CSCE 222: Discrete Structures for Computing
Section 503
Fall 2016

YOUR NAME HERE

October 9, 2016

Problem Set 6

Due: 9 October 2016 (Sunday) before 11:59 p.m. on eCampus (`ecampus.tamu.edu`).
You must show your work in order to recieve credit.

Problem 1. (25 points)

1. Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, \ldots, a_n$ is a sequence of real numbers.
   *This type of sum is called **telescoping**.*

2. Sum both sides of the identity $k^2 - (k-1)^2 = 2k - 1$ from $k = 1$ to $k = n$ and use the previous step to find:

   a. a formula for $\sum_{k=1}^{n}(2k-1)$.

   b. a formula for $\sum_{k=1}^{n} k$.

3. Use the technique given in step 1, together with the results of step 2, to derive the formula for $\displaystyle\sum_{k=1}^{n} k^2$.

   Hint: take $a_k = k^3$ in the telescoping sum in step 1.

Solution.

1. $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_1 - a_0 + a_2 - a_1 + a_3 - a_2 + a_4 - a_3 \ldots + a_n - a_{n-1} + a_{n-1} - a_{n-2} + a_{n-2} - a_{n-3}$
   $\sum_{j=1}^{n}(a_j - a_{j-1}) = \cancel{a_1} - a_0 + \cancel{a_2} - \cancel{a_1} + \cancel{a_3} - \cancel{a_2} + \cancel{a_4} - \cancel{a_3} \ldots + a_n - \cancel{a_{n-1}} + \cancel{a_{n-1}} - \cancel{a_{n-2}} + \cancel{a_{n-2}} - \cancel{a_{n-3}}$
   $= a_n - a_0$

2. sum

Problem 2. (25 points)
Solve the recurrence relation:

1. $A_n = 2 \cdot A_{n-1} + 3$, where $A_0 = 1$
2. $A_n = A_{n-1} + 4n - 2$, where $A_0 = 1$

Solution.

Problem 3. (25 points)
Let $V = \{S, A, B, a, b\}$ and $T = \{a, b\}$. Find the language generated by the grammar (V, T, S, P) when the set P of production rules consists of

1. $S \to AB$, $A \to ab$, $B \to bb$.
2. $S \to AB$, $S \to aA$, $A \to a$, $B \to ba$.
3. $S \to AB$, $S \to AA$, $A \to aB$, $A \to ab$, $B \to b$.
4. $S \to AA$, $S \to B$, $A \to aaA$, $A \to aa$, $B \to bB$, $B \to b$.
5. $S \to AB$, $A \to aAb$, $B \to bBa$, $A \to \lambda$, $B \to \lambda$.

Solution.

Problem 4. (25 points)
Find a phrase-structure grammar for each of these languages.

1. the set consisting of the bitstrings 00, 11, and 010.
2. the set of bitstrings that start with 10 and end with one or more 1s.
3. the set of bitstrings consisting of an odd number of 0s followed by a final 1.
4. the set of bitstrings that have neither two consecutive 0s nor two consecutive 1s.

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist: Did you...
1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to eCampus?