

CSCE 222: Discrete Structures for Computing
Section 503
Fall 2016

Joseph Martinsen

November 20, 2016

Problem Set 12

Due: 20 November 2016 (Sunday) before 11:59 p.m. on eCampus (ecampus.tamu.edu).
You must show your work in order to receive credit.

Problem 1. (20 points)

Verify that the following program segment is correct with respect to the initial assertion $y = 3$ and the final assertion $z = 6$.

Algorithm 1: program segment

```
1  $x := 2$ 
2  $z := x + y$ 
3 if  $y > 0$  then
4   |  $z := z + 1$ 
5 else
6   |  $z := 0$ 
```

Solution.

$$\begin{aligned} p &\Rightarrow y := 3 \\ x &:= 2 \\ z &:= x + y \Rightarrow 5 \\ \text{Condition: } y &> 0 \\ z &:= 5 + 1 = 6 \\ \therefore p\{S\}q &\text{ because } z = 6 \wedge z = 6 \text{ with given } p \end{aligned}$$

Problem 2. (30 points)

Use a loop invariant to prove that the following program segment for computing the n -th power, where n is a positive integer, of a real number x is correct.

Algorithm 2: program segment

```

1  $power := 1$ 
2  $i := 1$ 
3 while  $i \leq n$  do
4    $power := power * x$ 
5    $i := i + 1$ 

```

Solution.

To show p is a loop invariant, if p is true at the beginning of the loop, then p must still hold true after the execution of the loop

Let $p \Rightarrow i \leq n + 1 \wedge power = x^{i-1}$

Assume p is true,

Check if p is true at the end of the loop

$$\begin{aligned}
 i_{loop} &= i + 1 \\
 power_{loop} &= power * x \\
 &= x^{i-1} * x \\
 &= x^i \\
 &= x^{i_{loop}-1}
 \end{aligned}$$

With condition $i \leq n$

$$i_{loop} \leq n + 1 \text{ follows}$$

$\therefore p$ is true at the end of the loop

Show p is true before loop is executed

It is given that $n \geq 1$ so, $i \leq n + 1$

$$\begin{aligned}
 power &= 1 \\
 &= x^0 \\
 &= x^{i-1}
 \end{aligned}$$

$\therefore p$ is true before the loop executes

Show loop terminates correctly

Loop terminates when p is true and $i \leq n$ is false

$$\begin{aligned}
 i &= n + 1 \text{ so} \\
 power &= x^{(n+1)-i} \\
 &= x^n
 \end{aligned}$$

\therefore Loop terminates correctly

Show loop terminates

$$\begin{aligned}
 i_0 &= 1 \\
 i &= i + 1
 \end{aligned}$$

i will continue to increase until $i > n$ resulting in $i \leq n$ being false

\therefore loop terminates after n iterations

\therefore program is correct

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist: Did you...

1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to eCampus?