CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 2

Due dates: Electronic submission of *yourLastName-yourFirstName-hw2.tex* and *yourLastName-yourFirstName-hw2.pdf* files of this homework is due on **Friday, 2/3/2017 before 11:00 a.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 2/3/2017** *at the beginning of class.* **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph Martinsen **Section: 503**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ______________________________

Problem 1. Section 1.4, Exercise 30, page 54.

Solution.
$x \in \{1, 2, 3\} \quad y \in \{1, 2, 3\}$

a)

$$\exists x\ P(x, 3) \equiv P(1, 3) \lor P(2, 3) \lor P(3, 3)$$

b)

$$\forall y\ P(1, y) \equiv P(1, 1) \land P(1, 2) \land P(1, 3)$$

c)

$$\exists y\ \neg P(2, y) \equiv \neg P(2, 1) \lor \neg P(2, 2) \lor \neg P(2, 3)$$

d)

$$\forall x\ \neg P(x, 2) \equiv \neg P(1, 2) \land \neg P(2, 2) \land \neg P(3, 2)$$

Problem 2. Section 1.4, Exercise 36, page 55.

Solution.

a) When $x = 1,\ 1^2 = 1$ and $x = 0,\ 0^2 = 0$
$\therefore$ The counter examples are $x = 0, 1$

b) When $x = \sqrt{2},\ (\sqrt{2})^2 = 2$ and $x = -\sqrt{2},\ (-\sqrt{2})^2 = 2$
$\therefore$ The counter examples are $x = \pm\sqrt{2}$

c) When $x = 0,\ |0| = 0$ it follows that $0 \not> 0$
$\therefore$ The counter example is $x = 0$

Problem 3. Section 1.5, Exercise 28 b), c), e), and i), page 67. *Justify your answer or give a counterexample.*

Solution.
b) $\forall x \exists y (x = y^2)$ is **false** for when $x = -1$ because there is no real number that satisfies $y^2 = -1$

c) $\exists x \forall y (xy = 0)$ is **true** for all y when $x = 0$

e) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$ for all x that does not equal 0 there exists a real number y that is equivalent to $\frac{1}{x}$ and it can be shown that $x \cdot \frac{1}{x} = 1$ $\therefore$ the statement is **true**

i) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$
For when $x = 0$ the equation $x + y = 2$ the solution for y is 2. Also for when $x = 0$ the equation $2x - y = 1$ the solution for y is -1
$\therefore$ the statement is **false** with $x = 0$ as a counterexample.

Problem 4. Section 1.5, Exercise 46, page 68. *Justify your answer or give a counterexample.*

Solution.
$\exists x \forall y\ (x \leq y^2)$

a) $\mathbb{D} = \{x, y \in \mathbb{R}^+\}$ for when $x = 1$

b) $\mathbb{D} = \{x, y \in \mathbb{Z}\}$

c) $\mathbb{D} = \{x, y \in \mathbb{R} | (\text{x, y}) \neq 0\}$

Problem 5. Section 1.6, Exercise 6, page 78.

Solution.

Problem 6. Section 1.6, Exercise 14 d), page 79.

Solution.
There is someone in this class who has been to France.
Everyone who goes to France visits the Louvre.
Therefore, someone in this class has visited the Louvre

$$
\begin{aligned}
P(x) &: x \text{ is in this class} \\
Q(x) &: x \text{ has been to France} \\
R(x) &: x \text{ has visited the Lourve}
\end{aligned}
$$

$$
\begin{array}{rrr}
\exists x(P(x) \wedge Q(x) & \textbf{Given} & (1) \\
\forall x(Q(x) \rightarrow R(x)) & \textbf{Given} & (2) \\
P(a) \wedge Q(a) & \textbf{Exisitential instantiation on (1)} & (3) \\
Q(a) \rightarrow R(a) & \textbf{Universal instatiation on (2)} & (4) \\
P(a) & \textbf{Simplification on (3)} & (5) \\
Q(a) & \textbf{Simplification on (3)} & (6) \\
R(a) & \textbf{Modus Ponnes on (4) and (6)} & (7) \\
P(a) \wedge R(a) & \textbf{Conjunction on (5) and (7)} & (8) \\
\exists x(P(x) \wedge R(x)) \quad \square & \textbf{Existential generation on (8)} & (9)
\end{array}
$$

Problem 7. Section 1.7, Exercise 18, page 91.

Solution.

Problem 8. Section 1.7, Exercise 22, page 91. *Prove by contradiction.*

Solution.

Problem 9. Section 1.7, Exercise 24, page 91. *Prove by contradiction.*

Solution.

Problem 10. Let $n > 1$ be an integer. *Prove by contradiction* that if n is a perfect square, then $n + 3$ cannot be a perfect square.

Solution.

Checklist:
- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?