CSCE 222: Discrete Structures for Computing
Section 503
Fall 2016

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Problem Set 4

Due: 25 September 2016 (Sunday) before 11:59 p.m. on eCampus (`ecampus.tamu.edu`).
You must show your work in order to recieve credit.

Problem 1. (30 points)
Consider the sets $P = (A - B) - C$ and $Q = (A - C) - (B - C)$.
Determine which relationship $(\subseteq, =, \supseteq)$ holds between the two sets P and Q.
Your answer will be either $P \subseteq Q$, or $P = Q$, or $P \supseteq Q$.
Justify your answer three ways by

1. drawing the Venn diagram,
2. constructing the membership table, and
3. proving it (using set identities with set builder notation).

Solution.

1. Drawing the Venn diagram

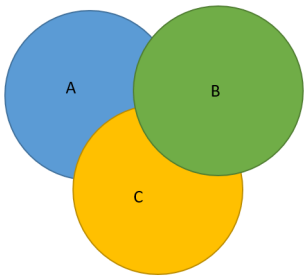

Figure 1: Venn Diagram of both sets

2. Constructing the membership table

A	B	C	$A-B$	$A-C$	$B-C$	$(A-B)-C$	$(A-C)-(B-C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	0	1	0	0
0	1	1	0	0	0	0	0
1	0	0	1	1	0	1	1
1	0	1	1	0	0	0	0
1	1	0	0	1	1	0	0
1	1	1	0	0	0	0	0

$(A-B)-C = (A-C)-(B-C)$ because they have the same truth values

$$\therefore P = Q$$

3. Using set identities with set builder notation.

$$\begin{aligned} A-B &= \{x \mid x \in A \wedge x \notin B\} \\ (A-B)-C &= \{x \mid x \in (A-B) \wedge x \notin C\} \\ &= \{x \mid x \in A \wedge x \notin B \wedge x \notin C\} \end{aligned}$$

$$\begin{aligned} (A-C) &= \{x \mid x \in A \wedge x \notin C\} \\ (B-C) &= \{x \mid x \in B \wedge x \notin C\} \\ (A-C)-(B-C) &= \{x \mid x \in (A-C) \wedge x \notin (B-C)\} \\ &= \{x \mid x \in A \wedge x \notin B \wedge x \notin C\} \\ (A-C)-(B-C) &= (A-B)-C = \{x \mid x \in A \wedge x \notin B \wedge x \notin C\} \\ \therefore P &= Q \end{aligned}$$

Problem 2. (20 points)
Show that if A, B, and C are sets, then $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

Solution.

$$\begin{aligned}
|A \cup B \cup C| &= |A| \cup |B \cup C| \\
&= |A| + |B \cup C| - |A \cap |B \cup C|| \\
&= |A| + |B + C - B \cap C| - ||A \cap B| \cup |A \cap C|| \\
&= |A| + |B| + |C| - |B \cap C| - ||A \cap B| + |A \cap C| - |A \cap B| \cap |A \cap C|| \\
&= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|
\end{aligned}$$

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist: Did you...

1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to eCampus?