CSCE 222: Discrete Structures for Computing
Section 503
Fall 2016

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Problem Set 5

Due: 2 October 2016 (Sunday) before 11:59 p.m. on eCampus (`ecampus.tamu.edu`).
You must show your work in order to recieve credit.

Problem 1. (25 points)
Suppose that A, B, and C are sets. Prove or disprove that $(A - B) - C = (A - C) - B$.

Solution.

$$\begin{aligned}
\text{Suppose } A &= \{1, 2, 3\} \\
B &= \{1, 2, 3, 4\} \\
C &= \{1\} \\
\\
\text{Then } B - C &= \{2, 3, 4\} \\
A - (B - C) &= 1 \\
\text{Also } A - B &= \emptyset \\
\text{Then } (A - B) - C &= \emptyset \\
\therefore (A - B) - C &\neq (A - C) - B
\end{aligned}$$

Problem 2. (25 points)
Determine whether the symmetric difference is associative; that is, if A, B, and C are sets, does it follow that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$?

a. Use a Venn diagram.

b. Use a membership table.

c. Use set identities.

Solution.

a. Use a Venn diagram.

b. Membership table

A	B	C	$A \oplus B$	$B \oplus C$	$(A \oplus B) \oplus C$	$A \oplus (B \oplus C)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	0	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	0	1	0	0
1	1	1	0	0	1	1

$\therefore A \oplus (B \oplus C) = (A \oplus B) \oplus C$ because they have the same truth values

c. Use set identities.

Problem 3. (25 points)
Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$ if

a. $f(n) = \pm n$

b. $f(n) = \left\lceil \frac{n}{2} \right\rceil$

c. $f(n) = \sqrt{n^2 + 1}$

d. $f(n) = \sqrt{n}$

e. $f(n) = \dfrac{1}{n^2 - 4}$

Solution.

a. $f(n) = \pm n$

b. $f(n) = \left\lceil \frac{n}{2} \right\rceil$

c. $f(n) = \sqrt{n^2 + 1}$

d. $f(n) = \sqrt{n}$

e. $f(n) = \dfrac{1}{n^2 - 4}$

Problem 4. (25 points)

Consider the function $f : \mathbb{Z} \to (\mathbb{N} - \{0\})$ where $f(n) = \begin{cases} 1 - 2n & n \leq 0 \\ 2n & n > 0 \end{cases}$

a. Prove that f is a bijection by showing that it is both injective and surjective.

b. Find the inverse function f^{-1}.

Solution.

a. Prove that f is a bijection by showing that it is both injective and surjective.

b. Find the inverse function f^{-1}.

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist: Did you...

1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to eCampus?