

CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 5

Due dates: Electronic submission of *yourLastName-yourFirstName-hw5.tex* and *yourLastName-yourFirstName-hw5.pdf* files of this homework is due on **Friday, 3/3/2017 before 11:00 a.m.** on <http://ecampus.tamu.edu>. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 3/3/2017 at the beginning of class.** **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph Martinsen

Section: 506

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1. (10 points) Section 2.4, Exercise 6 (b)–(f), pages 167–168

Solution.

- b) 3 6 10 15 21 28 36 45 55
- c) 1 5 19 65 211 665 2059 6305 19171 58025
- d) 1 1 1 2 2 2 2 2 3 3
- e) 1 5 6 11 17 28 45 73 118 191
- f) 1 3 7 15 31 63 127 255 511 1023

Problem 2. (10 points) Section 2.4, Exercise 10, page 168

Solution.

- a) -1 -2 -4 -8 -16 -32
- b) 2 -1 -3 -2 1 3
- c) 1 3 27 2187 14348907 617673396283947
- d) -1 0 1 3 13 74

e) 1 1 2 2 1 1

Problem 3. (20 points) Section 2.4, Exercise 16 (a)–(e), page 168

Solution.

a)

$$a_n = -a_{n-1} \quad a_0 = 5$$

$$a_1 = -5$$

$$a_2 = 5$$

$$a_3 = -5$$

$$a_4 = 5$$

$$\vdots$$

$$a_n = 5(-1)^n$$

b)

$$a_n = a_{n-1} + 3 \quad a_0 = 1$$

$$a_1 = 1 + 3 = 4$$

$$a_2 = 4 + 3 = 7$$

$$a_3 = 7 + 3 = 10$$

$$\vdots$$

$$a_n = 3n + 1$$

c)

$$a_n = a_{n-1} - n \quad a_0 = 4$$

$$a_1 = 4 - 1$$

$$a_2 = 4 - 1 - 2$$

$$a_3 = 4 - 1 - 2 - 3$$

$$a_4 = 4 - 1 - 2 - 3 - 4$$

$$\vdots$$

$$a_n = 4 - \sum_{i=1}^n i$$

$$a_n = 4 - \frac{n(n+1)}{2}$$

d)

$$\begin{aligned}
 a_n &= 2a_{n-1} - 3 \quad a_0 = -1 \\
 a_1 &= 2a_0 - 3 \\
 a_2 &= 2(2a_0 - 3) - 3 \\
 a_3 &= 2(2(2a_0 - 3) - 3) - 3 \\
 a_3 &= 2^3 a_0 - 4 \cdot 3 - 2 \cdot 3 - 3 \\
 a_3 &= 2^3 a_0 - 2^2 \cdot 3 - 2^1 \cdot 3 - 2^0 \cdot 3 \\
 &\vdots \\
 a_n &= 2^n - 3 \frac{(2^n - 1)}{2 - 1} \\
 a_n &= 2^n - 3 \cdot 2^n - 3
 \end{aligned}$$

e)

$$\begin{aligned}
 a_n &= (n+1)a_{n-1} \quad a_0 = 2 \\
 a_n &= (n+1)na_{n-2} \\
 a_n &= (n+1)(n)(n-1)a_{n-3} \\
 a_n &= (n+1)(n)(n-1)(n-2)a_{n-4} \\
 a_n &= (n+1)(n)(n-1)(n-2) \dots (n-(n)+1)a_{n-n} \\
 a_n &= (n+1)(n)(n-1)(n-2) \dots 2 \\
 a_n &= 2(n+1)!
 \end{aligned}$$

Problem 4. (10 points) Section 2.4, Exercise 34, page 169

Solution. *does not ask to show*

- a) 3
- b) 78
- c) 9
- d) 180

Before attempting the problems below on proof-by-induction, make sure that you have carefully read Section 5.1.

Problem 5. (10 points) *Prove by mathematical induction that*

$$\sum_{i=0}^n 3^i = \frac{3^{n+1} - 1}{2}$$

holds for every non-negative integer n .

Solution.

Assume $\sum_{i=0}^n 3^i = \frac{3^{n+1} - 1}{2}$ is true for all $n \geq 0$

Basis Step: Show $P(0)$

$$3^0 = 1$$
$$\frac{3^1 - 1}{2} = 1$$

$\therefore P(0)$ holds

Inductive Step: Show $P(k) \rightarrow P(k+1)$

Assume $P(k)$ for arbitrary $k \geq 0$: $\sum_{i=0}^k 3^i$

Show $P(k+1)$: $\sum_{i=0}^{k+1} 3^i$

$$\begin{aligned} \sum_{i=0}^{k+1} 3^i &= 3^{k+1} + \sum_{i=0}^k 3^i \\ &= 3^{k+1} + \frac{3^{k+1} - 1}{2} && \text{by Inductive Hypothesis} \\ &= \frac{2 \cdot 3^{k+1} + 3^{k+1} - 1}{2} \\ &= \frac{3^{k+1}(2+1) - 1}{2} \\ &= \frac{3^{k+1}(3) - 1}{2} \\ &= \frac{3^{k+2} - 1}{2} \end{aligned}$$

$\therefore P(k) \rightarrow P(k+1)$ holds

\therefore the statement holds for all $n \geq 0$ by mathematical induction

Problem 6. (15 points) Section 5.1, Exercise 8, page 329 (use mathematical induction)

Solution.

Problem 7. (15 points) Section 5.1, Exercise 24, page 330 (use mathematical induction)

Solution.

Problem 8. (10 points) Section 5.1, Exercise 32, page 330 (use mathematical induction)

Solution.

Problem 9. (Extra credit 10 points) Section 5.1, Exercise 14, page 330 (use mathematical induction)

Solution.

Checklist:

- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?