

CSCE 222 [Sections 502, 503] Discrete Structures for Computing  
Spring 2017 – Hyunyoung Lee

**Problem Set 7**

**Due dates:** Electronic submission of *yourLastName-yourFirstName-hw7.tex* and *yourLastName-yourFirstName-hw7.pdf* files of this homework is due on **Monday, 3/27/2017 before class begins** on <http://ecampus.tamu.edu>. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Monday, 3/27/2017 at the beginning of class**. **If any of the three submissions are missing, your work will not be graded.**

**Name:** Joseph

**Section:** Martinsen

**Resources.** (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

**Signature:** \_\_\_\_\_

**Problem 1.** (16 points) Section 6.1, Exercise 24, page 396

**Solution.**

How many positive integers between 1000 and 9999 inclusive

a) are divisible by 9?

$$\left\lfloor \frac{9999}{9} \right\rfloor - \left\lfloor \frac{1000-1}{9} \right\rfloor = \left\lfloor \frac{9000}{9} \right\rfloor = 1000$$

b) are even?

$$\left\lfloor \frac{9999}{2} \right\rfloor - \left\lfloor \frac{1000-1}{2} \right\rfloor = \left\lfloor \frac{9000}{2} \right\rfloor = 4500$$

c) have distinct digits?

$$9 \cdot 9 \cdot 8 \cdot 7 = 4536$$

d) are not divisible by 3?

$$9999 - 999 - \left\lfloor \frac{9999}{3} \right\rfloor + \left\lfloor \frac{1000-1}{3} \right\rfloor = 1000 - \left\lfloor \frac{9000}{3} \right\rfloor = 6000$$

e) are divisible by 5 or 7?

Divisible by 5:

$$\left\lfloor \frac{9999}{5} \right\rfloor - \left\lfloor \frac{1000}{5} \right\rfloor = 1799$$

Divisible by 7:

$$\left\lfloor \frac{9999}{7} \right\rfloor - \left\lfloor \frac{1000}{7} \right\rfloor = 1286$$

Divisible by 5 and 7:

$$\left\lfloor \frac{9999}{5 \cdot 7} \right\rfloor - \left\lfloor \frac{1000}{5 \cdot 7} \right\rfloor = 257$$

divisible by 5 or 7:

$$1799 + 1286 - 257 = 2828$$

f) are not divisible by either 5 or 7?

$$9000 - 2828 = 6172$$

g) are divisible by 5 but not by 7?

$$1799 - 257 = 1542$$

h) are divisible by 5 and 7?

$$257$$

**Problem 2.** (16 points) Section 6.1, Exercise 32, page 397

**Solution.**

How many strings of eight uppercase English letters are there

a) if letters can be repeated?

$$26^8$$

b) if no letter can be repeated?

$$\frac{26!}{18!}$$

c) that start with X, if letters can be repeated?

$$26^7$$

d) that start with X, if no letter can be repeated?

$$\frac{25!}{18!}$$

e) that start and end with X, if letters can be repeated?

$$26^6$$

f) that start with the letters BO (in that order), if letters can be repeated?

$$26^6$$

g) that start and end with the letters BO (in that order), if letters can be repeated?

$$26^4$$

h) that start or end with the letters BO (in that order), if letters can be repeated?

$$2 \cdot 26^6 - 26^4$$

**Problem 3.** (10 points) Section 6.1, Exercise 46, page 397

**Solution.**

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?

$$6 \cdot C(9, 5) = 90720$$

b) both the bride and groom must be in the picture?

$$6 \cdot 5 \cdot C(8, 4) = 50400$$

c) exactly one of the bride and the groom is in the picture?

$$6 \cdot C(8, 5) + 6 \cdot C(8, 5) = 80640$$

**Problem 4.** (10 points) Section 6.1, Exercise 62, page 398. Explain.

**Solution.**

Inclusion-Exclusion Principle:

$$|P \cup Q| = |P| + |Q| - |P \cap Q|$$

WolframAlpha defines relatively prime as: Two integers are relatively prime if they share no common positive factors (divisors) except 1

$$n = pq$$

$$|P| = \left\lfloor \frac{n}{p} \right\rfloor$$

$$|Q| = \left\lfloor \frac{n}{q} \right\rfloor$$

$$|P \cap Q| = 1$$

$$|P \cup Q| = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{q} \right\rfloor - 1$$

Since  $P \cup Q$  represent the values of  $p$  and  $q$  that go into  $n$ , it must be the case that  $n - |P \cup Q|$  represent the number of values that are relatively prime to  $n$ .

$$n - |P \cup Q| = n - \left\lfloor \frac{n}{p} \right\rfloor - \left\lfloor \frac{n}{q} \right\rfloor + 1$$

**Problem 5.** (10 points) Section 6.2, Exercise 12, page 405. Explain.

**Solution.**

**Problem 6.** (10 points) Section 6.2, Exercise 14, page 405.

**Solution.**

**Problem 7.** (10 points) Section 6.3, Exercise 20, page 413.

**Solution.**

**Problem 8.** (10 points) Section 6.3, Exercise 22 b), c), d), e), and f), page 414.

**Solution.**

**Problem 9.** (10 points) Section 6.4, Exercise 12, page 421.

**Solution.**

**Problem 10. (Extra credit 10 points)** Section 6.2, Exercise 46, page 407. Prove the claim by contradiction (that is similar to the proof for the generalized pigeonhole principle shown in slide #18 in the lecture slides counting.pdf).

**Solution.**

**Checklist:**

- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?  
(This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?