CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 5

Due dates: Electronic submission of *yourLastName-yourFirstName-hw5.tex* and *yourLastName-yourFirstName-hw5.pdf* files of this homework is due on **Friday, 3/3/2017 before 11:00 a.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 3/3/2017** *at the beginning of class.* **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph Martinsen **Section: 506**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____

Problem 1. (10 points) Section 2.4, Exercise 6 (b)–(f), pages 167–168

Solution.

b) 3 6 10 15 21 28 36 45 55

c) 1 5 19 65 211 665 2059 6305 19171 58025

d) 1 1 1 2 2 2 2 2 3 3

e) 1 5 6 11 17 28 45 73 118 191

f) 1 3 7 15 31 63 127 255 511 1023

Problem 2. (10 points) Section 2.4, Exercise 10, page 168

Solution.

a) -1 -2 -4 -8 -16 -32

b) 2 -1 -3 -2 1 3

c) 1 3 27 2187 14348907 617673396283947

d) -1 0 1 3 13 74

e) 1 1 2 2 1 1

Problem 3. (20 points) Section 2.4, Exercise 16 (a)–(e), page 168

Solution.

a)

$$
\begin{aligned}
a_n &= -a_{n-1} \; a_0 = 5 \\
a_1 &= -5 \\
a_2 &= 5 \\
a_3 &= -5 \\
a_4 &= 5 \\
&\vdots \\
a_n &= 5(-1)^n
\end{aligned}
$$

b)

$$
\begin{aligned}
a_n &= a_{n-1} + 3 \; a_0 = 1 \\
a_1 &= 1 + 3 = 4 \\
a_2 &= 4 + 3 = 7 \\
a_3 &= 7 + 3 = 10 \\
&\vdots \\
a_n &= 3n + 1
\end{aligned}
$$

c)

$$
\begin{aligned}
a_n &= a_{n-1} - n \; a_0 = 4 \\
a_1 &= 4 - 1 \\
a_2 &= 4 - 1 - 2 \\
a_3 &= 4 - 1 - 2 - 3 \\
a_4 &= 4 - 1 - 2 - 3 - 4 \\
&\vdots \\
a_n &= 4 - \sum_{i=1}^{n} i \\
a_n &= 4 - \frac{n(n+1)}{2}
\end{aligned}
$$

d)

$$
\begin{aligned}
a_n &= 2a_{n-1} - 3 \; a_0 = -1 \\
a_1 &= 2a_0 - 3 \\
a_2 &= 2(2a_0 - 3) - 3 \\
a_3 &= 2(2(2a_0 - 3) - 3) - 3 \\
a_3 &= 2^3 a_0 - 4 \cdot 3 - 2 \cdot 3 - 3 \\
a_3 &= 2^3 a_0 - 2^2 \cdot 3 - 2^1 \cdot 3 - 2^0 \cdot 3 \\
&\vdots \\
a_n &= 2^n - 3\frac{(2^n - 1)}{2 - 1} \\
a_n &= 2^n - 3 \cdot 2^n - 3
\end{aligned}
$$

e)

$$
\begin{aligned}
a_n &= (n+1)a_{n-1} \; a_0 = 2 \\
a_n &= (n+1)na_{n-2} \\
a_n &= (n+1)(n)(n-1)a_{n-3} \\
a_n &= (n+1)(n)(n-1)(n-2)a_{n-4} \\
a_n &= (n+1)(n)(n-1)(n-2)\ldots(n-(n)+1)a_{n-n} \\
a_n &= (n+1)(n)(n-1)(n-2)\ldots 2 \\
a_n &= 2(n+1)!
\end{aligned}
$$

Problem 4. (10 points) Section 2.4, Exercise 34, page 169

Solution. *does not ask to show*

a) 3

b) 78

c) 9

d) 180

Before attempting the problems below on proof-by-induction, make sure that you have carefully read Section 5.1.

Problem 5. (10 points) *Prove by mathematical induction* that

$$\sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}$$

holds for every non-negative integer n.

Solution.

Assume $\sum_{i=0}^{n} 3^i = \dfrac{3^{n+1}-1}{2}$ **is true for all** $n \geq 0$

Basis Step: Show $P(0)$

$$3^0 = 1$$
$$\frac{3^1-1}{2} = 1$$

$\therefore P(0)$ holds

Inductive Step: Show $P(k) \to P(k+1)$

Assume $P(k)$ **for arbitrary** $k \geq 0$**:** $\sum_{i=0}^{k} 3^i$

Show P(k+1): $\sum_{i=0}^{k+1} 3^i$

$$\begin{aligned}
\sum_{i=0}^{k+1} 3^i &= 3^{k+1} + \sum_{i=0}^{k} 3^i \\
&= 3^{k+1} + \frac{3^{k+1}-1}{2} && \textbf{by Inductive Hypothesis} \\
&= \frac{2 \cdot 3^{k+1} + 3^{k+1} - 1}{2} \\
&= \frac{3^{k+1}(2+1) - 1}{2} \\
&= \frac{3^{k+1}(3) - 1}{2} \\
&= \frac{3^{k+2} - 1}{2}
\end{aligned}$$

$\therefore P(k) \to P(k+1)$ **holds**

$\therefore$ **the statement holds for all** $n \geq 0$ **by mathematical induction**

Problem 6. (15 points) Section 5.1, Exercise 8, page 329 (use mathematical induction)

Solution.

Assume $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = \dfrac{(1-(-7)^{n+1})}{4}$ **is true for all** $n \geq 1$

Basis Step: Show $P(0)$

$$\frac{(1-(-7)^{0+1})}{4} = 2$$
$$2(-7)^0 = 2$$

$\therefore P(0)$ holds
Inductive Step: Show $P(k) \rightarrow P(k+1)$
Assume $P(k)$ **for arbitrary** $k \geq 0$
Show P(k+1):

$$
\begin{aligned}
2 - 2\cdot 7 + 2\cdot 7^2 - \cdots + 2(-7)^k + 2(-7)^{k+1} & \\
\frac{(1-(-7)^{k+1})}{4} + 2(-7)^{k+1} & \quad \textbf{by Inductive Hypothesis} \\
\frac{1-(-7)^{k+1} + 8(-7)^{k+1}}{4} & \\
\frac{1-(8-1)(-7)^{k+1}}{4} & \\
\frac{1-(-7)^{k+2}}{4} &
\end{aligned}
$$

$\therefore P(k) \rightarrow P(k+1)$ **holds**
$\therefore$ **the statement holds for all** $n \geq 0$ **by mathematical induction**

Problem 7. (15 points) Section 5.1, Exercise 24, page 330 (use mathematical induction)

Solution.
Assume $\frac{1}{2n} \leq \frac{[1\cdot 3\cdot 5\ldots(2n-1)]}{2\cdot 4\ldots 2n}$ **is true for all** $n \geq 1$
Basis Step: Show $P(1)$

$$
\begin{aligned}
\frac{1}{2(1)} &= \frac{1}{2} \\
\frac{2(1)-1}{2(1)} &= \frac{[1]}{2}
\end{aligned}
$$

$\therefore P(1)$ holds
Inductive Step: Show $P(k) \rightarrow P(k+1)$
Assume $P(k)$ **for arbitrary** $k \geq 1$
Show P(k+1):

$$\frac{[1 \cdot 3 \cdot 5 \ldots (2k-1)]}{2 \cdot 4 \ldots 2k} \cdot \frac{2(k+1)-1}{2(k+1)}$$
$$\frac{1}{2k} \frac{2(k+1)-1}{2(k+1)} \textbf{by Inductive Hypothesis}$$
$$\frac{2k+1}{4k(k+1)}$$
$$\frac{2k}{4k(k+1)} + \frac{1}{4k(k+1)}$$
$$\frac{1}{2(k+1)} + \frac{1}{4k(k+1)}$$
$$\frac{1}{2(k+1)} \leq \frac{1}{2(k+1)} + \frac{1}{4k(k+1)}$$

$\therefore P(k) \to P(k+1)$ **holds**
$\therefore$ **the statement holds for all** $n \geq 1$ **by mathematical induction**

Problem 8. (10 points) Section 5.1, Exercise 32, page 330 (use mathematical induction)

Solution.
$3m = n^3 + 2n$ **is true for all integers** $n,\ m \geq 1$
Basis Step: Show $P(1)$

$$(1)^3 + 2(1) = 3$$
$$3m = 3 \to m = 1$$

$\therefore P(1)$ holds
Inductive Step: Show $P(k) \to P(k+1)$
Assume $P(k)$ **for arbitrary** $k \geq 1$
Show P(k+1):

$$(k+1)^3 + 2(k+1)$$
$$k^3 + 2k + 3k^2 + 3k + 3$$
$$3m + 3(k^2 + k + 1) \textbf{by Inductive Hypothesis}$$
$$3(m + k^2 + k + 1)$$
$$3n \quad \textbf{where } n = m + k^2 + k + 1$$

$\therefore P(k) \to P(k+1)$ **holds**
$\therefore$ **the statement holds for all** $n \geq 1$ **by mathematical induction**

Problem 9. (**Extra credit 10 points**) Section 5.1, Exercise 14, page 330 (use mathematical induction)

Solution.
$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$ **is true for all** $n \geq 1$
Basis Step: Show $P(1)$

$$\sum_{k=1}^{1} k2^k = 1 \cdot 2^1 = 2$$
$$(1-1)2^{1+1} + 2 = 0 + 2 = 2$$

$\therefore P(1)$ holds
Inductive Step: Show $P(k) \to P(k+1)$
Assume $P(k)$ **for arbitrary** $k \geq 1$
Show P(k+1):

$$\sum_{i=1}^{k+1} i2^i$$
$$\sum_{i=1}^{k} i2^i + (k+1)2^{k+1}$$
$$(k-1)2^{k+1} + 2 + (k+1)2^{k+1} \textbf{by Inductive Hypothesis}$$
$$(k-1+k+1)2^{k+1} + 2$$
$$(2k)2^{k+1} + 2$$
$$k2^{k+2} + 2$$

$\therefore P(k) \to P(k+1)$ **holds**
$\therefore$ **the statement holds for all** $n \geq 1$ **by mathematical induction**

Checklist:
☐ Did you type in your name and section?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
☐ Did you submit a signed hardcopy of the pdf file in class?