CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 3

Due dates: Electronic submission of *yourLastName-yourFirstName-hw3.tex* and *yourLastName-yourFirstName-hw3.pdf* files of this homework is due on **Friday, 2/10/2017 before 11:00 a.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 2/10/2017** *at the beginning of class.* **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph Martinsen **Section: 503**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.) https://www.youtube.com/watch?v=Uzlj6N5OYcM

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____

Problem 1. Section 2.1, Exercise 24, page 126. *Explain.*

Solution. Cardinality of set $|A| = k$ and of power set is $|P| = 2^k$ given by lecture and book

a) The power set of every set contains the empty set. A power set then cannot be empty.$\therefore$ this is not a power set

b) Consider in this case $A = \{a\}$
$|A| = 1$
$|P(A)| = 2^1 = 2$ consisting of $\{a\}$ and the empty set. $\therefore$ the given $\{\emptyset, \{a\}\}$ is the power set of $\{a\}$

c) The given power set contains 3 elements. There are no number of elements k that satisfy $2^k = 3$ where $|A| = k$ $\therefore$ this cannot be a power set

d) Consider in this case $A = \{a, b\}$
$|A| = 2$ so it follows that power set must contain $2^2 = 4$ elements.
These elements are $\{a, b\}, \{a\}, \{b\}, \emptyset$ $\therefore$ the given must be the power set of $\{a, b\}$

Problem 2. Show that a set which is a subset of every set must be the empty set.

Solution. $A \subset B$ if and only if every element of A is also a member of B. If A is the empty set, meaning that A has no members within its set, all 0 members of A are also all members of B, no mater what members of B has.
$\therefore$ the empty set is a subset of every set.

Problem 3. Let A and B be sets. Show that $P(A) = P(B)$ implies $A = B$.

Solution.

$$(P(A) = P(B)) \to (A = B)$$

$$\begin{aligned} P(A) &= P(B) \\ \forall C(C \in P(A) \land C \in P(B)) & \qquad \textbf{Universal instantiation} \\ \forall C(C \subseteq A \land CB) \\ A &= B \end{aligned}$$

Problem 4. (**20 Points**) Section 2.2, Exercise 16, page 136.

Solution.

a)

$$\begin{aligned} A \cap B \equiv& \{x | x \in (A \cap B)\} & \textbf{By definition} \\ \equiv& x \text{ must be members of } A \text{ and } B \\ \subseteq& x \text{ is a member of } A & \textbf{This is a proper subset of the previous} \\ \subseteq& A & \textbf{By generalization} \quad \square \end{aligned}$$

b)

$$\begin{aligned} A &= \{x \in A\} & \textbf{By instatiation} \\ &\subseteq \{x \in A \lor x \in B\} & \textbf{By addition} \\ &\subseteq \{x \in A \cup B\} & \textbf{By combination} \\ &\subseteq \{A \cup B\} & \textbf{By generalization} \quad \square \end{aligned}$$

c)

$$\begin{aligned} A - B &= \{x \in (A - B)\} \\ &= \{x \in A \land x \notin B\} \\ &\subseteq \{x \in A\} \\ &\subseteq A \quad \square \end{aligned}$$

d)

$$\begin{aligned}
A \cap (B - A) &= A \cap \{x \in B \land x \notin A\} \\
&= A \cap \{x \in B \land x \in \bar{A}\} \\
&= A \cap (B \cap \bar{A}) \\
&= (A \cap \bar{A}) \cap B && \textbf{By communative property} \\
&= \emptyset \cap B \\
&= \emptyset
\end{aligned}$$

e)

$$\begin{aligned}
A \cup (B - A) &= A \cup \{x \in B \land x \notin A\} \\
&= A \cup \{x \in B \land x \in \bar{A}\} \\
&= A \cup (B \cap \bar{A}) \\
&= (A \cup B) \cap (A \cup \bar{A}) && \textbf{By distribution} \\
&= (A \cup B) \cap U && A \cup \bar{A} = \textbf{Power set} \\
&= A \cup B && \textbf{By } A \cap U = A
\end{aligned}$$

Problem 5. Show that $A \cap (B - C) = (A \cap B) - C$. [Hint: Start out by expanding the definition of $(B - C)$.]

Solution.

$$\begin{aligned}
A \cap (B - C) &= A \cap (x \in B | x \notin C) \\
&= A \cap (B \cap \bar{C}) \\
&= (A \cap B) \cap \bar{C} && \textbf{By communative property} \\
&= \{x \in (A \cap B) | x \in \bar{C}\} \\
&= \{x \in (A \cap B) | x \notin C\} \\
&= (A \cap B) - C \quad \square
\end{aligned}$$

Problem 6. Section 2.3, Exercise 12, page 153. *Explain.*

Solution. a)

$$\begin{aligned}
f(a) &= f(b) \\
a - 1 &= b - 1 \\
a &= b
\end{aligned}$$

$\therefore$ this function is one-to-one

b)

$$\begin{aligned}
f(a) &= f(b) \\
a^2 + 1 &= b^2 + 1 \\
a^2 &= b^2 \\
a &= \pm b
\end{aligned}$$

$\therefore$ this function is NOT one-to-one

c)

$$
\begin{aligned}
f(a) &= f(b) \\
a^3 &= b^3 \\
a &= b
\end{aligned}
$$

For all this holds $\therefore$ this function is one-to-one

d)

$$
\begin{aligned}
f(2) &= \left\lceil \frac{2}{2} \right\rceil = 1 \\
f(1) &= \left\lceil \frac{1}{2} \right\rceil = 1
\end{aligned}
$$

$\therefore$ this function is not one-to-one

Problem 7. Section 2.3, Exercise 14 a), b), c) and d), page 153. *Explain.*

Solution.

a)

$$k = 2m - n \tag{1}$$

k is always $k \in \mathbb{Z}$ because any linear transformations of integers is an integer

$$f(m, n) = 2m - s = p \tag{2}$$

$\therefore$ this function is onto

b) There does not exists any integer solution that satisfy $m^2 - n^2 = 2$ $\therefore$ this function is not onto

c)

$$k = m + n + 1 \tag{3}$$

k is always $k \in \mathbb{Z}$ because any linear transformations of integers is an integer

$$f(a, b) = a + b + 1 = p \tag{4}$$

$\therefore$ this function is onto for any integer value

d) If $m = 0$, $f(0, n) = -|n|$. This equation covers all negative integer values < 0. If $n = 0$, $f(m, 0) = |m|$. This equation covers all positive integer values. If $m = n$, $f(m, m) = |m| - |m| = 0$. This covers 0. $\therefore$ this function is onto for any integer value

Problem 8. Section 2.3, Exercise 50, page 154.

Solution.

$$\begin{aligned} \lceil x+m \rceil &= \lceil x \rceil + m \\ \text{Suppose } \lceil x \rceil &= n \\ \text{It follows } n-1 < x &\le n \\ \text{Add } m,\ n-1+m < x+m &\le n+m \\ \text{By the ceil function definition } \lceil x+m \rceil &= n+m \\ \text{By } \lceil x \rceil = n,\ \lceil x+m \rceil &= \lceil x \rceil + m \quad \square \end{aligned}$$

Problem 9. Section 2.3, Exercise 58, page 154. *Explain.*

Solution. a) There are 8 bits in a byte. $\left\lceil \frac{4}{8} \right\rceil = 1$ To encode only 4 bits, only 1 byte is needed $\left\lceil \frac{10}{8} \right\rceil = 2$ To encode a 10 bits of data, 2 bytes are needed

b) $\left\lceil \frac{500}{8} \right\rceil = 63$ bytes are needed to encode 500 bits

c) $\left\lceil \frac{3000}{8} \right\rceil = 375$ Exactly 375 bytes are needed to encode 3000 bits

Problem 10. (**Extra credit 10 points**) Prove that

$$\left\lceil \left\lceil \frac{x}{2} \right\rceil /2 \right\rceil = \left\lceil \frac{x}{4} \right\rceil$$

holds for all real numbers x.

Solution.

Checklist:
- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?