CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 9

Due dates: Electronic submission of *yourLastName-yourFirstName-hw9.tex* and *yourLastName-yourFirstName-hw9.pdf* files of this homework is due on **Wednesday, 4/19/2017 *before* the beginning of class** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Wednesday, 4/19/2017** at the beginning of class. **If any of the three submissions are missing, your work will not be graded. Late submissions of any form will not be accepted.**

Name: Joseph Martinsen **Section: 503**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)
http://mathworld.wolfram.com/EquivalenceRelation.html

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____

Problem 1. (16 points) Section 9.1, Exercise 6, page 581

Solution.

(a)

$$R : x + y = 0$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = 1$, $1 + 1 = 2 \neq 0$
$\therefore R$ is not reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
For $x, y \in \mathbb{R}$, $x + y = 0 = y + x$
$\therefore R$ is symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $x = 1$, $y = -1$, it follows $1 + (-1) = 0$ and $-1 + 1 = 0$ eventhough $x \neq y$
$\therefore R$ is not anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
For 1 and -1, $R(-1, 1) : -1 + 1 = 0$ and $R(1, -1) : 1 + (-1) = 0$ yet $R(1, 1) : 1 + 1 \neq 0$
$\therefore R$ is not transitive

(b)

$$R : x = \pm y$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = x$, $x = x$
$\therefore R$ is reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
$x = \pm y$
$y = \pm x$
$\therefore R$ is symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $-1 = \pm 1$ and $1 = \pm(-1)$ yet $-1 \neq 1$
$\therefore R$ is not anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
For $x = \pm y$ and $y = \pm z$, it follows that $x = \pm z$
$\therefore R$ is transitive

(c)

$$R : x - y \quad \text{is rational}$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = x$, $x - x = 0$, it follows that $x = x$ and since x is rational, the difference must be rational
$\therefore R$ is reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
The difference between two rationals results in a rational number. It follows $x - y$ is rational and $y - x$ is rational.
$\therefore R$ is symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $R(1, 0)$ and $R(0, 1)$, it follows $1 - 0 = 1$ is rational and $0 - 1 = -1$ is rational yet $0 \neq 1$
$\therefore R$ is not anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
For $R(0, 1)$ $R(1, 2)$, it follows $0 - 1 = -1$ is rational and $1 - 2 = -1$ is rational yet $0 \neq 2$ $\therefore R$ is not transitive

(d)

$$R : x = 2y$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = 1$, $1 \neq 2 \cdot 1$
$\therefore R$ is not reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
For $x, y \in \mathbb{R}$, $x = 2y$ and $y = 2x$, it follows that $\frac{1}{2}y \neq 2y$
$\therefore R$ is not symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $x, y \in \mathbb{R}$, $x = 2y$ and $y = 2x$, it follows that $\frac{1}{2}y \neq 2y$
$\therefore R$ is not anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
$R(4, 2) : 4 = 2 \cdot 2$ and $R(2, 1) : 2 = 2 \cdot 1$ yet $4 \neq 1$
$\therefore R$ is not transitive

(e)

$$R : xy \geq 0$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = x$, $x^2 \geq 0$
$\therefore R$ is reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
For $xy \geq 0$, $yx \geq 0$
$\therefore R$ is symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $x = 1$, $y = 2$, it follows $1 \cdot 2 \geq 0$ and $1 \cdot 2 \geq 0$ eventhough $1 \neq 2$
$\therefore R$ is not anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
$R(2, 1) : 2 \cdot 1 \geq 0$ and $R(2, 3) : 2 \cdot 3 \geq 0$ yet $2 \neq 3$
$\therefore R$ is not transitive

(f)

$$R : xy = 0$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = 1$, $1 \cdot 1 \neq 0$
$\therefore R$ is not reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
For $x, y \in \mathbb{R}$, $xy = 0$ it must follow that $yx = 0$
$\therefore R$ is symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $x = 1$, $y = 0$, it follows $1 \cdot 0 = 0$ and $0 \cdot 1 = 0$ eventhough $1 \neq 0$
$\therefore R$ is not anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
$R(1, 0) : 1 \cdot 0 = 0$ and $R(0, 3) : 0 \cdot 3 = 0$ yet $3 \neq 1$
$\therefore R$ is not transitive

(g)

$$R : x = 1$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = 2$, $2 \neq 1$
$\therefore R$ is not reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
For $x, y \in \mathbb{R}$, $x + y = 0 = y + x$
$\therefore R$ is symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $x = 1$, $y = 1$ from R, it follows $x = y$
$\therefore R$ is anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
Since for R(1,1) is the only case that will hold true, $x = z$ because $1 = 1$
$\therefore R$ is transitive

(h)

$$R : x = 1 \text{ or } y = 1$$

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = 2$, $2 \neq 1$
$\therefore R$ is not reflexive

Symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
$x = 1$ or $y = 1$ implies that $x = 1$ and $y = 1$ $\therefore R$ is symmetric

Anit-Symmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all $a, b \in A$
For $R(1, 2)$ and $R(1, 1)$ holds yet $1 \neq 2$ $\therefore R$ is not anti-symmetric

Transitive if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
For $R(1, 2)$ and $R(1, 3)$ holds yet $R(2, 3)$ does not hold $\therefore$ R is not transitive

Problem 2. (10 points) We define on the set $\mathbf{N}_1 = \{1, 2, 3, \cdots\}$ of positive integers a relation $\sim$ such that two positive integers x and y satisfy $x \sim y$ if and only if $x/y = 2^k$ for some integer k. Show that $\sim$ is an equivalence relation.

Solution.

Reflexive if $(a, a) \in R$ for every element $a \in A$
For $x = y = x$, $x \sim x \rightarrow \dfrac{x}{x} = 1 = 2^0$
$\therefore$ R is reflexive **Symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$
$R(x, y) : x/y = 2^{\log x/y}$ where $k = \log x/y$ since x/y is always a number greater than 0
$R(y, x) : y/x = 2^{\log y/x}$ where $k = \log y/x$
$\therefore$ R is symmetric **Transitive** if whenever $(a, b) \in R$ and $(b, a) \in R$ then $(a, c) \in R$, for all $a, b, c \in A$
For $R(x, y)$ and $R(y, z)$ it follows that $R(x, z)$ holds where $k = \log x/z$ $\therefore$ R is transitive
Since the relation is transitive, symmetric, and reflexive, the relation is an equivalence relation.

Problem 3. (10 points) Section 9.5, Exercise 2, page 615

Solution.

(a) R is an euq. relation

(b) R is an euq. relation

(c) R is reflexive and symmetric but not transitive. a and b may share a common parent. b and c may share a common parent but it does not follow that a and c have a common parent.

(d) R is not transitive. a and b may have met. b and c may have met but it does not follow that a and c have met.

(e) R is not transitive. a and b may share a common language. b and c may share a common language but it does not follow that a and c share a common language.

Problem 4. (10 points) Section 9.5, Exercise 16, page 615

Solution.

$$((a, b), (c, d)) \in R \text{ iff } ad = bc$$

$a \cdot b = b \cdot a$ by communicative property of multiplication. It then follows $((a, b), (b, a))$ $\therefore$ R is reflexive

$ad = bc$ relates to $R((a, b), (c, d))$
$bc = ad$ relates to $R((c, d), (a, b))$
$\therefore R$ is symmetric.
Given $R0 : ((a, b), (c, d))$ or $ad = bc$ and $R0 : ((c, d), (e, f))$ or $cf = de$ it follows that $((a, b), (e, f))$ holds because of the following:

$$\begin{aligned} ad &= bc \\ \frac{a}{b}7 &= \frac{c}{d} \\ cf &= de \\ \frac{c}{d} &= \frac{e}{f} \\ \frac{a}{b} &= \frac{e}{f} \\ af &= eb \end{aligned}$$

$\therefore R$ is is transitive
$\therefore R$ is equivalence relation

Problem 5. (10 points) Section 9.5, Exercise 58, page 618

Solution.

Problem 6. (10 points) Section 9.6, Exercise 4, page 630

Solution.

(a) This is not a poset because it is not reflexive.

(b) This is not a poset because it is not antisymetric

(c) Is a poset

(d) Is not a poset because it is not antisymetric

Problem 7. (10 points) Section 9.6, Exercise 16, page 630.
You can answer the subproblem c) by either actually drawing the Hasse diagram or by clearly writing out the cover relation of the Hasse diagram instead of drawing the diagram.

Solution.

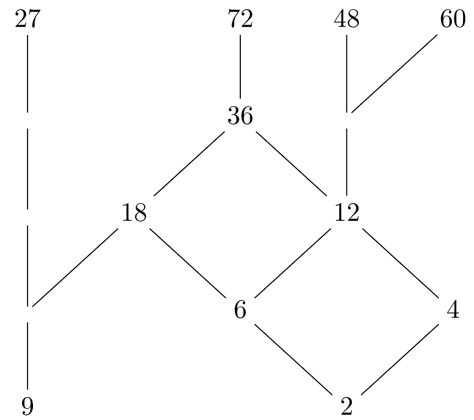
(a) $\{1, 1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 1\}, \{2, 2\}$

(b) $\{3, 2\}, \{3, 3\}, \{3, 4\}, \{4, 1\}, \{4, 2\}, \{4, 3\}, \{4, 4\}$

(c)

Problem 8. ($8 + 8 \times 2 = 24$ points) Section 9.6, Exercise 34, page 631.
First, draw the Hasse diagram or clearly write out the cover relation. This part is worth eight points. Each of the eight subproblems is worth two points. For subproblems c) and d), if it exists, give the number.

Solution.



(a) 27, 48, 60, 72

(b) 2, 9

(c) No greatest

(d) No least

(e) 18, 36, 72

(f) 18

(g) 2, 4, 6, 12

(h) 12

Checklist:
☐ Did you type in your name and section?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit both the .tex and .pdf files of your homework separately to the correct link on eCampus?
☐ Did you submit a signed hardcopy of the pdf file in class?