CSCE 222: Discrete Structures for Computing
Section 503
Fall 2016

Joseph Martinsen

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Problem Set 2

Due: 11 September 2016 (Sunday) before 11:59 p.m. on eCampus (`ecampus.tamu.edu`).

Problem 1. (20 points)
For each of the following functions, determine whether that function is of the same order as n^2 either by finding witnesses or showing that sufficient witnesses do not exist:

1. $13n + 12$

   $13n$ is $O(n)$
2. $n^2 + 1000n \log n$
3. 3^n
4. $3n^2 + n - 5$
5. $\dfrac{n^3 + 2n^2 - n + 3}{4n}$

Solution.

1. $13n + 12$

   Is $13n + 12$ $O(n^2)$?

   $13n + 12 \leq C(n^2)$

   $13n + 12 \leq C(13n^2 + 12)$

   $13n + 12 \leq 25n^2 \qquad C = 25;\ k \geq 0$

   Is $13n + 12$ $\Omega(n^2)$?

   $13n + 12 \geq C(n^2)$

   $13n + 12 \ngeq n^2$ for all values of C and k

   $13n + 12$ is $O(n^2)$ with $C = 25$ and $k \geq 0$ as witnesses but $13n + 12$ is not $\Omega(n^2)$ because there are no witnesses that satisify that. Thus $13n+12$ is not $\Theta(n^2)$ and therfore $13n+12$ is not of the same order as n^2

2. $n^2 + 1000n \log n$

   Is $n^2 + 1000n \log n$ $O(n^2)$?

   $n^2 + 1000n \log n \leq C(n^2)$

   $n^2 + 1000n \log n \leq C(n^2 + 1000n^2)$

   $n^2 + 1000n \log n \leq 1001n^2 \qquad C = 1001;\ k \geq 1$

   Is $n^2 + 1000n \log n$ $\Omega(n^2)$?

   $n^2 + 1000n \log n \geq C(n^2)$

   $n^2 + 1000n \log n \geq C(\frac{1}{2}n^2 + 0n^2)$

   $n^2 + 1000n \log n \geq \frac{1}{2}n^2 \qquad C = \frac{1}{2}; k \geq 1$

   $n^2 + 1000n \log n$ is $O(n^2)$ wtih $C = 1001$ and $k \geq 1$ as witnesses and $n^2 + 1000n \log n$ is $\Omega(n^2)$ with $C = \dfrac{1}{2}$ and $k \geq 1$ as witnesses. Thus $n^2 + 1000n \log n$ is $\Theta(n^2)$ and therfore $n^2 + 1000n \log n$ is of the same order as n^2

3. 3^n

Is 3^n $O(n^2)$? Is 3^n $\Omega(n^2)$?

$3^n \leq C(n^2)$ $3^n \geq C(n^2)$

$3^n \nleq n^2$ for all values of $C \geq 0$ and $k \geq 0$ $3^n \geq n^2$ $C = 1;\ k \geq 0$

3^n is not $O(n^2)$ because there are no witnesses that satisify that but 3^n is $\Omega(n^2)$ with $C = 1$ and $k \geq 0$ as witnesses. Thus 3^n is not $\Theta(n^2)$ and therfore 3^n is not of the same order as n^2

4. $3n^2 + n - 5$

Is $3n^2 + n - 5$ $O(n^2)$? Is $3n^2 + n - 5$ $\Omega(n^2)$?

$3n^2 + n - 5 \leq C(n^2)$ $3n^2 + n - 5 \geq C(n^2)$

$3n^2 + n - 5 \leq C(3n^2 + n^2 + n^2)$ $3n^2 + n - 5 \geq C(\frac{1}{2}n^2 + 0n^2 + 0n^2)$

$3n^2 + n - 5 \leq 5n^2$ $C = 5;\ k \geq 1$ $3n^2 + n - 5 \geq \frac{1}{2}n^2$ $C = \frac{1}{2}; k \geq 1$

$3n^2 + n - 5$ is $O(n^2)$ wtih $C = 5$ and $k \geq 1$ as witnesses and $3n^2 + n - 5$ is $\Omega(n^2)$ with $C = \frac{1}{2}$ and $k \geq 1$ as witnesses. Thus $3n^2 + n - 5$ is $\Theta(n^2)$ and therfore $3n^2 + n - 5$ is of the same order as n^2

5. $\dfrac{n^3 + 2n^2 - n + 3}{4n}$

Is $\frac{n^3+2n^2-n+3}{4n}$ $O(n^2)$? Is $\frac{n^3+2n^2-n+3}{4n}$ $\Omega(n^2)$?

$\frac{n^3+2n^2-n+3}{4n} \leq C(n^2)$ $\frac{n^3+2n^2-n+3}{4n} \geq C(n^2)$

$\frac{n^2}{4} + \frac{n}{2} + \frac{3}{4n} \leq C(n^2 + n^2 + n^2)$ $\frac{n^2}{4} + \frac{n}{2} + \frac{3}{4n} \geq C(\frac{1}{5}n^2 + 0n^2 + 0n^2)$

$\frac{n^2}{4} + \frac{n}{2} + \frac{3}{4n} \leq 3n^2$ $C = 3;\ k \geq 1$ $\frac{n^2}{4} + \frac{n}{2} + \frac{3}{4n} \geq \frac{1}{5}n^2$ $C = \frac{1}{5}; k \geq 1$

$\frac{n^3+2n^2-n+3}{4n}$ is $O(n^2)$ wtih $C = 3$ and $k \geq 1$ as witnesses and $\frac{n^3+2n^2-n+3}{4n}$ is $\Omega(n^2)$ with $C = \frac{1}{5}$ and $k \geq 1$ as witnesses. Thus $\frac{n^3+2n^2-n+3}{4n}$ is $\Theta(n^2)$ and therfore $\frac{n^3+2n^2-n+3}{4n}$ is of the same order as n^2

Problem 2. (20 points)
Do Supplementary Exercise 29 of Chapter 3 (page 234).

a) Use pseudocode to specify a brute-force algorithm that determines when given as input a sequence of n positive integers whether there are two distinct terms of the sequence that have as sum a third term. The algorithm should loop through all triples of terms of the sequence, checking whether the sum of the first two terms equals the third. b) Give a big-O estimate for the complexity of the bruteforce algorithm from part (a).

Solution.

```
procedure brute-force(a_1, a_2, a_3, ..., a_n)
for i = 1 to n                         O(n)
    for j = i + 1 to n                 O(n)
        for k = 1 to n                 O(n)
            if a_i + a_j = a_k then    O(1)
                return true
            else
            return false
```

The complexity for the algorithm is $O(n^3)$

Problem 3. (20 points)
Do Exercise 31 of Chapter 1.1 (page 15).

Solution.

a)

p	$\neg\, p$	$p\ and \neg\, p$
0	1	0
1	0	0

b)

p	$\neg\, p$	$p\ or \neg\, p$
0	1	1
1	0	1

c)

p	q	$(p \text{ or} \neg q) \to q$
1	1	1
1	0	0
0	1	1
0	0	0

d)

p	q	$(p \text{ or} \neg q) \to (p\ and\ q)$
1	1	1
1	0	0
0	1	0
0	0	1

e)

p	q	$(p \to q) \leftrightarrow (\neg p \to \neg\, q)$
1	1	1
1	0	1
0	1	1
0	0	1

f)

p	q	$(p \to q) \to (q \to p)$
1	1	1
1	0	1
0	1	0
0	0	1

Problem 4. (20 points)
Do Exercises 19, 21, and 23 of Chapter 1.2 (page 23).

Solution.

19) A is a knight and B is a knave
21) A is a knight and B is a knight
23) A is a knave and B is a knight

Problem 5. (20 points)
Do Exercises 50 and 51 of Chapter 1.3 (page 36).

Solution. 50)

a)

p	p	$\neg p$	$p \downarrow p$
1	1	0	0
1	1	0	0
0	0	1	1
0	0	1	1

$\neg p$ and $p \downarrow p$ have the same truth values

b)

p	q	$p \downarrow q$	$(p \downarrow q) \downarrow (p \downarrow q)$	p or q
1	1	0	1	1
1	0	0	1	1
0	1	0	1	1
0	0	1	0	0

$(p \downarrow q) \downarrow (p \downarrow q)$ and p or q have the same truth values.

c) $p \downarrow q$ is the same as $\neg(p \text{ } or \text{ } q)$. Also, the logical operators $\neg$ and *or* can be used to express any logical expression, $\downarrow$ is logically complete.

51)

p	q	$p \rightarrow q$	$p \downarrow q$	$(p \downarrow p) \downarrow q$	$((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$
1	1	1	0	0	1
1	0	0	0	1	0
0	1	1	1	0	0
0	0	1	1	0	0

Thus $((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$ is the same as $p \rightarrow q$

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist:

1. Did you abide by the Aggie Honor Code?
2. Did you solve all problems and start a new page for each?
3. Did you submit the PDF to eCampus?