CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 7

Due dates: Electronic submission of *yourLastName-yourFirstName-hw7.tex* and *yourLastName-yourFirstName-hw7.pdf* files of this homework is due on **Monday, 3/27/2017 before class begins** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Monday, 3/27/2017** *at the beginning of class*. **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph **Section: Martinsen**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________

Problem 1. (16 points) Section 6.1, Exercise 24, page 396

Solution.
How many positive integers between 1000 and 9999 inclusive

a) are divisible by 9?

$$\left\lfloor \frac{9999}{9} \right\rfloor - \left\lfloor \frac{1000-1}{9} \right\rfloor = \left\lfloor \frac{9000}{9} \right\rfloor = 1000$$

b) are even?

$$\left\lfloor \frac{9999}{2} \right\rfloor - \left\lfloor \frac{1000-1}{2} \right\rfloor = \left\lfloor \frac{9000}{2} \right\rfloor = 4500$$

c) have distinct digits?

$$9 \cdot 9 \cdot 8 \cdot 7 = 4536$$

d) are not divisible by 3?

$$9999 - 999 - \left\lfloor \frac{9999}{3} \right\rfloor - \left\lfloor \frac{1000-1}{3} \right\rfloor = 1000 - \left\lfloor \frac{9000}{3} \right\rfloor = 6000$$

e) are divisible by 5 or 7?
Divisible by 5:
$$\left\lfloor \frac{9999}{5} \right\rfloor - \left\lfloor \frac{1000}{5} \right\rfloor = 1799$$
Divisible by 7:
$$\left\lfloor \frac{9999}{7} \right\rfloor - \left\lfloor \frac{1000}{7} \right\rfloor = 1286$$
Divisible by 5 and 7:
$$\left\lfloor \frac{9999}{5 \cdot 7} \right\rfloor - \left\lfloor \frac{1000}{5 \cdot 7} \right\rfloor = 257$$
divisible by 5 or 7:
$$1799 + 1286 - 257 = 2828$$

f) are not divisible by either 5 or 7?
$$9000 - 2828 = 6172$$

g) are divisible by 5 but not by 7?
$$1799 - 257 = 1542$$

h) are divisible by 5 and 7?
$$257$$

Problem 2. (16 points) Section 6.1, Exercise 32, page 397

Solution.
How many strings of eight uppercase English letters are there

a) if letters can be repeated?
$$26^8$$

b) if no letter can be repeated?
$$\frac{26!}{18!}$$

c) that start with X, if letters can be repeated?
$$26^7$$

d) that start with X, if no letter can be repeated?
$$\frac{25!}{18!}$$

e) that start and end with X, if letters can be repeated?
$$26^6$$

f) that start with the letters BO (in that order), if letters can be repeated?

$$26^6$$

g) that start and end with the letters BO (in that order), if letters can be repeated?

$$26^4$$

h) that start or end with the letters BO (in that order), if letters can be repeated?

$$2 \cdot 26^6 - 26^4$$

Problem 3. (10 points) Section 6.1, Exercise 46, page 397

Solution.
In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?

$$6 \cdot C(9, 5) = 90720$$

b) both the bride and groom must be in the picture?

$$6 \cdot 5 \cdot C(8, 4) = 50400$$

c) exactly one of the bride and the groom is in the picture?

$$6 \cdot C(8, 5) + 6 \cdot C(8, 5) = 80640$$

Problem 4. (10 points) Section 6.1, Exercise 62, page 398. Explain.

Solution.
Inclusion-Explusion Principle:

$$\mid P \cup Q \mid = \mid P \mid + \mid Q \mid - \mid P \cap Q \mid$$

WolframAlpha defines relatively prime as: Two integers are relatively prime if they share no common positive factors (divisors) except 1

$$\begin{aligned} n &= pq \\ \mid P \mid &= \left\lfloor \frac{n}{p} \right\rfloor \\ \mid Q \mid &= \left\lfloor \frac{n}{q} \right\rfloor \\ \mid P \cap Q \mid &= 1 \\ \mid P \cup Q \mid &= \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{q} \right\rfloor - 1 \end{aligned}$$

Since $P \cup Q$ represent the values of p and q that go into n, it must be the case that $n- \mid P \cup Q \mid$ represent the number of values that are relatively prime to n.

$$n- \mid P \cup Q \mid = n - \left\lfloor \frac{n}{p} \right\rfloor - \left\lfloor \frac{n}{q} \right\rfloor + 1$$

Problem 5. (10 points) Section 6.2, Exercise 12, page 405. Explain.

Solution.
Let $p(n)$ be $n\%5$. The results of $p(n)$ are $0, 1, 2, 3, 4$. It follows that $\mid p(n) \mid = 5$. From the given statement, it must be the case that $(p(a_1), p(b_1)) = (p(a_2), p(b_2))$. There are $5 \cdot 5 = 25$ different ordered pars of the from $(p(a), p(b))$. By the pigeonhole principle, 26 ordered pairs are needed to guarantee that the statement is satisfied.

Problem 6. (10 points) Section 6.2, Exercise 14, page 405.

Solution.
a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11. b) Is the conclusion in part (a) true if six integers are selected rather than seven?

a) The combination of numbers from the first 10 positive integers that result in 11 are $11 = 10+1 = 9+2 = 8+3 = 7+4 = 6+5$, a total of 5 combinations. Once 5 numbers have been selected that are not from this combination. The resulting 2 choices will complete the set.

b) The statement from a) is false if there are only 6 numbers chosen. Only one can be guaranteed.

Problem 7. (10 points) Section 6.3, Exercise 20, page 413.

Solution. How many bit strings of length 10 have

1. exactly three 0s?

$$C10, 3) = 120$$

2. more 0s than 1s?

$$C(10, 6) + C(10, 7) + C(10, 8) + C(10, 9) + C(10, 10) = 386$$

3. at least seven 1s?

$$C(10, 7) + C(10, 8) + C(10, 9) + C(10, 10) = 176$$

4. at least three 1s?

$$2^{10} - C(10, 3) - C(10, 2) - C(10, 1) - C(10, 0) = 968$$

Problem 8. (10 points) Section 6.3, Exercise 22 b), c), d), e), and f), page 414.

Solution.
b) the string CDE?

$$5! = 720$$

c) the strings BA and FGH ?

$$5! = 120$$

d) the strings AB, DE, and GH ?

$$5! = 120$$

e) the strings CAB and BED?

$$4! = 24$$

f) the strings BCA and ABF ?

$$0$$

Problem 9. (10 points) Section 6.4, Exercise 12, page 421.

Solution.
$\binom{11}{n}$ for $0 \leq n \leq 11$

$$1 \quad 11 \quad 55 \quad 165 \quad 330 \quad 462 \quad 462 \quad 330 \quad 165 \quad 55 \quad 11 \quad 1$$

Problem 10. (**Extra credit 10 points**) Section 6.2, Exercise 46, page 407. Prove the claim by contradiction (that is similar to the proof for the generalized pigeonhole principle shown in slide #18 in the lecture slides counting.pdf).

Solution.
Seeking a contradiction, suppose that the i^{th} box has less than n_i objects. Let the ith box contain $n_i - 1$ objects. The total number of objects will then be $(n_1 - 1) + (n_2 - 1) + \cdots + (n_1 - 1)$ objects. Summed together, this results in $n_1 + n_2 + \cdots - t$. This contradicts $n_1 + n_2 + \ + n_t - t + 1$

Checklist:
- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?