CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 6

Due dates: Electronic submission of *yourLastName-yourFirstName-hw6.tex* and *yourLastName-yourFirstName-hw6.pdf* files of this homework is due on **Monday, 3/20/2017 before 11:00 a.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Monday, 3/20/2017** *at the beginning of class.* **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph Martinsen **Section: 503**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ______________________________

Problem 1. (5 points) Section 5.1, Exercise 50, page 331

Solution.
In the basis step of this proof, there is an error.

$$\frac{(1+\frac{1}{2})^2}{2} = \left(\frac{3}{2}\right)^2 \frac{1}{2} = \frac{9}{8}$$

and

$$\sum_{i=1}^{1} i = 1$$

are not true or equal as stated.

Problem 2. (15 points) Section 5.2, Exercise 4, pages 341–342

Solution.

a)

<u>**Basis Step**</u>
$P(18) : 1 \cdot 4$ cent stamps and $2 \cdot 7$ cent stamps
$P(19) : 3 \cdot 4$ cent stamps and $1 \cdot 7$ cent stamps
$P(20) : 5 \cdot 4$ cent stamps and $0 \cdot 7$ cent stamps
$P(21) : 0 \cdot 4$ cent stamps and $3 \cdot 7$ cent stamps

b) The inductive hypothesis of this proof is as follows. Any j¢ postage stamp can be made from a combination of 4¢ and 7¢ stamps for all j stamps with $18 \leq k$. where $k \geq 21$.

c) In order to prove the inductive step, show that $k + 1$¢ stamps can be made from 4¢ and 7¢ stamps.

d) With $k \geq 21$, $P(k-3)$ has been shown to be true by the inductive hypothesis. Adding another 4¢ stamp, it must follow that $P(k+1)$ is also true.

e) With the basis step and the inductive step shown to be valid, P has been proved to hold for $n \geq 21$ by strong induction.

Problem 3. (10 points) Section 5.2, Exercise 12, page 342

Solution.

Problem 4. (10 points) Section 5.2, Exercise 30, page 344

Solution.
The basis step shows that $a^0 = 1$. The correct induction hypothesis would be $a^0 = 1$ not $a^j = 1$. The incorrect inductive hypothesis is used in this proof.

Problem 5. (30 points) Section 5.3, Exercise 6, page 357. To prove that your formula is valid, use mathematical induction. For the invalid recursive definitions, explain why they are invalid.

Solution.

a)

Problem 6. (15 points) Section 5.3, Exercise 16, page 358 (use mathematical induction)

Solution.
f_n is the n^{th} Fibonacci number.
Assume $f_0 - f_1 + f_2 - \cdots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$ for $n \geq 1$
Basis Step: Show $P(1)$

$$P(1) = f_0 - f_1 + f_2 = 0 - 1 + 1 = 0$$
$$P(1) = f_{2(1)-1} - 1 = f_1 - 1 = 1 - 1 = 0$$

$\therefore P(1)$ holds
Inductive Step: Show $P(k) \rightarrow P(k+1)$
Assume $P(k)$ **for arbitrary** $k \geq 1$

Show P(k+1):

$$\begin{aligned}
f_0 - f_1 + f_2 - \cdots - f_{2(k+1)-1} + f_{2(k+1)} &= f_0 - f_1 + f_2 - \\
&\cdots - f_{2k-1} + f_{2k} - f_{2k+1} + f_{2k+2} \\
&= f_{2k-1} - 1 - f_{2k+1} + f_{2k+2} && \textbf{By Inductive Hypothesis} \\
&= f_{2k-1} - 1 + f_{2k} \\
&= f_{2k+1} - 1 && \textbf{By Fib.} \\
&= f_{2(k+1)-1} - 1
\end{aligned}$$

$\therefore P(k) \to P(k+1)$ **holds**
$\therefore$ **the statement holds for all** $n \geq 1$ **by mathematical induction**

Problem 7. (15 points) Section 5.3, Exercise 44, page 359

Solution.

Problem 8. (**Extra credit 10 points**) Section 5.3, Exercise 36, page 359

Solution.

Checklist:
- □ Did you type in your name and section?
- □ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- □ Did you sign that you followed the Aggie Honor Code?
- □ Did you solve all problems?
- □ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- □ Did you submit a signed hardcopy of the pdf file in class?