CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 7

Due dates: Electronic submission of *yourLastName-yourFirstName-hw7.tex* and *yourLastName-yourFirstName-hw7.pdf* files of this homework is due on **Monday, 3/27/2017 before class begins** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Monday, 3/27/2017** *at the beginning of class*. **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph **Section: Martinsen**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ______________________________

Problem 1. (16 points) Section 6.1, Exercise 24, page 396

Solution.
How many positive integers between 1000 and 9999 inclusive

a) are divisible by 9?

$$\left\lfloor \frac{9999}{9} \right\rfloor - \left\lfloor \frac{1000-1}{9} \right\rfloor = \left\lfloor \frac{9000}{9} \right\rfloor = 1000$$

b) are even?

$$\left\lfloor \frac{9999}{2} \right\rfloor - \left\lfloor \frac{1000-1}{2} \right\rfloor = \left\lfloor \frac{9000}{2} \right\rfloor = 4500$$

c) have distinct digits?

$$9 \cdot 9 \cdot 8 \cdot 7 = 4536$$

d) are not divisible by 3?

$$9999 - 999 - \left\lfloor \frac{9999}{3} \right\rfloor - \left\lfloor \frac{1000-1}{3} \right\rfloor = 1000 - \left\lfloor \frac{9000}{3} \right\rfloor = 6000$$

e) are divisible by 5 or 7?
Divisible by 5:
$$\left\lfloor \frac{9999}{5} \right\rfloor - \left\lfloor \frac{1000}{5} \right\rfloor = 1799$$
Divisible by 7:
$$\left\lfloor \frac{9999}{7} \right\rfloor - \left\lfloor \frac{1000}{7} \right\rfloor = 1286$$
Divisible by 5 and 7:
$$\left\lfloor \frac{9999}{5 \cdot 7} \right\rfloor - \left\lfloor \frac{1000}{5 \cdot 7} \right\rfloor = 257$$
divisible by 5 or 7:
$$1799 + 1286 - 257 = 2828$$

f) are not divisible by either 5 or 7?
$$9000 - 2828 = 6172$$

g) are divisible by 5 but not by 7?
$$1799 - 257 = 1542$$

h) are divisible by 5 and 7?
$$257$$

Problem 2. (16 points) Section 6.1, Exercise 32, page 397

Solution.
How many strings of eight uppercase English letters are there

a) if letters can be repeated?
$$26^8$$

b) if no letter can be repeated?
$$\frac{26!}{18!}$$

c) that start with X, if letters can be repeated?
$$26^7$$

d) that start with X, if no letter can be repeated?
$$\frac{25!}{18!}$$

e) that start and end with X, if letters can be repeated?
$$26^6$$

f) that start with the letters BO (in that order), if letters can be repeated?

$$26^6$$

g) that start and end with the letters BO (in that order), if letters can be repeated?

$$26^4$$

h) that start or end with the letters BO (in that order), if letters can be repeated?

$$2 \cdot 26^6 - 26^4$$

Problem 3. (10 points) Section 6.1, Exercise 46, page 397

Solution.
In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?

$$6 \cdot C(9, 5) = 90720$$

b) both the bride and groom must be in the picture?

$$6 \cdot 5 \cdot C(8, 4) = 50400$$

c) exactly one of the bride and the groom is in the picture?

$$6 \cdot C(8, 5) + 6 \cdot C(8, 5) = 80640$$

Problem 4. (10 points) Section 6.1, Exercise 62, page 398. Explain.

Solution.
Inclusion-Explusion Principle:

$$| P \cup Q | = | P | + | Q | - | P \cap Q |$$

WolframAlpha defines relatively prime as: Two integers are relatively prime if they share no common positive factors (divisors) except 1

$$\begin{aligned} n &= pq \\ | P | &= \left\lfloor \frac{n}{p} \right\rfloor \\ | Q | &= \left\lfloor \frac{n}{q} \right\rfloor \\ | P \cap Q | &= 1 \\ | P \cup Q | &= \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{q} \right\rfloor - 1 \end{aligned}$$

Since $P \cup Q$ represent the values of p and q that go into n, it must be the case that $n- \mid P \cup Q \mid$ represent the number of values that are relatively prime to n.

$$n- \mid P \cup Q \mid = n - \left\lfloor \frac{n}{p} \right\rfloor - \left\lfloor \frac{n}{q} \right\rfloor + 1$$

Problem 5. (10 points) Section 6.2, Exercise 12, page 405. Explain.

Solution.
Let $p(n)$ be $n\%5$. The results of $p(n)$ are $0, 1, 2, 3, 4$. It follows that $\mid p(n) \mid = 5$. From the given statement, it must be the case that $(p(a_1), p(b_1)) = (p(a_2), p(b_2))$. There are $5 \cdot 5 = 25$ different ordered pars of the from $(p(a), p(b))$. By the pigeonhole principle, 26 ordered pairs are needed to guarantee that the statement is satisfied.

Problem 6. (10 points) Section 6.2, Exercise 14, page 405.

Solution.

Problem 7. (10 points) Section 6.3, Exercise 20, page 413.

Solution.

Problem 8. (10 points) Section 6.3, Exercise 22 b), c), d), e), and f), page 414.

Solution.

Problem 9. (10 points) Section 6.4, Exercise 12, page 421.

Solution.

Problem 10. (**Extra credit 10 points**) Section 6.2, Exercise 46, page 407. Prove the claim by contradiction (that is similar to the proof for the generalized pigeonhole principle shown in slide #18 in the lecture slides counting.pdf).

Solution.

Checklist:
- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?