Problem Set D
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Math 308-510
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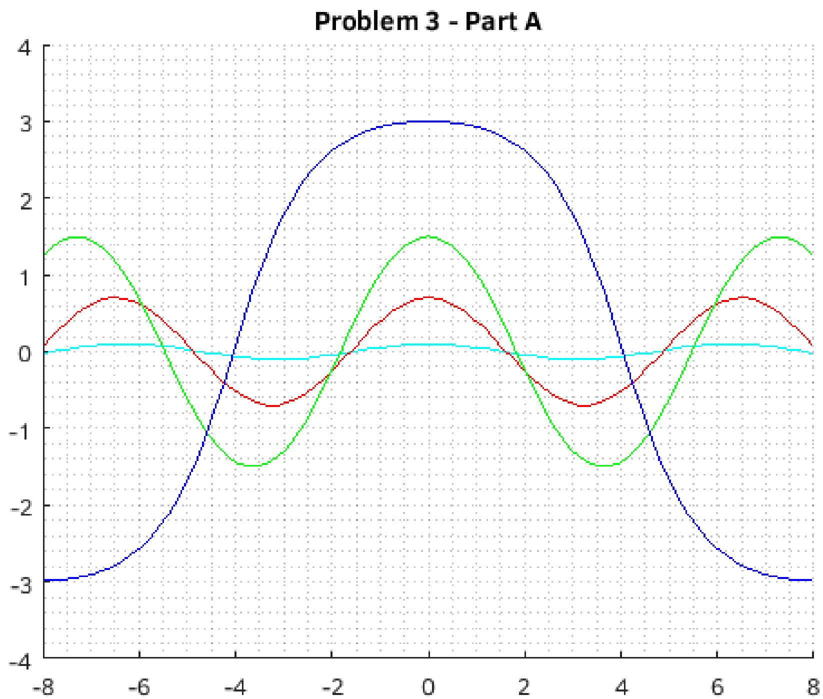
Contents

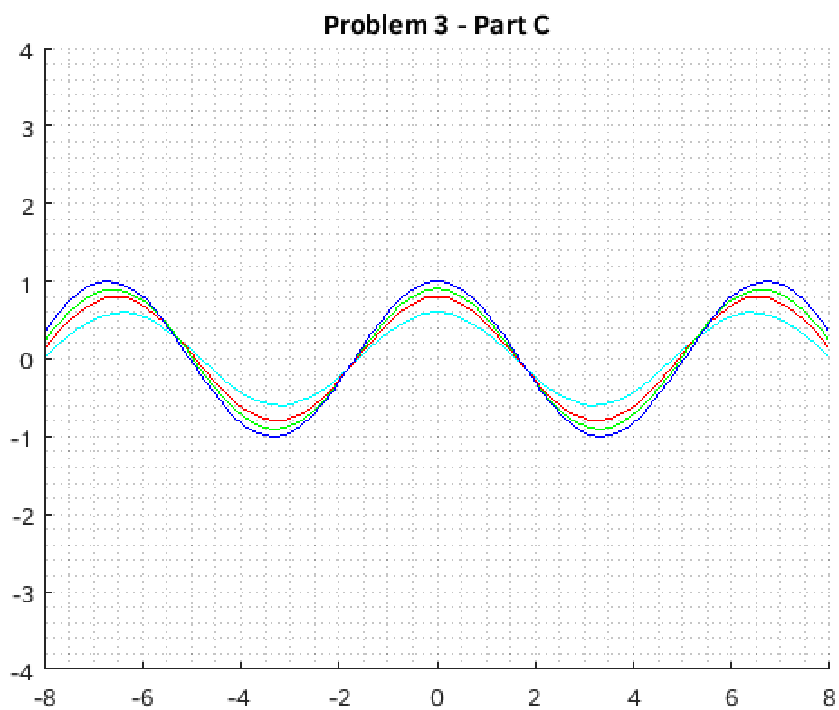
Problem 3

```
clear;

% Part A
% y'' + sin(y) = 0
rhs = @(t,y) [y(2); - sin(y(1))];
figure; hold on;
% Amplitude values
A = [.1 .7 1.5 3];
% Colors for the Graph
C = ['c' 'r' 'g' 'b' 'w'];
fprintf('Problem 3\n\nPart A\n\n')
for i = 1:4
    [xa, ya ]= ode45(rhs,[0 8], [A(i) ; 0]);
    p(i) = findPeriod(xa, ya(:,1),0);
    plot(xa,ya(:,1),C(i))
    [xa, ya ]= ode45(rhs,[0 -8], [A(i) ; 0]);
    plot(xa,ya(:,1),C(i))
    fprintf('Estimated Period for A = %.2f is %.5f\n\n', A(i), p(i))
end
axis([-8 8 -4 4])
title('Problem 3 - Part A')
grid minor

% Part B
fprintf('\nPart B\n\n')
for i = 1:4
    F = @(x) 1./sqrt(1 - sin(A(i)./2).^2 * sin(x).^2);
    Q = 4 * quad(F,0,pi/2);
    fprintf('Using elliptic integral for A = %.2f I get %.5f\n', A(i), Q)
    fprintf('This value is %.4f different from the value in A\n\n', abs(p(i) - Q))
end

% partc C
fprintf('Part C\n\n')
figure; hold on;
% Amplitude values
A = [.6 .8 .9 1];
% Colors for the Graph

for i = 1:4
    [xa, ya ]= ode45(rhs,[0 8], [A(i) ; 0]);
    p(i) = findPeriod(xa, ya(:,1),0);
    plot(xa,ya(:,1),C(i))
    [xa, ya ]= ode45(rhs,[0 -8], [A(i) ; 0]);
    plot(xa,ya(:,1),C(i))
    fprintf('Estimated Period for A = %.2f is %.5f\n\n', A(i), p(i))
end
axis([-8 8 -4 4])
```

```
title('Problem 3 - Part C')
grid minor

for i = 1:4
    F = @(x) 1./sqrt(1 - sin(A(i)./2).^2 * sin(x).^2);
    Q = 4 * quad(F,0,pi/2);
    fprintf('Using elliptic integral for A = %.2f I get %.5f\n', A(i), Q)
    fprintf('This value is %.4f different from the value in C before\n\n', abs(p(i) - Q))
end

% Part D
fprintf('Part D\n\n')
fprintf('The change in the period is not much for values of A < 2. Once A>2 the period shoots up\n')
```

```
Problem 3

Part A

Estimated Period for A = 0.10 is 5.90021

Estimated Period for A = 0.70 is 6.54814

Estimated Period for A = 1.50 is 7.54754

Estimated Period for A = 3.00 is 15.77415


Part B

Using elliptic integral for A = 0.10 I get 6.28711
This value is 0.3869 different from the value in A

Using elliptic integral for A = 0.70 I get 6.48119
This value is 0.0670 different from the value in A

Using elliptic integral for A = 1.50 I get 7.30087
This value is 0.2467 different from the value in A

Using elliptic integral for A = 3.00 I get 16.15556
This value is 0.3814 different from the value in A

Part C

Estimated Period for A = 0.60 is 6.67824

Estimated Period for A = 0.80 is 6.48574

Estimated Period for A = 0.90 is 6.48009

Estimated Period for A = 1.00 is 6.52801

Using elliptic integral for A = 0.60 I get 6.42754
This value is 0.2507 different from the value in C before

Using elliptic integral for A = 0.80 I get 6.54413
This value is 0.0584 different from the value in C before

Using elliptic integral for A = 0.90 I get 6.61687
This value is 0.1368 different from the value in C before

Using elliptic integral for A = 1.00 I get 6.69998
This value is 0.1720 different from the value in C before

Part D

The change in the period is not much for values of A < 2. Once A>2 the period shoots up
```





Problem 7

clear; G = 32; M = 195/G; % rhs = @(t,y) [y(2); -sqrt((1 + y(1)) * (32) / (0.005/M + 0.5))]; rhs = @(t,y) [y(2); -sqrt((M*G*y(1) - M*G) / (.5*M + 0.005))]; [xa, ya]= ode45( ); a = ode45(rhs,[0 5], [1000 ; 0]); deval(a,0)

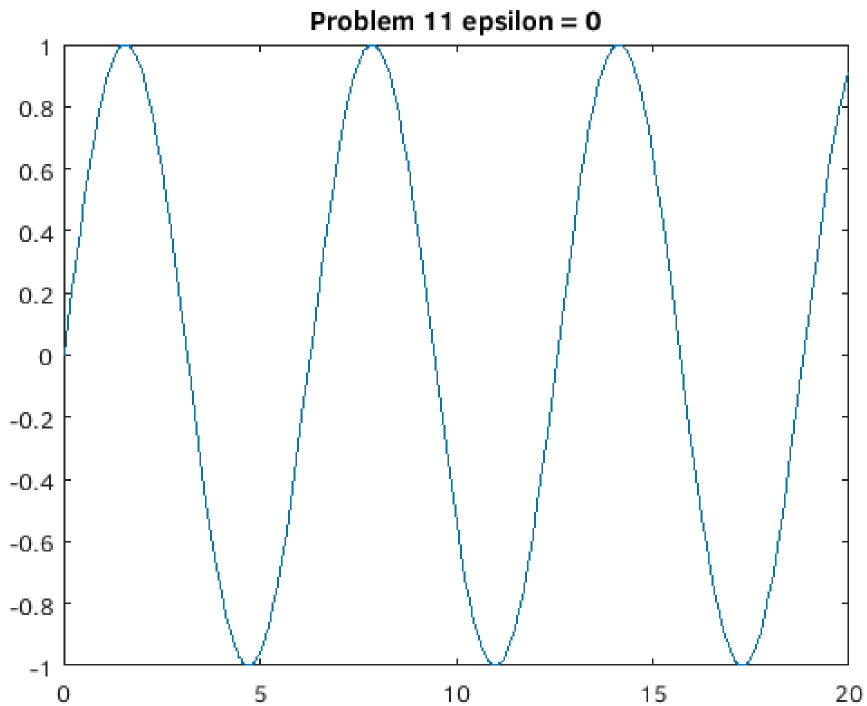
Problem 11

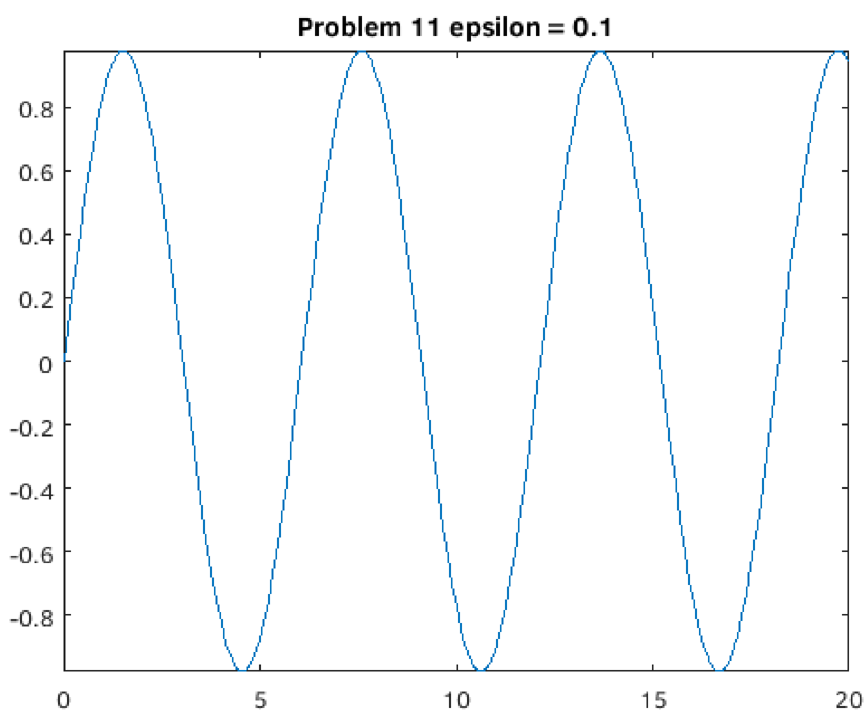
```
clear;
fprintf('Problem 11\n\n')
syms y(t)
```

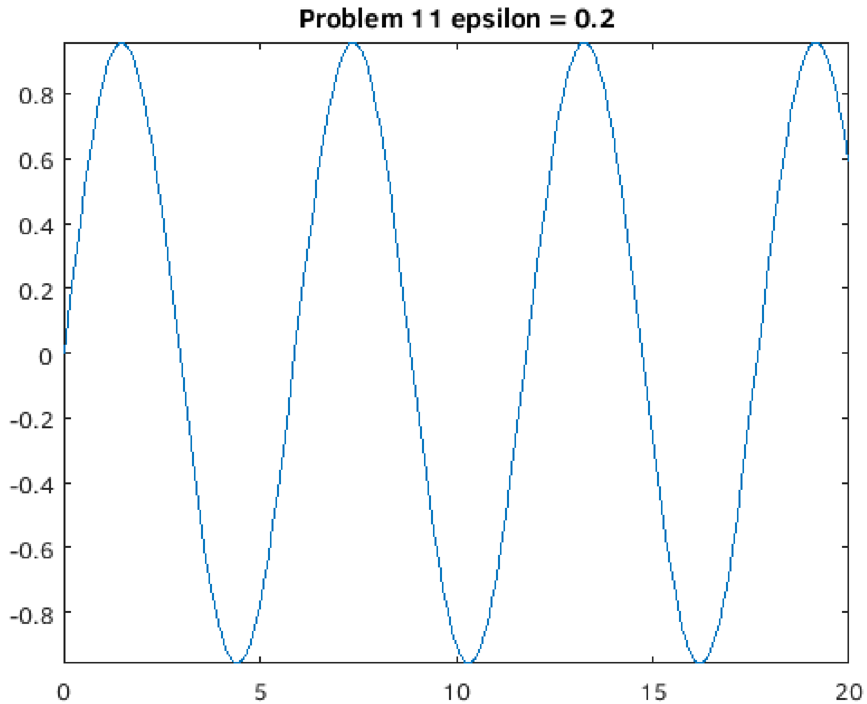
```
k = 1;
m = k;
lam = 0;
i = [0 .1 .2 .3];
j = 1;

for ep = i
    [V] = odeToVectorField(m * diff(y, 2) + lam * diff(y,1) + k * y + ep * y ^ 3 == 0);
    M = matlabFunction(V,'vars', {'t','Y'});
    sol = ode45(M,[0 20], [0;1]);
    [xa, ya] = ode45(M,[0 20], [0;1]);
    p(j) = findPeriod(xa, ya(:,1),11);
    a(j) = findAmplitude(ya);
    fprintf('For epsilon = %.1f the estimated Amplitude is %.3f and Period is %.3f\n\n', ep, a(j), p(j))
    j = j + 1;
    figure
    fplot(@(x)deval(sol,x,1), [0, 20])
    title(['Problem 11 epsilon = ' num2str(ep)]);
end

figure; hold on;
plot(p,i)
plot(a,i)
title('Period and Amplitude vs epsilon')
hold off

% repeat for negative values
fprintf('Now for negative Values of epsilon\n\n')
i = [0 -.1 -.2 -.3];
j = 1;
% for epsilon = 0

for ep = i
    [V] = odeToVectorField(m * diff(y, 2) + lam * diff(y,1) + k * y + ep * y ^ 3 == 0);
    M = matlabFunction(V,'vars', {'t','Y'});
    sol = ode45(M,[0 20], [0;1]);
    [xa, ya] = ode45(M,[0 20], [0;1]);
    p2(j) = findPeriod(xa, ya(:,1),11);
    a2(j) = findAmplitude(ya);
    fprintf('For epsilon = %.1f the estimated Amplitude is %.3f and Period is %.3f\n\n', ep, a2(j), p2(j))
    j = j + 1;
    figure
    fplot(@(x)deval(sol,x,1), [0, 20])
    title(['Problem 11 epsilon = ' num2str(ep)]);
end

figure; hold on;
plot(p2,i)
plot(a2,i)
title('Period and Amplitude vs -epsilon')
```

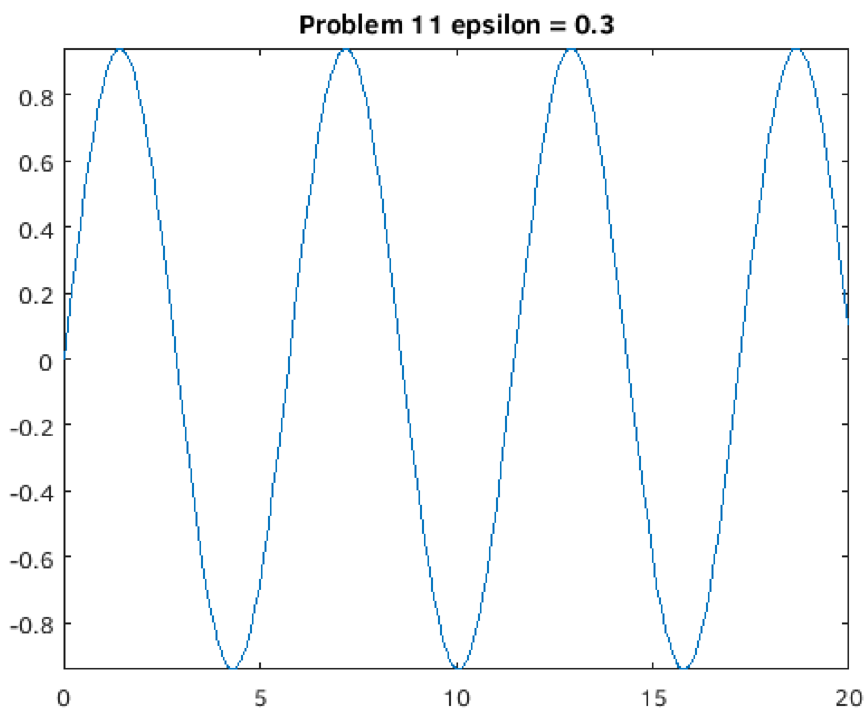
```
Problem 11

For epsilon = 0.0 the estimated Amplitude is 0.990 and Period is 6.487

For epsilon = 0.1 the estimated Amplitude is 0.962 and Period is 6.213

For epsilon = 0.2 the estimated Amplitude is 0.957 and Period is 6.012

For epsilon = 0.3 the estimated Amplitude is 0.939 and Period is 5.858

Now for negative Values of epsilon

For epsilon = 0.0 the estimated Amplitude is 0.990 and Period is 6.487

For epsilon = -0.1 the estimated Amplitude is 1.021 and Period is 6.470
```

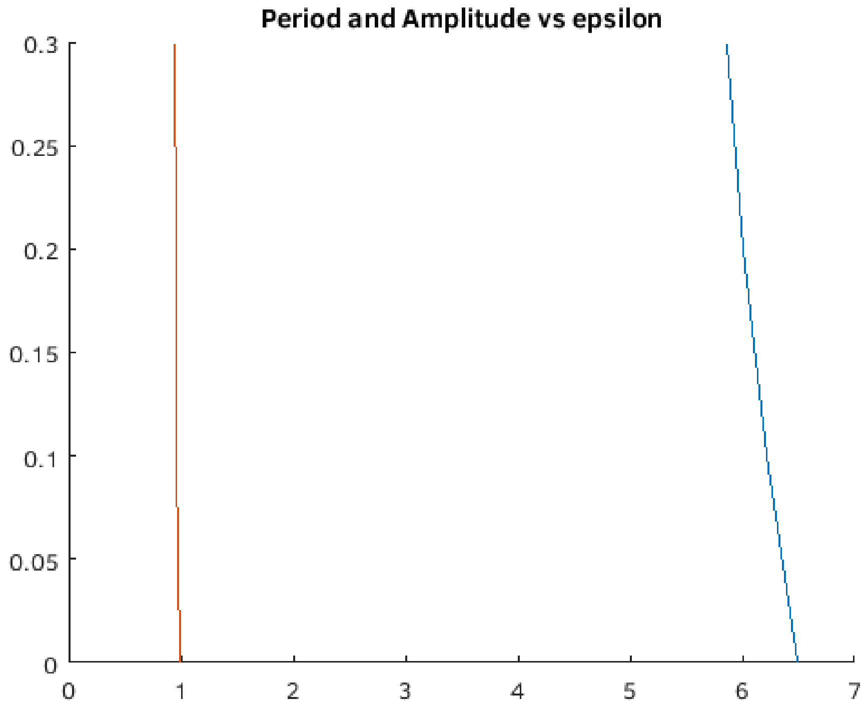
```
For epsilon = -0.2 the estimated Amplitude is 1.057 and Period is 6.984

For epsilon = -0.3 the estimated Amplitude is 1.093 and Period is 7.551
```

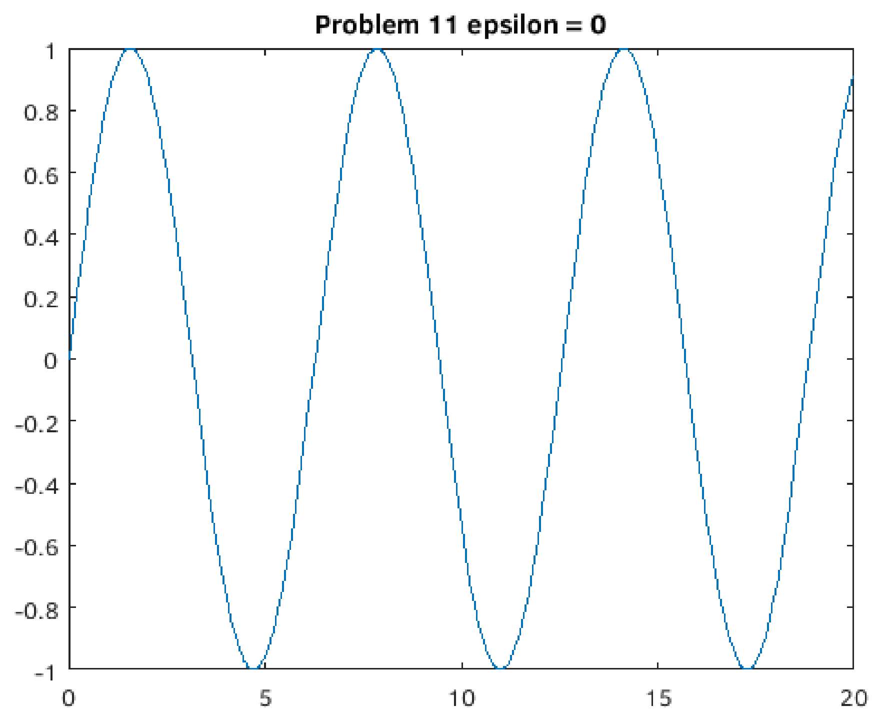


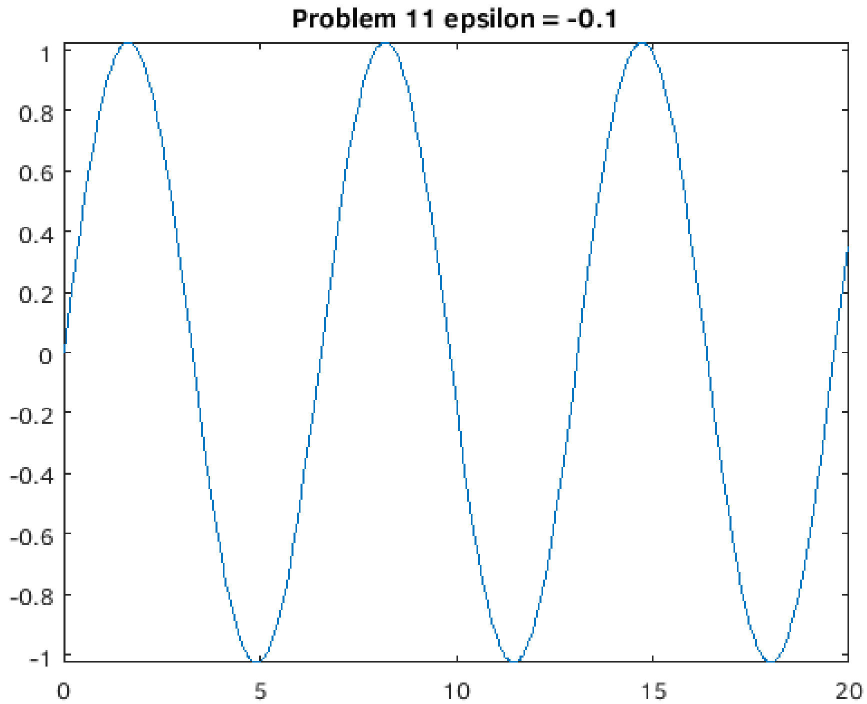


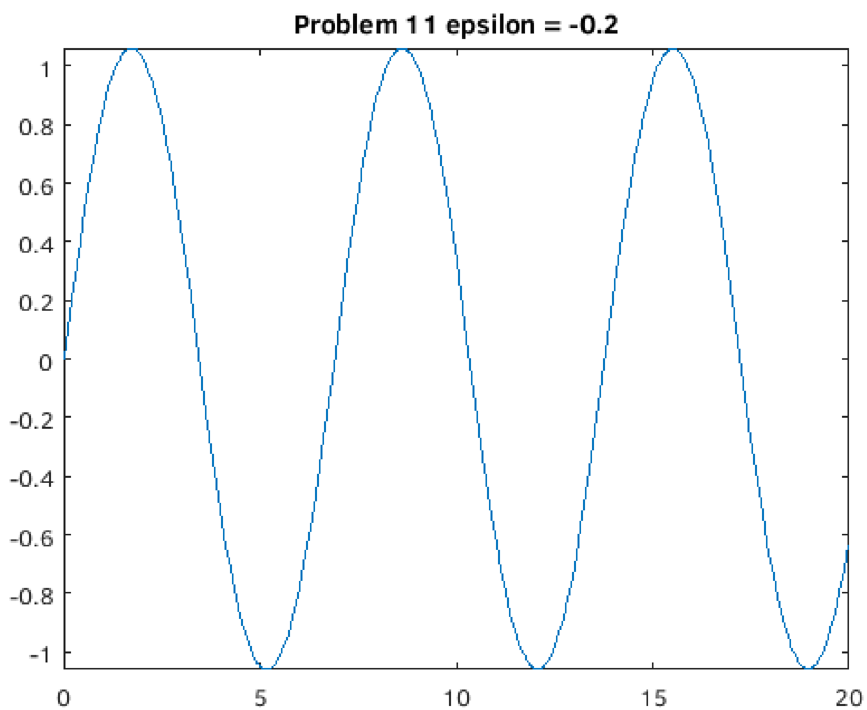

Problem 11 epsilon = 0.2
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20

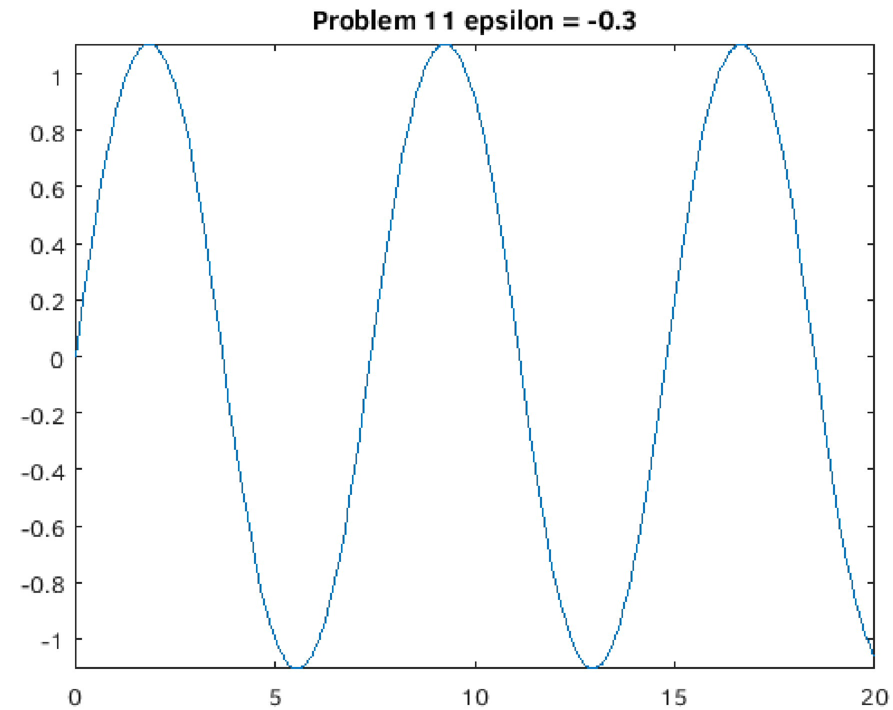

Problem 11 epsilon = 0.3
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20

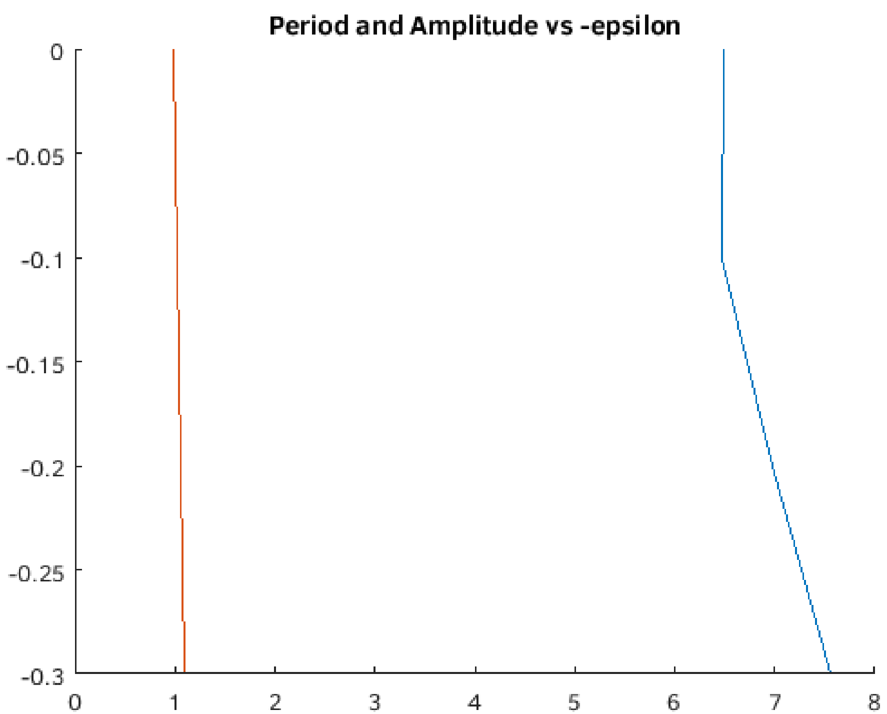

Period and Amplitude vs epsilon
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7


Problem 11 epsilon = 0
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20


Problem 11 epsilon = -0.1
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20


Problem 11 epsilon = -0.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20



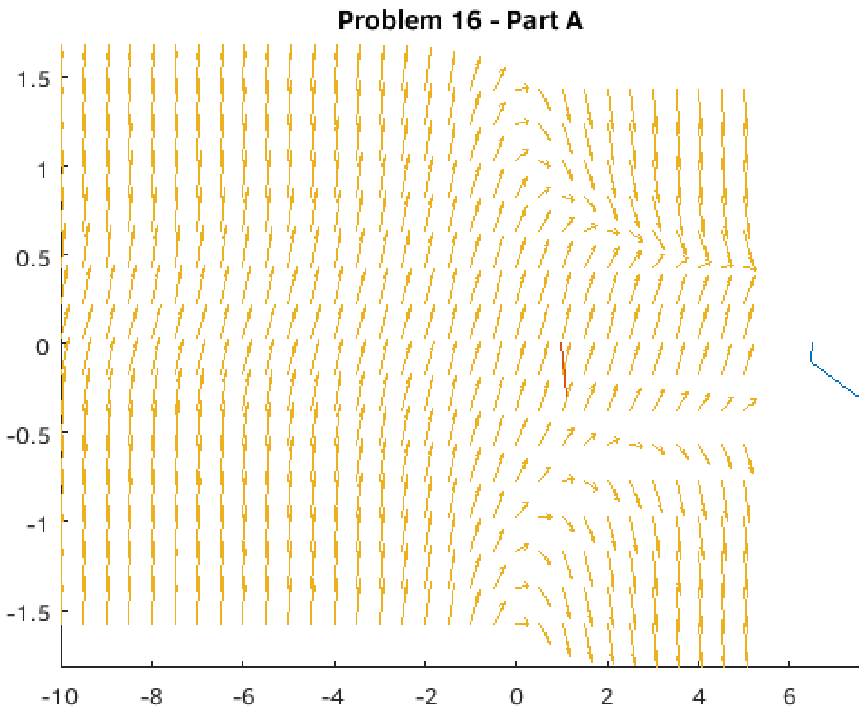


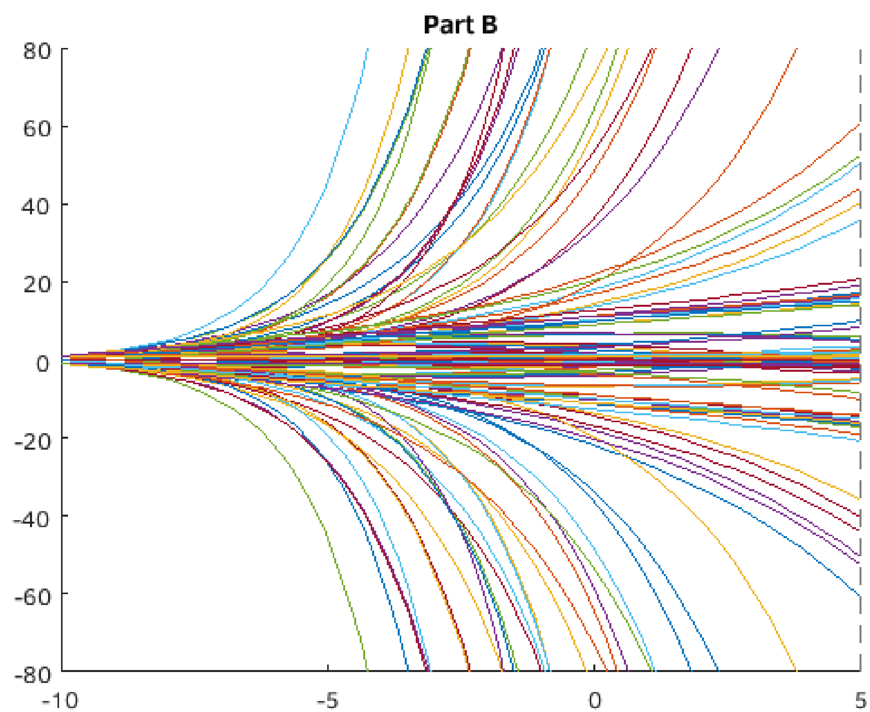
Problem 16

```
fprintf('Problem 16\n\n')
fprintf('Part A')
% Part A
syms y(x)
syms x
firstOrder = @(x, z) cos(z).^2 - ( x .* sin(z).^2);
[X, Z] = meshgrid(-10:.5:5, -pi/2:.2:pi/2);
W = firstOrder(X,Z);
l = sqrt(1 + W.^2);
```

```
quiver(X, Z, 1./l, W./l, .5)
axis tight
title('Problem 16 - Part A')
fprintf('To the left x-axis, the field looks the same. To the right, a \n')
fprintf('siniosodal nature occurs. The zeros of the solution occur \n')
fprintf('towards x-> -inf and happen in 0.5 increments. It appears one or less intersections would occur.\n')
fprintf(' The solutions oscillitate towards negative infinity but not infinity.')

% Part B
figure; hold on;
for x = -10:5
    [V] = odeToVectorField(diff(y, 2) == y*airy(0,x));
    M = matlabFunction(V,'vars', {'x','Y'});
    for i = -1:1
        for j = -1:1
            sol = ode45(M,[-10 5], [i;j]);
            fplot(@(x)deval(sol,x,1), [-10, 5])
        end
    end
end
axis([-10 5 -80 80])
title(['Part B'])

fprintf('The plots do not appear to be exactly the same')
```

```
Problem 16

Part ATo the left x-axis, the field looks the same. To the right, a
siniosodal nature occurs. The zeros of the solution occur
towards x-> -inf and happen in 0.5 increments. It appears one or less intersections would occur.
```





findPeriod() Function

```
function [ p ] = findPeriod( x, y, t)
%findPeriod finds the period of the input values by finding where y crosses
%   the y axis, then averaging the next and previous x coordinate value
%   then multiplies that by 4
    i = 2;
```

```
    while y(i) > 0
       i = i + 1;
    end

    if t == 11
        p = (x(i-1) + x(i));
    else
        p = (x(i-1) + x(i)) * 2;
    end


end
```

findAmplitdue() Function

```
function [ amp ] = findAmplitude( y )
% findAmplitude

    i = 1;

    while y(i,2) > 0
        i = i + 1;
    end

    amp = y(i,1);


end
```

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