CSCE 222: Discrete Structures for Computing
Section 503
Fall 2016

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November 20, 2016

Problem Set 11

Due: 13 November 2016 (Sunday) before 11:59 p.m. on eCampus (`ecampus.tamu.edu`). You must show your work in order to recieve credit.

Problem 1. (25 points)

1. Give a recursive definition for the set of bitstrings that have more 0s than 1s.
2. Give a recursive definition for the set of bitstrings that have twice as many 0s as 1s.

Solution.

1.

$$0 \in S$$
$$x, y \in S \to xy1, x1y, 1xy, xy \in S$$

2.

$$0 \in S$$
$$x, y \in S \to 1x00, 00x1, 0x1y0, 0xy \in S$$

Problem 2. (25 points)
The **reversal** of a string w, denoted w^R, is the string consisting of the symbols of w in reverse order.

1. Give a recursive definition for the reversal of a string. *Hint: First define the reversal of the empty string. Then write a string w of length $n+1$ as xy, where x is a string of length n and $y \in \Sigma$, and express the reversal of w in terms of x^R and y.*

2. Use structural induction to prove that $(w_1 w_2)^R = w_2^R w_1^R$.

Solution.

Problem 3. (25 points)
The set of leaves and the set of internal vertices of a full binary tree can be defined recursively.

Basis Step: The root r is a leaf of the full binary tree with exactly one vertex r. This tree has no internal vertices.

Recursive Step: The set of leaves of the tree $T = T_1 \cdot T_2$ is the union of the sets of leaves of T_1 and of T_2. The internal vertices of T are the root r of T and the union of the sets of internal vertices of T_1 and of T_2.

Use structural induction to prove that $\ell(T)$, the number of leaves of a full binary tree T, is 1 more than $i(T)$, the number of internal vertices of T.

Solution.

Problem 4. (25 points)

1. Give a recursive algorithm for finding the reversal of a string.
2. Prove that your recursive algorithm is correct.

Solution.
substr(a,b,c) is the substring of a consisting of the symbols in the b through c position

Algorithm 1: reversal(x: bit string)

```
1 l := length(x)
2 if l ≤ 1 then
3 |   reversal(x) := x
4 else
5 |   reversal(x) := substr(x, l, l)reversal(substr(x, 1, l − 1)
```

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist: Did you...

1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to eCampus?