CSCE 221 Cover Page
Homework #1
Due February 13 at midnight to eCampus

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Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero. According to the University Regulations, Section 42, scholastic dishonesty are including: acquiring answers from any unauthorized source, working with another person when not specifically permitted, observing the work of other students during any exam, providing answers when not specifically authorized to do so, informing any person of the contents of an exam prior to the exam, and failing to credit sources used. Disciplinary actions range from grade penalties to expulsion read more: Aggie Honor System Office

Type of sources			
People			
Web pages (provide URL)			
Printed material			
Other Sources			

I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.

"On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work."

Your Name   Joseph Martinsen   Date   02/11/2017

Type the solutions to the homework problems listed below using preferably L$_Y$X/L^{A}T$_E$X word processors, see the class webpage for more information about their installation and tutorial.

1. (10 points) Write a C++ program to implement the Binary Search algorithm for searching a target element in a sorted vector. Your program should keep track of the number of comparisons used to find the target.

   (a) (5 points) To ensure the correctness of the algorithm the input data should be sorted in ascending or descending order. An exception should be thrown when an input vector is unsorted.

   (b) (10 points) Test your program using vectors populated with consecutive (increasing or decreasing) integers in the ranges from 1 to powers of 2, that is, to these numbers:
   1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048.
   Select the target as the last integer in the vector.

   (c) (5 points) Tabulate the number of comparisons to find the target in each range.

Range [1,n]	Target for incr. values	# comp. for incr. values	Target for decr. values	# comp. for decr. values	Result of the formula in item 5
[1,1]	1	1	1	1	1
[1,2]	2	3	1	3	3
[1,4]	4	5	1	5	5
[1,8]	8	7	1	7	7
[1,16]	16	9	1	9	9
[1,32]	32	11	1	11	11
[1,64]	64	13	1	13	13
[1,128]	128	15	1	15	15
[1,256]	256	17	1	17	17
[1,512]	512	19	1	19	19
[1,1024]	1024	21	1	21	21
[1,2048]	2048	23	1	23	23

(d) (5 points) Plot the number of comparisons to find a target where the vector size $n = 2^k$, $k = 1, 2, \ldots, 11$ in each increasing/decreasing case. You can use any graphical package (including a spreadsheet).
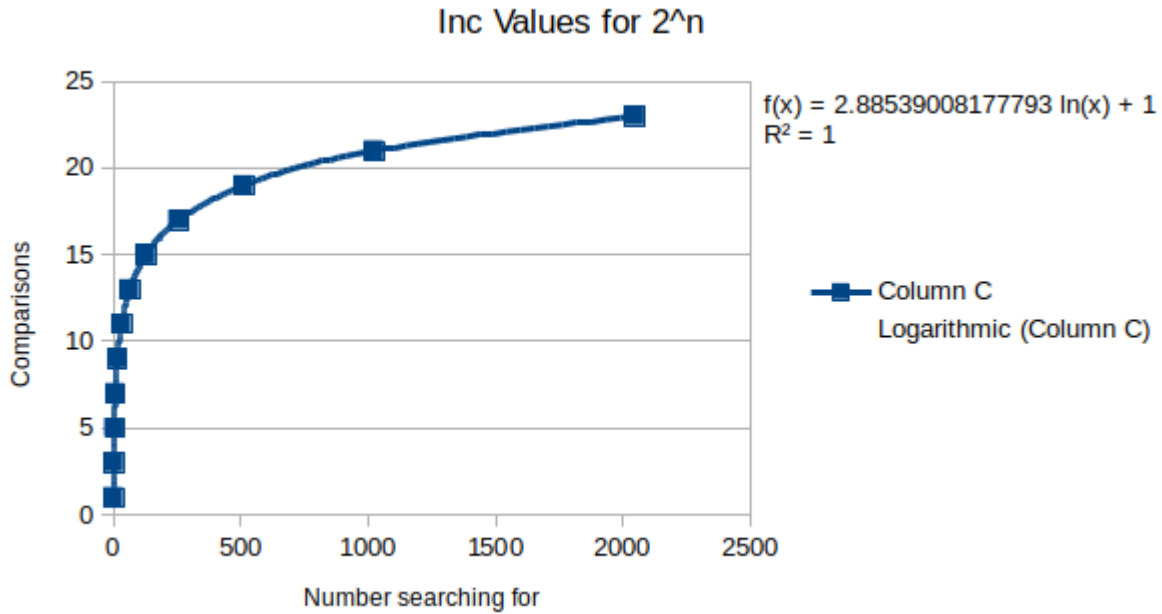

Figure 1: Plot for Increasing/Decreasing Even Numbers

(e) (5 points) Provide a mathematical formula/function which takes n as an argument, where n is the vector size and returns as its value the number of comparisons. Does your formula match the computed output for a given input? Justify your answer.

The equation for the best fit line of the graph is

$$f(x) = 2.88539 \ln(x) + 1 \tag{1}$$

The result is of plugging in the values to this equation is located in the first table. The numbers are going to be nearly identical to the number of comparisons due to the mean R^2 value being 1

(f) (5 points) How can you modify your formula/function if the largest number in a vector is not an exact power of two? Test your program using input in ranges from 1 to $2^k - 1$, $k = 1, 2, 3, \ldots, 11$.

Range [1,n]	Target for incr. values	# comp. for incr. values	Target for decr. values	# comp. for decr. values	Result of the formula in item 5
[1,1]	1	1	1	1	1
[1,3]	3	3	1	3	3
[1,7]	7	5	1	5	5
[1,15]	15	7	1	7	7
[1,31]	31	9	1	9	9
[1, 63]	63	11	1	11	11
[1,127]	127	13	1	13	13
[1,255]	255	15	1	15	15
[1,511]	511	17	1	17	17
[1,1023]	2023	19	1	19	19
[1,2047]	2047	21	1	21	21

(g) (5 points) Use Big-O asymptotic notation to classify this algorithm and justify your answer.

(h) Submit to CSNet an electronic copy of your code and results of all your experiments for grading.

2. (10 points) **(R-4.7 p. 185)** The number of operations executed by algorithms A and B is $8n\log n$ and $2n^2$, respectively. Determine n_0 such that A is better than B for $n \geq n_0$.

$$A = 8n\log(n) \quad B = 2n^2$$
$$8n\log(n) = 2n^2$$
$$4\log(n) = n$$
$$4 = \frac{n}{\log(n)}$$
$$4 = \frac{16}{\log_2(16)}$$
$$4 = \frac{16}{4}$$
$$4 = 4$$

The two lines intersect at 16. As n grows, B grows bigger than A. A is better than B for $n \geq 17$

3. (10 points) **(R-4.21 p. 186)** Bill has an algorithm, `find2D`, to find an element x in an $n \times n$ array A. The algorithm `find2D` iterates over the rows of A, and calls the algorithm `arrayFind`, of code fragment 4.5, on each row, until x is found or it has searched all rows of A. What is the worst-case running time of `find2D` in terms of n? What is the worst-case running time of `find2D` in terms of N, where N is the total size of A? Would it be correct to say that `find2D` is a linear-time algorithm? Why or why not?

The worse running time of `find2D` is $O(n^2)$ because it iterates from $i = 0$ to $i = n - 1$ for a total of n times. For each i iteration, `arrayFind` runs from $j = 0$ to $j = n$ until x is find. The worse-case would be if j ran n times for every i running all the way to n resulting in $n \cdot n = O(n^2)$

In terms of n where n is the total size of A, it is $O(N)$ because the element is searched in a single row of the array.

Yes it would be correct to say that `find2D` is a linear-time algorithm because it calls another function n times

4. (10 points) **(R-4.39 p. 188)** Al and Bob are arguing about their algorithms. Al claims his $O(n\log n)$-time method is always faster than Bob's $O(n^2)$-time method. To settle the issue, they perform a set of experiments. To Al's dismay, they find that if $n < 100$, the $O(n^2)$-time algorithm runs faster, and only when $n \geq 100$ then the $O(n\log n)$-time one is better. Explain how this is possible.

Let $A = n\log(n)$ and $B = n^2$
$A' = \log(n) + 1$ and $B' = 2n$
It is apparent from these derivatives that B will continue to increase at a faster rate than A at a certain point. Thanks to Al and Bob, they found that A and B intersect and 100 with B increasing more and more will A's rate of change begins to slowly decrease.

5. (20 points) Find the running time functions for the algorithms below and write their classification using Big-O asymptotic notation. The running time function should provide a formula on the number of operations performed on the variable s.

```
Algorithm Ex1(A):
   Input: An array A storing n ≥ 1 integers.
   Output: The sum of the elements in A.
  s ← A[0]
  for i ← 1 to n − 1 do
      s ← s + A[i]
  return s
```

This algorithm is $O(n)$. The running time function is $2n-1$

Algorithm Ex2(A):
  Input: An array A storing $n \geq 1$ integers.
  Output: The sum of the elements at even positions in A.
$s \leftarrow A[0]$
for $i \leftarrow 2$ **to** $n-1$ **by** increments of 2 **do**
  $s \leftarrow s + A[i]$
return s

$$1 + \frac{n}{2} \cdot 2$$
$$1 + n + 1$$
$$n + 1$$

The running time is $n+2$ which is $O(n)$

Algorithm Ex3(A):
  Input: An array A storing $n \geq 1$ integers.
  Output: The sum of the partial sums in A.
$s \leftarrow 0$
for $i \leftarrow 0$ **to** $n-1$ **do**
  $s \leftarrow s + A[0]$
  for $j \leftarrow 1$ **to** i **do**
    $s \leftarrow s + A[j]$
return s
The running time is $n^2 + 3n + 1$ which is $O(n^2)$

Algorithm Ex4(A):
  Input: An array A storing $n \geq 1$ integers.
  Output: The sum of the partial sums in A.
$t \leftarrow 0$
$s \leftarrow 0$
for $i \leftarrow 1$ **to** $n-1$ **do**
  $s \leftarrow s + A[i]$
  $t \leftarrow t + s$
return t
 The run time is $4n-2$ which is $O(n)$