons are categorized into two types: bright and dark solitons. While the generation and propagation of bright solitons are analytically and experimentally studied, the physics of dark solitons are less analyzed than bright solitons. It is predicted that pulse propagates within optical fibers with Raman gain form dark soliton[1]. However, from the numerical simulations, it is expected that an eruptive generation of dark solitons embraced in Raman shockwaves occurs[2,3], and analytical study is needed. In this paper, starting from Nonlinear Schrodinger Equation (NLSE) with both optical and Raman gain, the generation and its evolution of blackness parameter is derived, and the pulse shape is simulated using derived equation.

2. Handling NLSE with Taylor Expansion

The pulse propagation in optical fiber is expressed by the well-known NLSE. However, for simplicity, we only consider all-normal second-order-dispersion and the simplest form of nonlinearity with optical gain term[3,4].

$$\frac{\partial A}{\partial Z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + i\gamma|A|^2 A + \int_{-\infty}^{\infty} \frac{1}{2\pi}\, g(\omega-\omega_0)A(\omega-\omega_0)e^{-i(\omega-\omega_0)T}d\omega. \tag{1}$$

The Raman response is incorporated into the nonlinear term $i\gamma|A|^2A$ as in equation (2).

$$\gamma|A|^2 \rightarrow \gamma\left((1-f_R)|A|^2 + f_R\int_{-\infty}^{t} f(t-t')|A(t')|^2dt' \right) \tag{2}$$

Here f_R denotes Raman fraction and $f(t-t')$ denotes Raman response function. As Raman response function can be treated local[1], the Taylor expansion is applied to deal with the convolution form of Raman response and optical amplitude. Likewise, the gain term in the form of convolution of gain profile is treated. Additional parameter conversion is introduced as in equation (3) to make parameters be dimensionless:

$$Z \rightarrow \frac{t_0^2}{\beta_2}x, \quad T \rightarrow t_0\, t, \quad A(Z,T) \rightarrow \sqrt{\frac{\beta_2}{t_0^2}\frac{1}{\gamma}}\,u(x,t) \tag{3}$$

The t_0 denotes arbitrary time scale, and we chose typical pulsewidth about 10 ps. After all Taylor expansion, the NLSE in equation (1) can be converted to:

$$i\frac{\partial u}{\partial x} - \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + u|u|^2 = \bar{\alpha}\, u\frac{\partial}{\partial t}|u|^2 + i\bar{\beta}u + i\bar{\gamma}\frac{\partial u}{\partial t} + i\bar{\delta}\frac{\partial^2 u}{\partial t^2} \tag{4}$$

where

$$\bar{\alpha} = f_R\int_0^{\infty} t' t_0 h_R(t_0 t')dt' \tag{5}$$

$$\bar{\beta} = \eta\frac{t_0^2}{\beta_2}\left(1 + \int_0^{\infty} t_0\zeta(t_0 t')dt'\right) \tag{6}$$

$$\bar{\gamma} = \eta\frac{t_0^2}{\beta_2}\int_0^{\infty} -t' t_0\zeta(t_0 t')dt' \tag{7}$$

$$\bar{\delta} = \eta\frac{t_0^2}{\beta_2}\int_0^{\infty} \frac{1}{2}t'^2 t_0\zeta(t_0 t')dt' \tag{8}$$

Given that $h_R(t_0 t')$ is Raman response and η is gain with $\zeta(t_0 t')$ being gain-related function, the Raman effect is applied by $\overline{\alpha}$ term, and optical gain effect is applied by $\bar{\beta}, \bar{\gamma}, \bar{\delta}$ terms in Equation (4).

3. Applying Perturbative Inverse Scattering Transform Method

In order to introduce the similar method which Kivshar[1] has used, the perturbation of optical filed is introduced. However, it is worth noting that the perturbation should not be directly introduced due to optical gain; the optical gain makes the overall magnitude increasing. Therefore, one more substitution is applied.

$$u(x,t) = \bar{A}(x,t)\, exp\left(\bar{\beta}x + i\int u_0^2\left(e^{2\bar{\beta}x'} - 1\right)dx'\right) \tag{9}$$

The governing equation (4) is changed correspondingly. Then, the optical amplitude is perturbed.

$$\bar{A}(x,t) = \left(u_0 + a(x,t)\right) exp\left(iu_0^2 x + i\phi(x,t)\right) \tag{10}$$

From there, the perturbation terms can be reduced to form of Korteweg-de Vries Equation which gives soliton solution. It also gives the evolution of blackness parameter of a dark soliton, and the derived form is expressed as:

$$\frac{1}{v^2} = \frac{1}{v_0^2} + \frac{8}{45}\frac{\beta_2}{\eta}\frac{\bar{\alpha}}{\bar{\beta}} sign(C)\left(e^{3\bar{\beta}x} - e^{3\bar{\beta}x_0}\right) \tag{11}$$

Note that the evolution of a dark soliton depends on both Raman effect $\overline{\alpha}$ and optical gain effect $\bar{\beta}$. It is also consistent with the previous research of Kivshar[1] when optical gain is set to be zero.

4. Numerical Simulation of Dark Soliton Generation

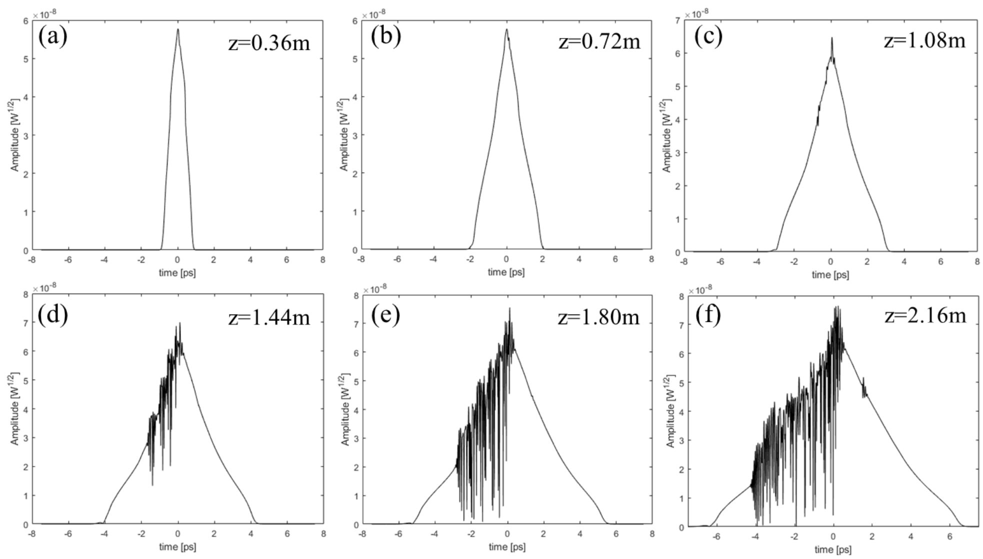

Fig 1. Simulated pulse shape propagating nonlinear gain fiber.

Using Equation (4), the pulse shape is simulated along propagation length. The generated Raman shockwaves and dark soliton groups embraced in the Raman shockwaves can only travel directionally, which is determined by sign of velocity C as in equation (11).

5. References

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