

CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 1

Due dates: Electronic submission of *yourLastName-yourFirstName-hw1.tex* and *yourLastName-yourFirstName-hw1.pdf* files of this homework is due on **Friday, 1/27/2017 before 11:00 a.m.** on <http://ecampus.tamu.edu>. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 1/27/2017 at the beginning of class. If any of the three submissions are missing, your work will not be graded.**

Name: Joseph Martinsen

Section: 503

Resources. The Textbook *Discrete Mathematics and Its Applications*
Also, familiar with the topics because I am retaking the course...

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1. Section 1.1, Exercise 6 on page 13.

Solution.

Smartphone	RAM (MB)	ROM (GB)	PIXELS (MP)
A	256	32	8
B	288	64	4
C	128	32	5

- (a) **Phrase:** Smartphone B has the most RAM of these three smart-phones.

Logic: $(B_{RAM} > A_{RAM}) \wedge (B_{RAM} > C_{RAM})$

Analysis:

$$(288 > 256) \wedge (288 > 128)$$

$$T \wedge T$$

$$T$$

\therefore The **Phrase** is T

- (b) **Phrase:** Smartphone C has more ROM or a higher resolution camera than Smartphone B.

Logic: $(C_{ROM} > B_{ROM}) \vee (C_{PIX} > B_{PIX})$

Analysis:

$$(32 > 64) \vee (5 > 4)$$

$$F \vee T$$

$$T$$

\therefore The **Phrase** is T

- (c) **Phrase:** Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

Logic: $(B_{RAM} > A_{RAM}) \wedge (B_{ROM} > A_{ROM}) \wedge (B_{PIX} > A_{PIX})$

Analysis:

$$(288 > 256) \wedge (64 > 32) \wedge (4 > 8)$$

$$T \wedge T \wedge F$$

$$F$$

\therefore The **Phrase** is F

- (d) **Phrase:** If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

Logic: $((B_{RAM} > C_{RAM}) \wedge (B_{ROM} > C_{ROM})) \rightarrow (B_{PIX} > C_{PIX})$

Analysis:

$$((288 > 128) \wedge (64 > 32)) \rightarrow (4 > 5)$$

$$(T \wedge T) \rightarrow F$$

$$T \rightarrow F$$

$$F$$

\therefore The **Phrase** is F

- (e) **Phrase:** Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

Logic: $(A_{RAM} > B_{RAM}) \leftrightarrow (B_{RAM} > A_{RAM})$

Analysis:

$$(256 > 288) \leftrightarrow (288 > 256)$$

$$F \leftrightarrow T$$

$$F$$

\therefore The **Phrase** is F

Problem 2. Section 1.1, Exercise 16 on page 14.

Solution.

- (a) $2 + 2 = 4$ if and only if $1 + 1 = 2$
 $T \leftrightarrow T$
 $\therefore T$
- (b) $1 + 1 = 2$ if and only if $2 + 3 = 4$
 $T \leftrightarrow F$
 $\therefore F$
- (c) $1 + 1 = 3$ if and only if monkeys can fly.
 $F \leftrightarrow F$
 $\therefore T$
- (d) $0 > 1$ if and only if $2 > 1$
 $F \leftrightarrow T$
 $\therefore F$

Problem 3. Section 1.1, Exercise 22 a) – e) on page 14.

Solution.

- (a) It is necessary to wash the boss's car to get promoted.
 p : one washes the boss's car
 q : one gets promoted
if p , then q : if one washes the boss's car then one gets promoted
- (b) Winds from the south imply a spring thaw.
 p : there are winds from the south
 q : there will be a spring thaw
if p , then q : if there are winds from the south then there will be a spring thaw
- (c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
 p : you bought the computer less than a year ago
 q : sufficient condition for the warranty is good
if p , then q : if you bought the computer less than a year ago then sufficient condition for the warranty is good
- (d) Willy gets caught whenever he cheats.
 p : he cheats
 q : gets caught
if p , then q : if Willy cheats then he gets caught.
- (e) You can access the website only if you pay a subscription fee.
 p : you pay a subscription fee
 q : you can access the website
if p , then q : if you pay a subscription fee then you can access the website

Problem 4. Section 1.1, Exercise 40 on page 16.

Solution.

Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p , q , and r have the same truth value and it is false otherwise.

Case 1: If p, q , and r are all T, then the results is

$$\begin{aligned} & (T \vee \neg T) \wedge (T \vee \neg T) \wedge (T \vee \neg T) \\ & \quad T \wedge T \wedge T \\ & \quad T \end{aligned}$$

Case 2: If p , q , and r are all F, then the result is

$$\begin{aligned} & (F \vee \neg F) \wedge (F \vee \neg F) \wedge (F \vee \neg F) \\ & \quad T \wedge T \wedge T \\ & \quad T \end{aligned}$$

Case 3: If p is T and q is F, r will be set to F, then the result is

$$\begin{aligned} & (T \vee \neg F) \wedge (T \vee \neg F) \wedge (F \vee \neg T) \\ & \quad T \wedge T \wedge F \\ & \quad F \end{aligned}$$

Regardless of r , the result is the same.

Case 4: If p is F and q is T, r will be set to F, then the result is

$$\begin{aligned} & (F \vee \neg T) \wedge (T \vee \neg F) \wedge (F \vee \neg F) \\ & \quad F \wedge T \wedge T \\ & \quad F \end{aligned}$$

Regardless of r , the result is the same.

\therefore If p , q , and r are not the same truth value, the proposition will result in false.

Problem 5. Section 1.1, Exercise 42 on page 16. *Explain.*

Solution. $x := 1$

(a) if $x + 2 = 3$ **then** $x := x + 1$

Since x is 1 to begin with, $1 + 2 = 3$ is true

$\therefore x := x + 1$ will occur and x will result in $x = 2$

(b) if $(x + 1 = 3)$ OR $(2x + 2 = 3)$ **then** $x := x + 1$

Since x is 1 to begin with, $(1 + 1 = 3)$ is false and $(2(1) + 2 = 3)$ is false

\therefore the statement will not enter and x will stay 1

(c) if $(2x + 3 = 5)$ AND $(3x + 4 = 7)$ **then** $x := x + 1$

Since x is 1 to begin with, $(2(1) + 3 = 5)$ is true and $(3(1) + 4 = 7)$ is true

$\therefore x := x + 1$ will execute and the result of x is 2

- (d) if $(x + 1 = 2)$ XOR $(x + 2 = 3)$ **then** $x := x + 1$
 Since x is 1 to begin with, $(1 + 1 = 2)$ is true and $(1 + 2 = 3)$ is true
 \therefore the statement $x := x + 1$ will not execute and x will stay as 1
- (e) if $x < 2$ then $x := x + 1$
 Since x is 1 to begin with, $1 < 2$ is true
 \therefore the statement will execute and x will result in being 2

Problem 6. Section 1.2, Exercise 6 on page 22.

Solution.

u : You can upgrade your operating system
 b_{32} : You have a 32-bit processor
 b_{64} : You have a 64-bit processor
 g_1 : Your processor runs at 1 GHz or faster
 g_2 : Your processor runs at 2 GHz or faster
 r_1 : Your processor has at least 1 GB RAM
 r_2 : Your processor has at least 2 GB RAM
 h_{16} : You have at least 16 GB free hard disk space
 h_{32} : You have at least 32 GB free hard disk space
 $u \rightarrow (b_{32} \wedge g_1 \wedge r_1 \wedge h_{16}) \vee (b_{64} \wedge g_2 \wedge r_2 \wedge h_{32})$

Problem 7. Section 1.2, Exercise 22 on page 23. *Explain.*

Solution.

Since both A and B both said that they are knights the following conclusions can be drawn:

A is either a knight or a knave and B is knight or a knave

Problem 8. Section 1.3, Exercise 10 b) and c), page 35.

Solution. (a)

(b) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
F	F	F	T	T	T	T	T
F	F	T	T	F	T	F	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

This is the definition of hypothetical syllogism

$$(c) ((p \wedge (p \rightarrow q)) \rightarrow q$$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$((p \wedge (p \rightarrow q)) \rightarrow q$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

This is the definition of modus ponens

Solve the following two problems by developing a series of logical equivalences, as shown in class and also in Examples 7 and 8 on page 30.

Problem 9. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

Solution.

$$\begin{aligned}
 (p \rightarrow q) \vee (p \rightarrow r) &\equiv (\neg p \vee q) \vee (\neg p \vee r) && \text{By } A \rightarrow B \equiv \neg A \vee B \\
 &\equiv \neg p \vee \neg p \vee q \vee r && \text{By Associative Laws} \\
 &\equiv \neg p \vee q \vee r && \text{By Simplification} \\
 &\equiv \neg p \vee (q \vee r) && \text{By Associative Laws} \\
 &\equiv p \rightarrow (q \vee r) \quad \square && \text{By } \neg A \vee B \equiv A \rightarrow B
 \end{aligned}$$

Problem 10. Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

Solution.

$$\begin{aligned}
 (p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r) &\equiv \neg((p \vee q) \wedge (\neg p \vee r)) \vee (q \vee r) && \text{By } A \rightarrow B \equiv \neg A \vee B \\
 &\equiv (\neg(p \vee q) \vee \neg(\neg p \vee r)) \vee (q \vee r) && \text{By DeMorgan's} \\
 &\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \vee r) && \text{By DeMorgan's} \\
 &\equiv (\neg p \wedge \neg q) \vee q \vee (p \wedge \neg r) \vee r && \text{By Associative Laws} \\
 &\equiv ((\neg p \vee q) \wedge (\neg q \vee q)) \vee (p \wedge \neg r) \vee r && \text{By Distributive Laws} \\
 &\equiv ((\neg p \vee q) \wedge T) \vee (p \wedge \neg r) \vee r && \text{By } \neg A \vee A \equiv T \\
 &\equiv \neg p \vee q \vee (p \wedge \neg r) \vee r && \text{By } A \wedge T \equiv A \\
 &\equiv \neg p \vee q \vee ((p \vee r) \wedge (r \vee \neg r)) && \text{By Distributive Laws} \\
 &\equiv \neg p \vee q \vee ((p \vee r) \wedge T) && \text{By } \neg A \vee A \equiv T \\
 &\equiv \neg p \vee q \vee p \vee r && \text{By } A \wedge T \equiv A \\
 &\equiv T \vee q \vee r && \text{By } \neg A \vee A \equiv T \\
 &\equiv T \vee r && \text{By } T \vee A \equiv T \\
 &\equiv T \quad \square && \text{By } T \vee A \equiv T
 \end{aligned}$$

Checklist:

- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?