

CSCE 222 [Sections 502, 503] Discrete Structures for Computing  
Spring 2017 – Hyunyoung Lee

**Problem Set 5**

**Due dates:** Electronic submission of *yourLastName-yourFirstName-hw5.tex* and *yourLastName-yourFirstName-hw5.pdf* files of this homework is due on **Friday, 3/3/2017 before 11:00 a.m.** on <http://ecampus.tamu.edu>. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. A signed paper copy of the pdf file is due on **Friday, 3/3/2017 at the beginning of class.** **If any of the three submissions are missing, your work will not be graded.**

**Name:** Joseph Martinsen

**Section:** 506

**Resources.** (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework.)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

**Signature:** \_\_\_\_\_

**Problem 1.** (10 points) Section 2.4, Exercise 6 (b)–(f), pages 167–168

**Solution.**

- b) 3 6 10 15 21 28 36 45 55
- c) 1 5 19 65 211 665 2059 6305 19171 58025
- d) 1 1 1 2 2 2 2 2 3 3
- e) 1 5 6 11 17 28 45 73 118 191
- f) 1 3 7 15 31 63 127 255 511 1023

**Problem 2.** (10 points) Section 2.4, Exercise 10, page 168

**Solution.**

- a) -1 -2 -4 -8 -16 -32
- b) 2 -1 -3 -2 1 3
- c) 1 3 27 2187 14348907 617673396283947
- d) -1 0 1 3 13 74

e) 1 1 2 2 1 1

**Problem 3.** (20 points) Section 2.4, Exercise 16 (a)–(e), page 168

**Solution.**

a)

$$a_n = -a_{n-1} \quad a_0 = 5$$

$$a_1 = -5$$

$$a_2 = 5$$

$$a_3 = -5$$

$$a_4 = 5$$

$$\vdots$$

$$a_n = 5(-1)^n$$

b)

$$a_n = a_{n-1} + 3 \quad a_0 = 1$$

$$a_1 = 1 + 3 = 4$$

$$a_2 = 4 + 3 = 7$$

$$a_3 = 7 + 3 = 10$$

$$\vdots$$

$$a_n = 3n + 1$$

c)

$$a_n = a_{n-1} - n \quad a_0 = 4$$

$$a_1 = 4 - 1$$

$$a_2 = 4 - 1 - 2$$

$$a_3 = 4 - 1 - 2 - 3$$

$$a_4 = 4 - 1 - 2 - 3 - 4$$

$$\vdots$$

$$a_n = 4 - \sum_{i=1}^n i$$

$$a_n = 4 - \frac{n(n+1)}{2}$$

d)

$$\begin{aligned}
 a_n &= 2a_{n-1} - 3 \quad a_0 = -1 \\
 a_1 &= 2a_0 - 3 \\
 a_2 &= 2(2a_0 - 3) - 3 \\
 a_3 &= 2(2(2a_0 - 3) - 3) - 3 \\
 a_3 &= 2^3 a_0 - 4 \cdot 3 - 2 \cdot 3 - 3 \\
 a_3 &= 2^3 a_0 - 2^2 \cdot 3 - 2^1 \cdot 3 - 2^0 \cdot 3 \\
 &\vdots \\
 a_n &= 2^n - 3 \frac{(2^n - 1)}{2 - 1} \\
 a_n &= 2^n - 3 \cdot 2^n - 3
 \end{aligned}$$

e)

$$\begin{aligned}
 a_n &= (n+1)a_{n-1} \quad a_0 = 2 \\
 a_n &= (n+1)na_{n-2} \\
 a_n &= (n+1)(n)(n-1)a_{n-3} \\
 a_n &= (n+1)(n)(n-1)(n-2)a_{n-4} \\
 a_n &= (n+1)(n)(n-1)(n-2) \dots (n-(n)+1)a_{n-n} \\
 a_n &= (n+1)(n)(n-1)(n-2) \dots 2 \\
 a_n &= 2(n+1)!
 \end{aligned}$$

**Problem 4.** (10 points) Section 2.4, Exercise 34, page 169

**Solution.** *does not ask to show*

- a) 3
- b) 78
- c) 9
- d) 180

*Before attempting the problems below on proof-by-induction, make sure that you have carefully read Section 5.1.*

**Problem 5.** (10 points) *Prove by mathematical induction that*

$$\sum_{i=0}^n 3^i = \frac{3^{n+1} - 1}{2}$$

holds for every non-negative integer  $n$ .

**Solution.**

Assume  $\sum_{i=0}^n 3^i = \frac{3^{n+1} - 1}{2}$  is true for all  $n \geq 0$

Basis Step: Show  $P(0)$

$$3^0 = 1$$

$$\frac{3^1 - 1}{2} = 1$$

$\therefore P(0)$  holds

Inductive Step: Show  $P(k) \rightarrow P(k+1)$

Assume  $P(k)$  for arbitrary  $k \geq 0$ :  $\sum_{i=0}^k 3^i$

Show  $P(k+1)$ :  $\sum_{i=0}^{k+1} 3^i$

$$\begin{aligned} \sum_{i=0}^{k+1} 3^i &= 3^{k+1} + \sum_{i=0}^k 3^i \\ &= 3^{k+1} + \frac{3^{k+1} - 1}{2} && \text{by Inductive Hypothesis} \\ &= \frac{2 \cdot 3^{k+1} + 3^{k+1} - 1}{2} \\ &= \frac{3^{k+1}(2+1) - 1}{2} \\ &= \frac{3^{k+1}(3) - 1}{2} \\ &= \frac{3^{k+2} - 1}{2} \end{aligned}$$

$\therefore P(k) \rightarrow P(k+1)$  holds

$\therefore$  the statement holds for all  $n \geq 0$  by mathematical induction

**Problem 6.** (15 points) Section 5.1, Exercise 8, page 329 (use mathematical induction)

**Solution.**

Assume  $2 - 2 \cdot 7 + 2 \cdot 7^2 - \dots + 2(-7)^n = \frac{(1 - (-7)^{n+1})}{4}$  is true for all  $n \geq 1$

Basis Step: Show  $P(0)$

$$\begin{aligned} \frac{(1 - (-7)^{0+1})}{4} &= 2 \\ 2(-7)^0 &= 2 \end{aligned}$$

$\therefore P(0)$  holds

**Inductive Step:** Show  $P(k) \rightarrow P(k+1)$

**Assume  $P(k)$  for arbitrary  $k \geq 0$**

**Show  $P(k+1)$ :**

$$\begin{aligned}
 & 2 - 2 \cdot 7 + 2 \cdot 7^2 - \dots + 2(-7)^k + 2(-7)^{k+1} \\
 & \quad \frac{(1 - (-7)^{k+1})}{4} + 2(-7)^{k+1} \text{ by Inductive Hypothesis} \\
 & \quad \frac{1 - (-7)^{k+1} + 8(-7)^{k+1}}{4} \\
 & \quad \frac{1 - (8 - 1)(-7)^{k+1}}{4} \\
 & \quad \frac{1 - (-7)^{k+2}}{4}
 \end{aligned}$$

$\therefore P(k) \rightarrow P(k+1)$  holds

$\therefore$  the statement holds for all  $n \geq 0$  by mathematical induction

**Problem 7.** (15 points) Section 5.1, Exercise 24, page 330 (use mathematical induction)

**Solution.**

**Assume**  $\frac{1}{2n} \leq \frac{[1 \cdot 3 \cdot 5 \dots (2n-1)]}{2 \cdot 4 \dots 2n}$  **is true for all  $n \geq 1$**

**Basis Step:** Show  $P(1)$

$$\begin{aligned}
 \frac{1}{2(1)} &= \frac{1}{2} \\
 \frac{2(1) - 1}{2(1)} &= \frac{[1]}{2}
 \end{aligned}$$

$\therefore P(1)$  holds

**Inductive Step:** Show  $P(k) \rightarrow P(k+1)$

**Assume  $P(k)$  for arbitrary  $k \geq 1$**

**Show  $P(k+1)$ :**

$$\begin{aligned}
& \frac{[1 \cdot 3 \cdot 5 \dots (2k-1)]}{2 \cdot 4 \dots 2k} \cdot \frac{2(k+1)-1}{2(k+1)} \\
& \quad \frac{1}{2k} \frac{2(k+1)-1}{2(k+1)} \text{by Inductive Hypothesis} \\
& \quad \frac{2k+1}{4k(k+1)} \\
& \quad \frac{2k}{4k(k+1)} + \frac{1}{4k(k+1)} \\
& \quad \frac{1}{2(k+1)} + \frac{1}{4k(k+1)} \\
& \quad \frac{1}{2(k+1)} \leq \frac{1}{2(k+1)} + \frac{1}{4k(k+1)}
\end{aligned}$$

$\therefore P(k) \rightarrow P(k+1)$  holds

$\therefore$  the statement holds for all  $n \geq 1$  by mathematical induction

**Problem 8.** (10 points) Section 5.1, Exercise 32, page 330 (use mathematical induction)

**Solution.**

$3m = n^3 + 2n$  is true for all integers  $n, m \geq 1$

**Basis Step:** Show  $P(1)$

$$\begin{aligned}
& (1)^3 + 2(1) = 3 \\
& 3m = 3 \rightarrow m = 1
\end{aligned}$$

$\therefore P(1)$  holds

**Inductive Step:** Show  $P(k) \rightarrow P(k+1)$

Assume  $P(k)$  for arbitrary  $k \geq 1$

Show  $P(k+1)$ :

$$\begin{aligned}
& (k+1)^3 + 2(k+1) \\
& k^3 + 2k + 3k^2 + 3k + 3 \\
& 3m + 3(k^2 + k + 1) \text{by Inductive Hypothesis} \\
& 3(m + k^2 + k + 1) \\
& 3n \quad \text{where } n = m + k^2 + k + 1
\end{aligned}$$

$\therefore P(k) \rightarrow P(k+1)$  holds

$\therefore$  the statement holds for all  $n \geq 1$  by mathematical induction

**Problem 9.** (Extra credit 10 points) Section 5.1, Exercise 14, page 330 (use mathematical induction)

**Solution.**

$$\sum_{k=1}^n k2^k = (n-1)2^{n+1} + 2 \text{ is true for all } n \geq 1$$

**Basis Step: Show  $P(1)$**

$$\begin{aligned} \sum_{k=1}^1 k2^k &= 1 \cdot 2^1 = 2 \\ (1-1)2^{1+1} + 2 &= 0 + 2 = 2 \end{aligned}$$

$\therefore P(1)$  holds

**Inductive Step: Show  $P(k) \rightarrow P(k+1)$**

**Assume  $P(k)$  for arbitrary  $k \geq 1$**

**Show  $P(k+1)$ :**

$$\begin{aligned} &\sum_{i=1}^{k+1} i2^i \\ &\sum_{i=1}^k i2^i + (k+1)2^{k+1} \\ &(k-1)2^{k+1} + 2 + (k+1)2^{k+1} \text{ by Inductive Hypothesis} \\ &(k-1+k+1)2^{k+1} + 2 \\ &(2k)2^{k+1} + 2 \\ &k2^{k+2} + 2 \end{aligned}$$

$\therefore P(k) \rightarrow P(k+1)$  holds

$\therefore$  the statement holds for all  $n \geq 1$  by mathematical induction

**Checklist:**

- ☐ Did you type in your name and section?
- ☐ Did you disclose all resources that you have used?  
(This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?
- ☐ Did you submit a signed hardcopy of the pdf file in class?