CSCE 222 [Sections 502, 503] Discrete Structures for Computing
Spring 2017 – Hyunyoung Lee

Problem Set 5

Due dates: Electronic submission of *yourLastName-yourFirstName-hw5.tex* and *yourLastName-yourFirstName-hw5.pdf* files of this homework is due on **Friday, 3/3/2017 before 11:00 a.m.** on http://ecampus.tamu.edu. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 3/3/2017** *at the beginning of class.* **If any of the three submissions are missing, your work will not be graded.**

Name: Joseph Martinsen   **Section: 506**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1. (10 points) Section 2.4, Exercise 6 (b)–(f), pages 167–168

Solution.

b) 3 6 10 15 21 28 36 45 55

c) 1 5 19 65 211 665 2059 6305 19171 58025

d) 1 1 1 2 2 2 2 2 3 3

e) 1 5 6 11 17 28 45 73 118 191

f) 1 3 7 15 31 63 127 255 511 1023

Problem 2. (10 points) Section 2.4, Exercise 10, page 168

Solution.

a) -1 -2 -4 -8 -16 -32

b) 2 -1 -3 -2 1 3

c) 1 3 27 2187 14348907 617673396283947

d) -1 0 1 3 13 74

e) 1 1 2 2 1 1

Problem 3. (20 points) Section 2.4, Exercise 16 (a)–(e), page 168

Solution.

a)

$$a_n = -a_{n-1} \quad a_0 = 5$$
$$a_1 = -5$$
$$a_2 = 5$$
$$a_3 = -5$$
$$a_4 = 5$$
$$\vdots$$
$$a_n = 5(-1)^n$$

b)

$$a_n = a_{n-1} + 3 \quad a_0 = 1$$
$$a_1 = 1 + 3 = 4$$
$$a_2 = 4 + 3 = 7$$
$$a_3 = 7 + 3 = 10$$
$$\vdots$$
$$a_n = 3n + 1$$

c)

$$a_n = a_{n-1} - n \quad a_0 = 4$$
$$a_1 = 4 - 1$$
$$a_2 = 4 - 1 - 2$$
$$a_3 = 4 - 1 - 2 - 3$$
$$a_4 = 4 - 1 - 2 - 3 - 4$$
$$\vdots$$
$$a_n = 4 - \sum_{i=1}^{n} i$$
$$a_n = 4 - \frac{n(n+1)}{2}$$

d)

$$a_n = 2a_{n-1} - 3 \quad a_0 = -1$$
$$a_1 = 2a_0 - 3$$
$$a_2 = 2(2a_0 - 3) - 3$$
$$a_3 = 2(2(2a_0 - 3) - 3) - 3$$
$$a_3 = 2^3 a_0 - 4 \cdot 3 - 2 \cdot 3 - 3$$
$$a_3 = 2^3 a_0 - 2^2 \cdot 3 - 2^1 \cdot 3 - 2^0 \cdot 3$$
$$\vdots$$
$$a_n = 2^n - 3\frac{(2^n - 1)}{2 - 1}$$
$$a_n = 2^n - 3 \cdot 2^n - 3$$

e)

$$a_n = (n+1)a_{n-1} \quad a_0 = 2$$
$$a_n = (n+1)na_{n-2}$$
$$a_n = (n+1)(n)(n-1)a_{n-3}$$
$$a_n = (n+1)(n)(n-1)(n-2)a_{n-4}$$
$$a_n = (n+1)(n)(n-1)(n-2)\ldots(n-(n)+1)a_{n-n}$$
$$a_n = (n+1)(n)(n-1)(n-2)\ldots 2$$
$$a_n = 2(n+1)!$$

Problem 4. (10 points) Section 2.4, Exercise 34, page 169

Solution. *does not ask to show*

a) 3

b) 78

c) 9

d) 180

Before attempting the problems below on proof-by-induction, make sure that you have carefully read Section 5.1.

Problem 5. (10 points) *Prove by mathematical induction* that

$$\sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}$$

holds for every non-negative integer n.

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Solution.
Assume $\displaystyle\sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}$ **is true for all** $n \geq 0$
Basis Step: Show $P(0)$

$$3^0 = 1$$
$$\frac{3^1 - 1}{2} = 1$$

$\therefore P(0)$ holds
Inductive Step: Show $P(k) \rightarrow P(k+1)$

Assume $P(k)$ **for arbitrary** $k \geq 0$: $\displaystyle\sum_{i=0}^{k} 3^i$

Show P(k+1): $\displaystyle\sum_{i=0}^{k+1} 3^i$

$$\sum_{i=0}^{k+1} 3^i = 3^{k+1} + \sum_{i=0}^{k} 3^i$$

$$= 3^{k+1} + \frac{3^{k+1} - 1}{2} \qquad \textbf{by Inductive Hypothesis}$$

$$= \frac{2 \cdot 3^{k+1} + 3^{k+1} - 1}{2}$$

$$= \frac{3^{k+1}(2 + 1) - 1}{2}$$

$$= \frac{3^{k+1}(3) - 1}{2}$$

$$= \frac{3^{k+2} - 1}{2}$$

$\therefore P(k) \rightarrow P(k+1)$ **holds**
$\therefore$ **the statement holds for all** $n \geq 0$ **by mathematical induction**

Problem 6. (15 points) Section 5.1, Exercise 8, page 329 (use mathematical induction)

Solution.
Assume $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = \dfrac{(1 - (-7)^{n+1})}{4}$ **is true for all** $n \geq 1$
Basis Step: Show $P(0)$

$$\frac{(1 - (-7)^{0+1})}{4} = 2$$
$$2(-7)^0 = 2$$

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$\therefore P(0)$ holds
Inductive Step: Show $P(k) \to P(k+1)$
Assume $P(k)$ **for arbitrary** $k \geq 0$
Show P(k+1):

$$2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^k + 2(-7)^{k+1}$$

$$\frac{(1 - (-7)^{k+1})}{4} + 2(-7)^{k+1} \text{ by Inductive Hypothesis}$$

$$\frac{1 - (-7)^{k+1} + 8(-7)^{k+1}}{4}$$

$$\frac{1 - (8 - 1)(-7)^{k+1}}{4}$$

$$\frac{1 - (-7)^{k+2}}{4}$$

$\therefore P(k) \to P(k+1)$ **holds**
$\therefore$ **the statement holds for all** $n \geq 0$ **by mathematical induction**

Problem 7. (15 points) Section 5.1, Exercise 24, page 330 (use mathematical induction)

Solution.

Problem 8. (10 points) Section 5.1, Exercise 32, page 330 (use mathematical induction)

Solution.

Problem 9. (**Extra credit 10 points**) Section 5.1, Exercise 14, page 330 (use mathematical induction)

Solution.

Checklist:
☐ Did you type in your name and section?
☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
☐ Did you sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit the .tex and .pdf files of your homework to the correct link
  on eCampus?
☐ Did you submit a signed hardcopy of the pdf file in class?