# CSCE 222: Discrete Structures for Computing
# Section 503
# Fall 2016

YOUR NAME HERE

August 28, 2016

**Problem Set 1**

**Due: 4 September 2016 (Sunday) before 11:59 p.m.** on eCampus (`ecampus.tamu.edu`).

**Problem 1.** (20 points)
You have $n$ coins, exactly one of which is counterfeit. You know counterfeit coins weigh more than authentic coins. Devise an algorithm for finding the counterfeit coin using a balance scale[1]. Express your algorithm in pseudocode. For $n = 12$, how many weighings does your algorithm use?

**Problem 2.** (20 points)
Devise an algorithm that takes as input a list of $n$ integers in unsorted order, where the integers are not necessarily distinct, and outputs the location (index of first element) and length of the longest contiguous non-decreasing subsequence in the list. If there is a tie, it outputs the location of the first such subsequence. Express your algorithm in pseudocode. For the list $9, 7, 9, 4, 5, 8, 1, 0, 5, 9$, what is the algorithm's output? How many comparison operations between elements of the list are used?

**Problem 3.** (20 points)
Arrange the following functions in order such that each function is big-$O$ of the next function: $2 \cdot 3^n$, $3n!$, $2019 \log n$, $\dfrac{n^3}{10^6}$, $n \log n$, $\sqrt{n}$, $3 \cdot 2^n$. Prove your answer is correct by giving the witnesses for each pair of consecutive functions.

**Problem 4.** (20 points)
For each of the following functions, give a big-$O$ estimate, including witnesses, using a simple function $g(n)$ of the smallest order:

1. $(n^2 + 8)(n + 1)$
2. $(n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$
3. $n^{2^n} + n^{n^2}$
4. $\dfrac{n^4 + 5 \log n}{x^3 + 1}$
5. $2x^4 + 7x^3 + 5x + 3$

[1] A balance scale compares the weight of objects placed on it. The result of the comparison is either left side heavier, right side heavier, or both sides equal.

**Problem 5.** (20 points)
For each of the following functions, determine whether that function is of the same order as $n^2$ either by finding witnesses or showing that sufficient witnesses do not exist:

1. $13n + 12$
2. $n^2 + 1000n \log n$
3. $3^n$
4. $3n^2 + n - 5$
5. $\dfrac{n^3 + 2n^2 - n + 3}{4n}$

**Aggie Honor Statement:** On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

**Checklist:**
1. Did you abide by the Aggie Honor Code?
2. Did you solve all problems and start a new page for each?
3. Did you submit the PDF to eCampus?