

CSCE 222 [Sections 502, 503] Discrete Structures for Computing  
Spring 2017 – Hyunyoung Lee

**Problem Set 3**

**Due dates:** Electronic submission of *yourLastName-yourFirstName-hw3.tex* and *yourLastName-yourFirstName-hw3.pdf* files of this homework is due on **Friday, 2/10/2017 before 11:00 a.m.** on <http://ecampus.tamu.edu>. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 2/10/2017 at the beginning of class. If any of the three submissions are missing, your work will not be graded.**

**Name:** Joseph Martinsen

**Section:** 503

**Resources.** (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.) <https://www.youtube.com/watch?v=Uzlj6N5OYcI>

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

**Signature:** \_\_\_\_\_

**Problem 1.** Section 2.1, Exercise 24, page 126. *Explain.*

**Solution.** Cardinality of set  $|A| = k$  and of power set is  $|P| = 2^k$  given by lecture and book

- a) The power set of every set contains the empty set. A power set then cannot be empty.  $\therefore$  this is not a power set
- b) Consider in this case  $A = \{a\}$   
 $|A| = 1$   
 $|P(A)| = 2^1 = 2$  consisting of  $\{a\}$  and the empty set.  $\therefore$  the given  $\{\emptyset, \{a\}\}$  is the power set of  $\{a\}$
- c) The given power set contains 3 elements. There are no number of elements  $k$  that satisfy  $2^k = 3$  where  $|A| = k$   $\therefore$  this cannot be a power set
- d) Consider in this case  $A = \{a, b\}$   
 $|A| = 2$  so it follows that power set must contain  $2^2 = 4$  elements.  
These elements are  $\{a, b\}, \{a\}, \{b\}, \emptyset$   $\therefore$  the given must be the power set of  $\{a, b\}$

**Problem 2.** Show that a set which is a subset of every set must be the empty set.

**Solution.**  $A \subset B$  if and only if every element of  $A$  is also a member of  $B$ . If  $A$  is the empty set, meaning that  $A$  has no members within its set, all 0 members of  $A$  are also all members of  $B$ , no matter what members of  $B$  has.  
 $\therefore$  the empty set is a subset of every set.

**Problem 3.** Let  $A$  and  $B$  be sets. Show that  $P(A) = P(B)$  implies  $A = B$ .

**Solution.**

$$(P(A) = P(B)) \rightarrow (A = B)$$

$$\begin{array}{ll} P(A) = P(B) & \\ \forall C(C \in P(A) \wedge C \in P(B)) & \text{Universal instantiation} \\ \forall C(C \subseteq A \wedge C \subseteq B) & \\ A = B & \end{array}$$

**Problem 4. (20 Points)** Section 2.2, Exercise 16, page 136.

**Solution.** a)

$$\begin{array}{ll} A \cap B \equiv \{x | x \in (A \cap B)\} & \text{By instantiation} \\ \equiv x \in A \wedge x \in B & \text{By Distributive property} \\ \equiv x \in A & \text{By Simplification} \\ \equiv A & \text{By generalization} \end{array}$$

**Problem 5.** Show that  $A \cap (B - C) = (A \cap B) - C$ . [Hint: Start out by expanding the definition of  $(B - C)$ .]

**Solution.**

**Problem 6.** Section 2.3, Exercise 12, page 153. *Explain.*

**Solution.** a)

$$\begin{array}{l} f(a) = f(b) \\ a - 1 = b - 1 \\ a = b \end{array}$$

$\therefore$  this function is one-to-one

b)

$$\begin{array}{l} f(a) = f(b) \\ a^2 + 1 = b^2 + 1 \\ a^2 = b^2 \\ a = \pm b \end{array}$$

$\therefore$  this function is NOT one-to-one

c)

$$\begin{aligned}f(a) &= f(b) \\ a^3 &= b^3 \\ a &= b\end{aligned}$$

For all this holds  $\therefore$  this function is one-to-one

d)

$$\begin{aligned}f(2) &= \left\lceil \frac{2}{2} \right\rceil = 1 \\ f(1) &= \left\lceil \frac{1}{2} \right\rceil = 1\end{aligned}$$

$\therefore$  this function is not one-to-one

**Problem 7.** Section 2.3, Exercise 14 a), b), c) and d), page 153. *Explain.*

**Solution.**

a)

$$k = 2m - n \tag{1}$$

$k$  is always  $k \in \mathbb{Z}$  because any linear transformations of integers is an integer

$$f(m, n) = 2m - s = p \tag{2}$$

$\therefore$  this function is onto

b) There does not exist any integer solution that satisfy  $m^2 - n^2 = 2$   $\therefore$  this function is not onto

c)

**Problem 8.** Section 2.3, Exercise 50, page 154.

**Solution.**

**Problem 9.** Section 2.3, Exercise 58, page 154. *Explain.*

**Solution.** a) There are 8 bits in a byte.  $\left\lceil \frac{4}{8} \right\rceil = 1$  To encode only 4 bits, only

1 byte is needed  $\left\lceil \frac{10}{8} \right\rceil = 2$  To encode a 10 bits of data, 2 bytes are needed

b)  $\left\lceil \frac{500}{8} \right\rceil = 63$  bytes are needed to encode 500 bits

c)  $\left\lceil \frac{3000}{8} \right\rceil = 375$  Exactly 375 bytes are needed to encode 3000 bits

**Problem 10. (Extra credit 10 points)** Prove that

$$\left\lceil \left\lceil \frac{x}{2} \right\rceil / 2 \right\rceil = \left\lceil \frac{x}{4} \right\rceil$$

holds for all real numbers  $x$ .

**Solution.**

**Checklist:**

- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?  
(This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?