Final Project by Joseph Martinsen

Math 308-510 - Dr. Pilant

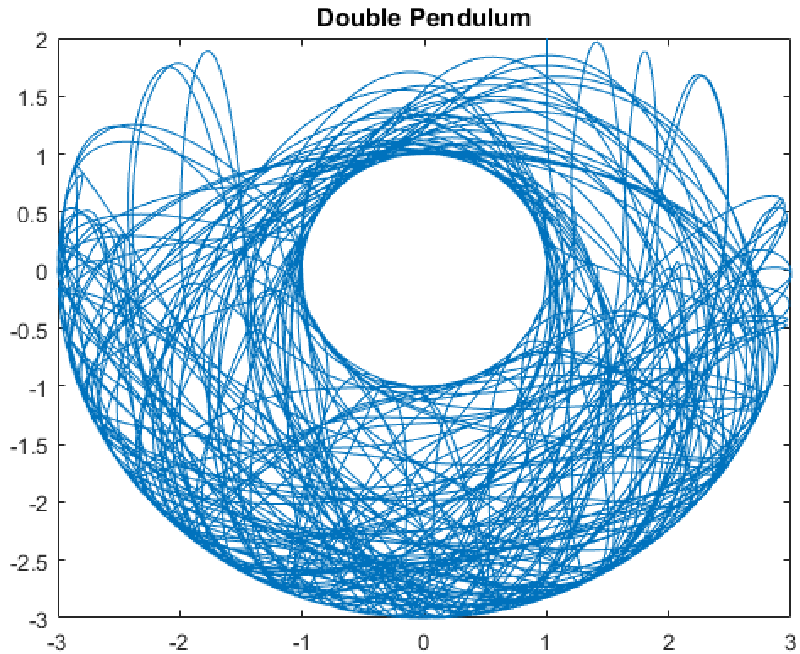
Contents

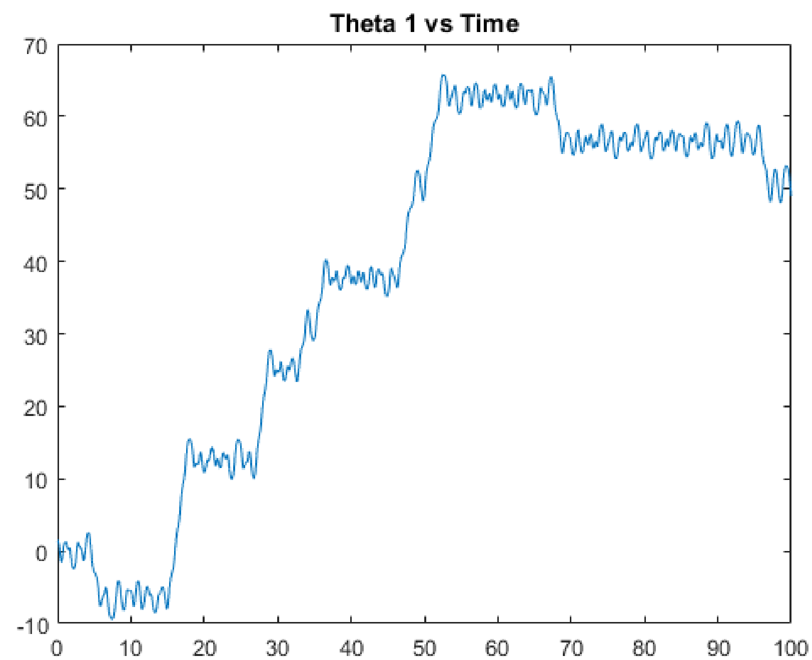
Case 1

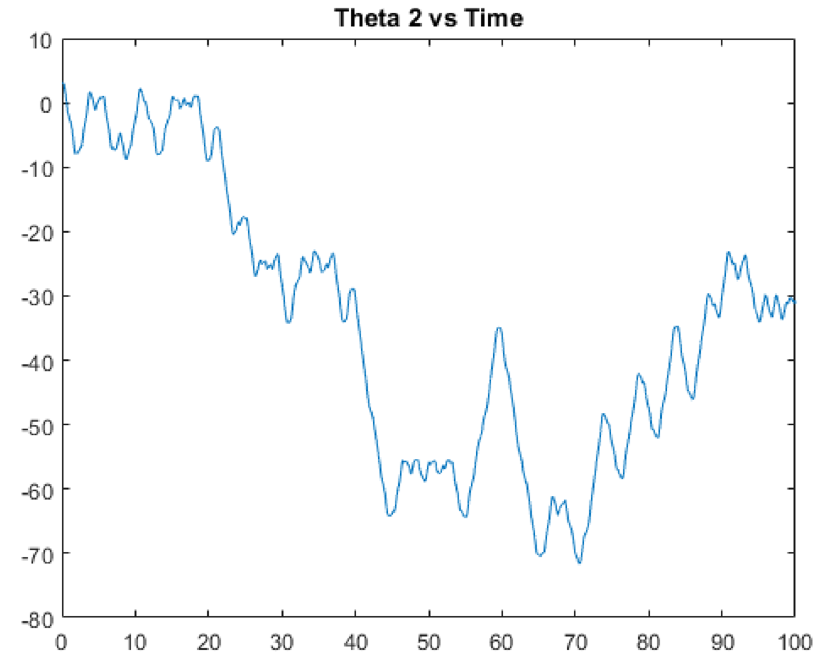
```
clear;
close all;
clc;

% Constents
g  = 32;
m1 = 2;
m2 = 1;
L1 = 1;
L2 = 2;

fprintf('For case 1, the following inital values are used\nm1 = 2\n')
fprintf('m2 = 1\nL1 = 1\nL2 = 2\nu1 = 1.57\nu1prime = 0\nv1 = 3.14\nv1prime = 0')

u1       = 1.57;
u1_prime = 0;
v1       = 3.14;
v1_prime = 0;

% Solve Differential Equation
options = odeset('AbsTol',1e-6,'RelTol',1e-6);
[t,y]= ode45('pend_func' , [ 0 100], [ u1 ; u1_prime ; v1 ; v1_prime ],options);

% Potential Energy
PE = -(m1+m2)*g*L1*cos(y(:,1))-m2*g*L2*cos(y(:,3));
% Kinetic Energy
KE = 1/2*m1*L1.^2*y(:,2).^2 + 0.5*m2*(L1.^2*y(:,2).^2+L2.^2*y(:,4).^2 + 2* L1*L2*y(:,2).*y(:,4).*cos(y(:,1)-y(:,3)));

% Total Energy
E=PE+KE;

% Position Values
x = L1* sin( y(:,1)) + L2*sin(y(:,3));
yplot = -L1* cos(y(: ,1)) - L2*cos(y(:,3));

% Plots
plot(x, yplot);
 axis([-3 3 -3 2])
title('Double Pendulum')
figure;
plot(t,y(:,1));
title('Theta 1 vs Time')
figure;
plot(t,y(:,3));
title('Theta 2 vs Time')
figure;
plot(t,PE);
title('Potential Energy')
figure;
plot(t,KE);
title('Kinetic Energy')
figure;
plot(t,E);
title('Total Energy')
figure;
```

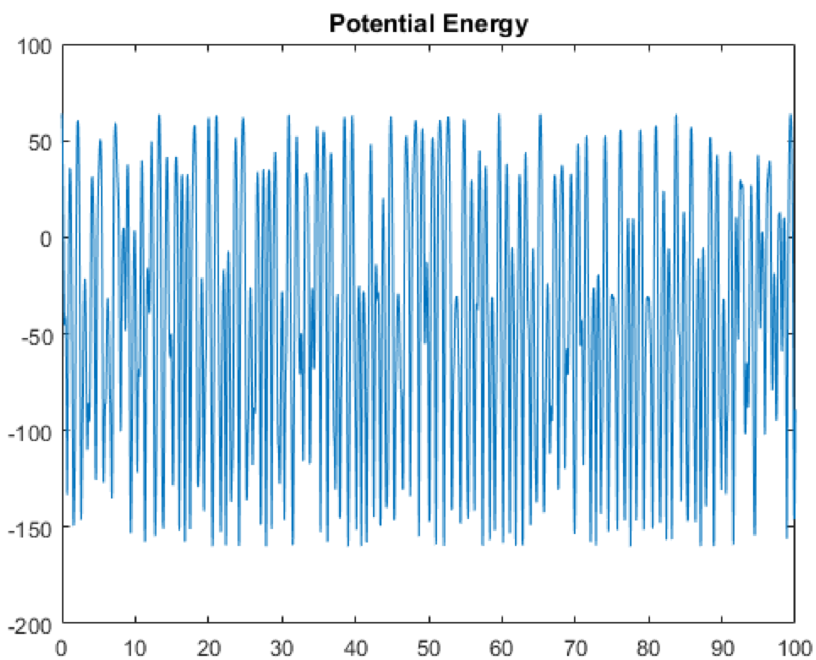
```
plot(t,E);
title('Total Energy')
axis([0 100 0 100])

fprintf('Total energy deviation %f %% \n', 100*(max(E)-min(E))/(max(E)))
```

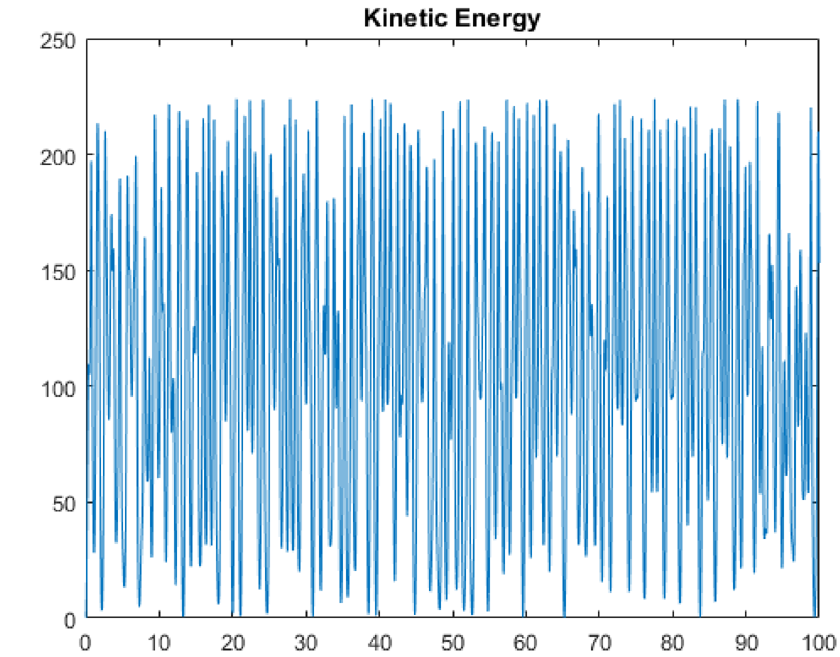
```
For case 1, the following inital values are used
m1 = 2
m2 = 1
L1 = 1
L2 = 2
u1 = 1.57
u1prime = 0
v1 = 3.14
v1prime = 0Total energy deviation 0.022006 %
```

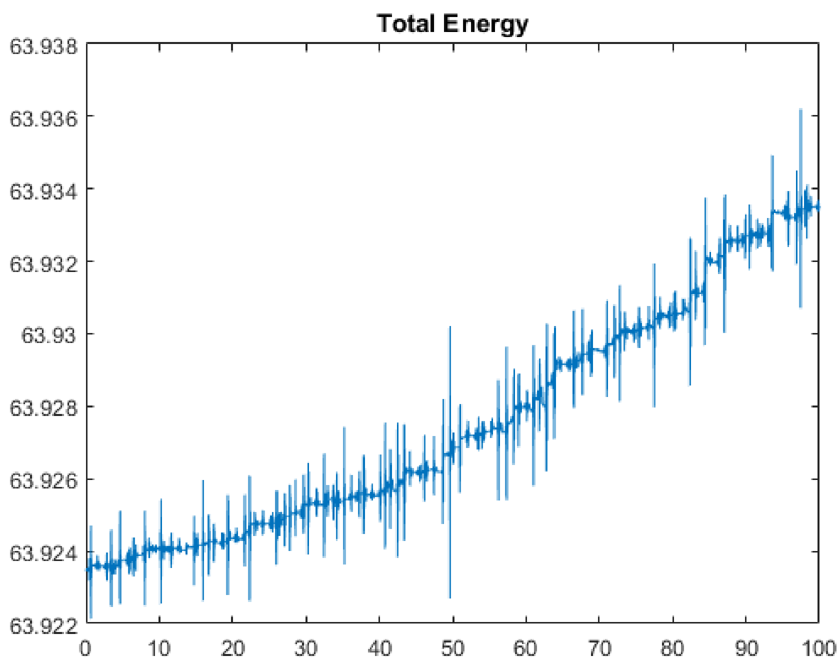

Double Pendulum
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3
-2
-1
0
1
2
3

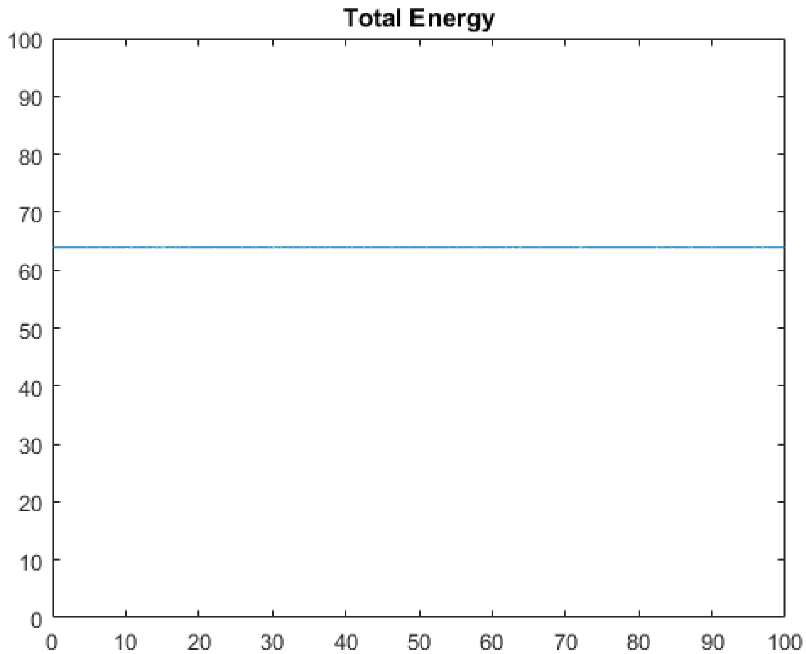

Theta 1 vs Time
70
60
50
40
30
20
10
0
-10
0
10
20
30
40
50
60
70
80
90
100


Theta 2 vs Time
10
0
-10
-20
-30
-40
-50
-60
-70
-80
0
10
20
30
40
50
60
70
80
90
100


Potential Energy
100
50
0
-50
-100
-150
-200
0
10
20
30
40
50
60
70
80
90
100


Kinetic Energy
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100


Total Energy
63.938
63.936
63.934
63.932
63.93
63.928
63.926
63.924
63.922
0
10
20
30
40
50
60
70
80
90
100

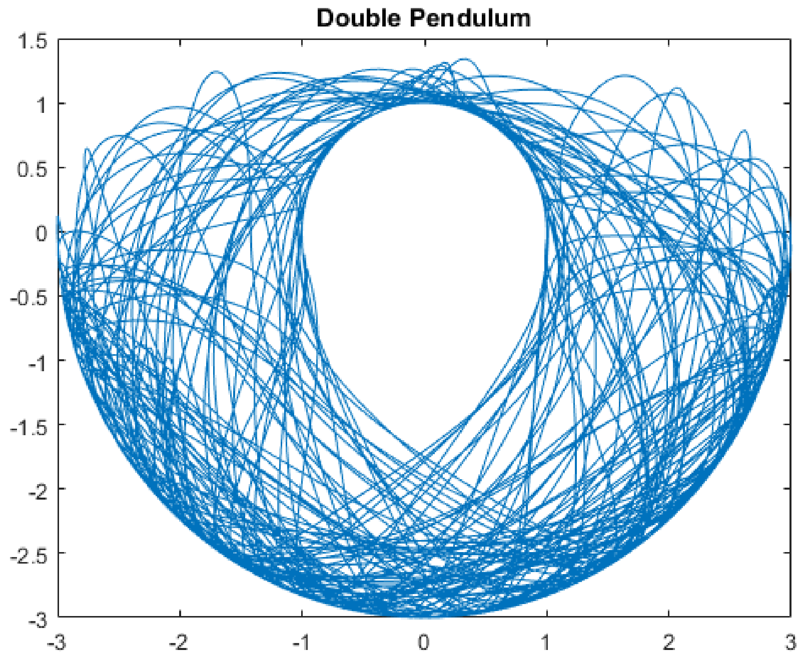


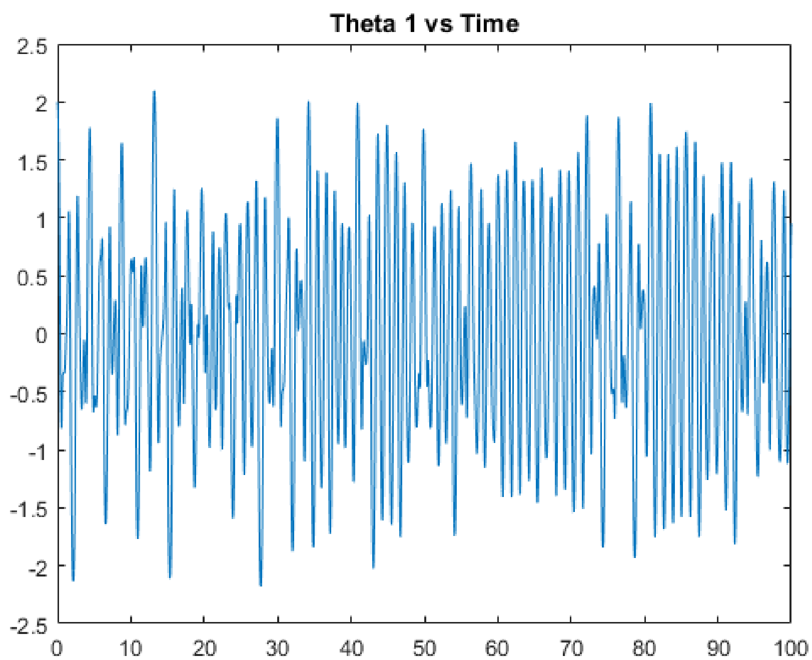
Case 2

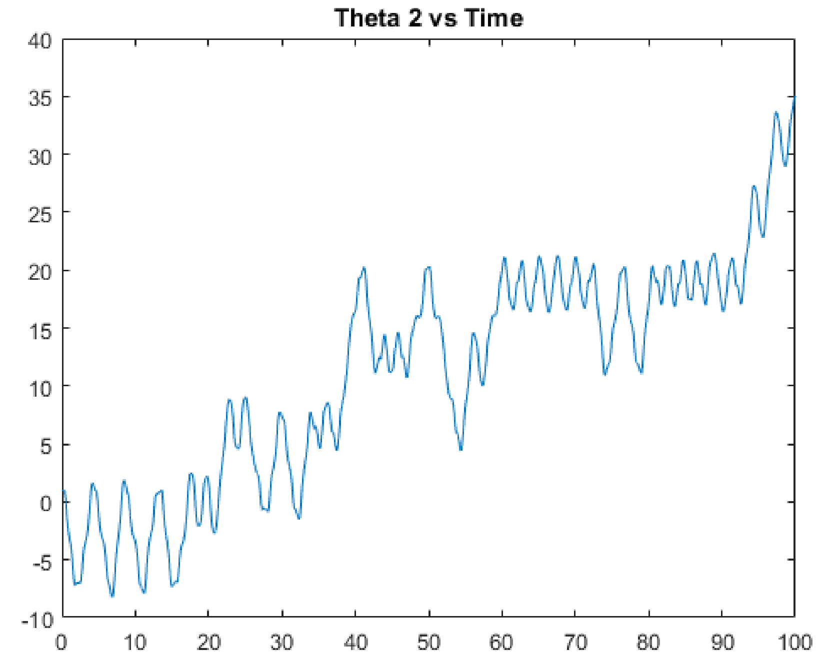
```
clear;
clc;

% Constents
g  = 32;
m1 = 2;
m2 = 1;
L1 = 1;
L2 = 2;

fprintf('For case 2, the angles will differ from Case 1.\n')
fprintf('The following inital values are used\nm1 = 2\n')
fprintf('m2 = 1\nL1 = 1\nL2 = 2\nu1 = 2\nu1prime = 0\nv1 = 1\nv1prime = 0')

u1       = 2;
u1_prime = 0;
v1       = 1;
v1_prime = 0;

% Solve Differential Equation
options = odeset('AbsTol',1e-6,'RelTol',1e-6);
[t,y]= ode45('pend_func' , [ 0 100], [ u1 ; u1_prime ; v1 ; v1_prime ],options);

% Potential Energy
PE = -(m1+m2)*g*L1*cos(y(:,1))-m2*g*L2*cos(y(:,3));
% Kinetic Energy
KE = 1/2*m1*L1.^2*y(:,2).^2 + 0.5*m2*(L1.^2*y(:,2).^2+L2.^2*y(:,4).^2 + 2* L1*L2*y(:,2).*y(:,4).*cos(y(:,1)-y(:,3)));

% Total Energy
E=PE+KE;

% Position Values
x = L1* sin( y(:,1)) + L2*sin(y(:,3));
yplot = -L1* cos(y(: ,1)) - L2*cos(y(:,3));

% Plots
plot(x, yplot);
title('Double Pendulum')
figure;
plot(t,y(:,1));
title('Theta 1 vs Time')
figure;
```

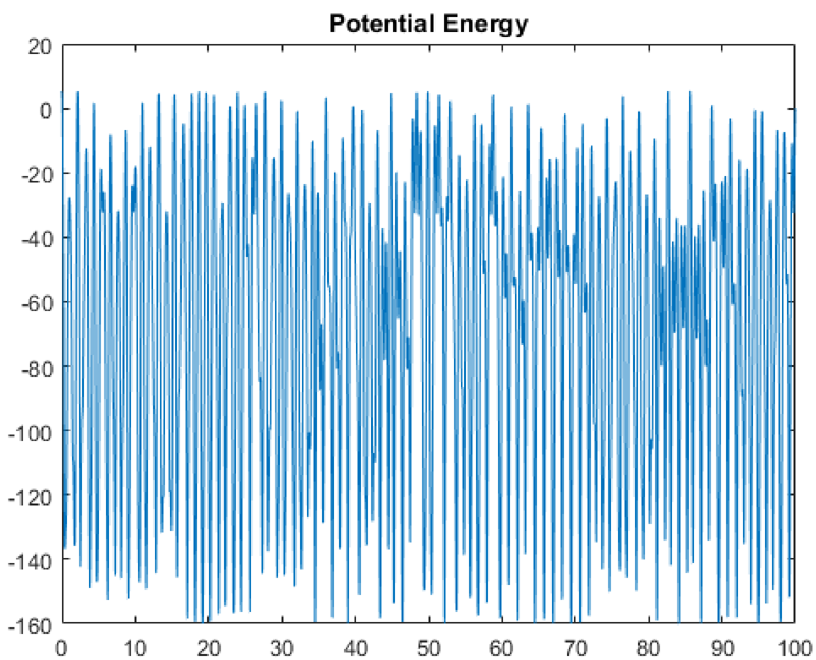
```
plot(t,y(:,3));
title('Theta 2 vs Time')
figure;
plot(t,PE);
title('Potential Energy')
figure;
plot(t,KE);
title('Kinetic Energy')
figure;
plot(t,E);
title('Total Energy')
figure;
plot(t,E);
title('Total Energy')
axis([0 100 0 100])

fprintf('Total energy deviation %f %% \n', 100*(max(E)-min(E))/(max(E)))
```

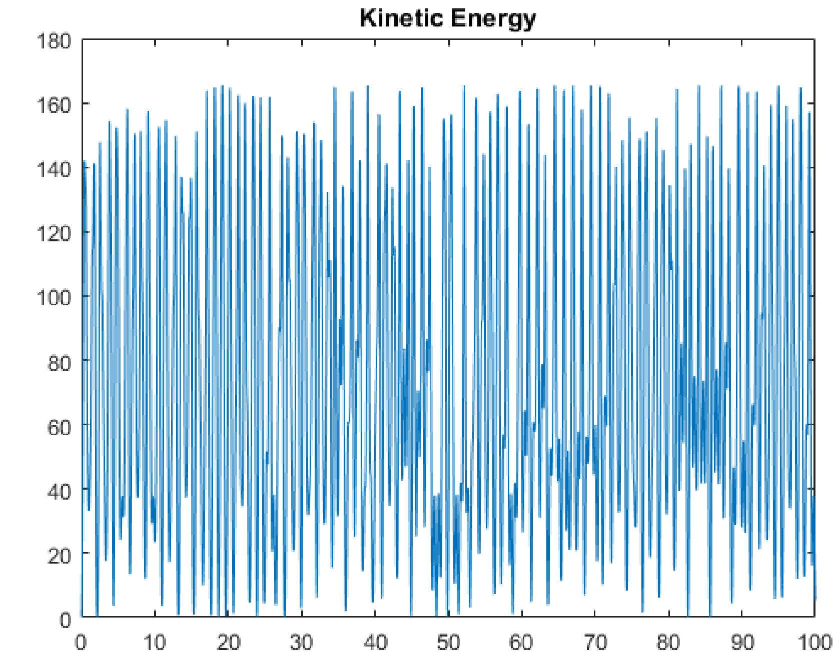
```
For case 2, the angles will differ from Case 1.
The following inital values are used
m1 = 2
m2 = 1
L1 = 1
L2 = 2
u1 = 2
u1prime = 0
v1 = 1
v1prime = 0Total energy deviation 0.076580 %
```

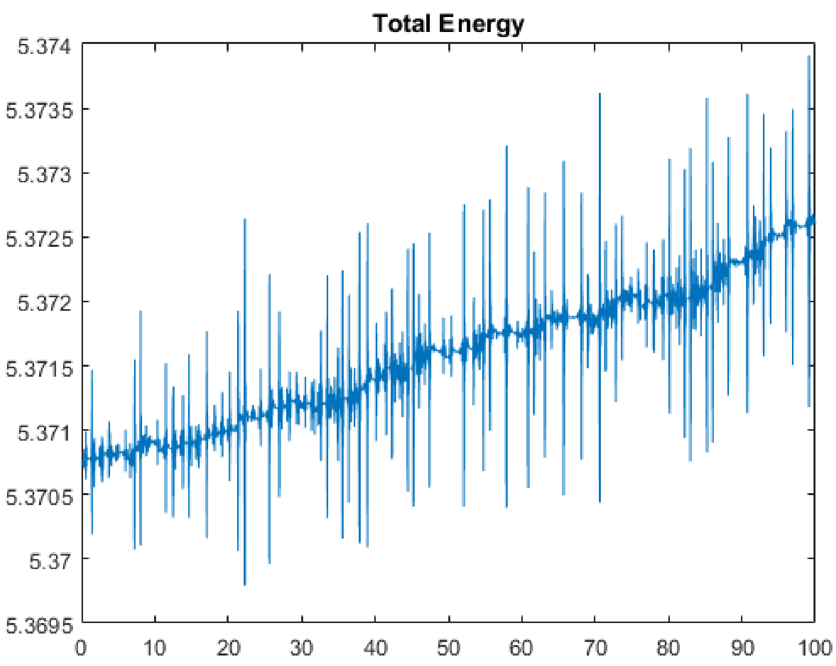

Double Pendulum
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3
-2
-1
0
1
2
3

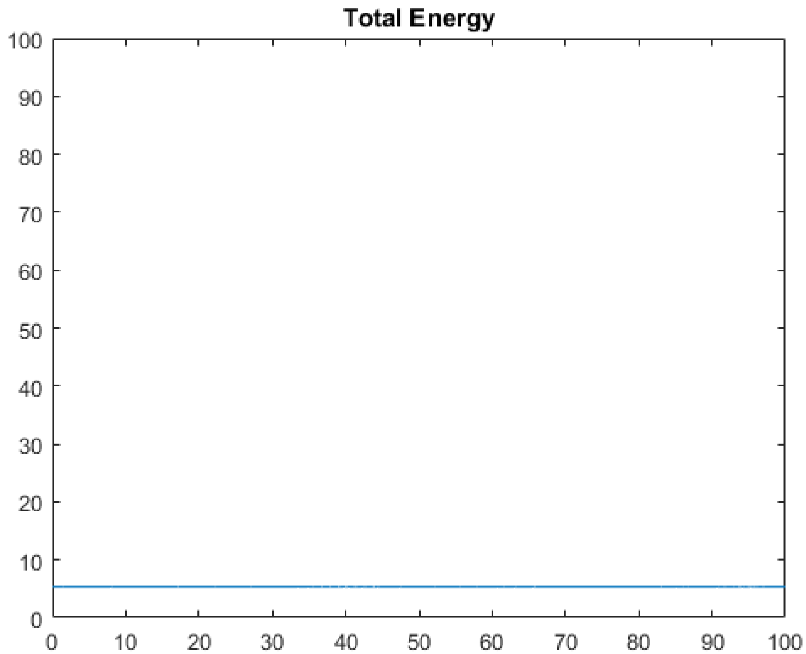

Theta 1 vs Time
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
10
20
30
40
50
60
70
80
90
100


Theta 2 vs Time
40
35
30
25
20
15
10
5
0
-5
-10
0
10
20
30
40
50
60
70
80
90
100


Potential Energy
20
0
-20
-40
-60
-80
-100
-120
-140
-160
0
10
20
30
40
50
60
70
80
90
100


Kinetic Energy
180
160
140
120
100
80
60
40
20
0
0 10 20 30 40 50 60 70 80 90 100


Total Energy
5.374
5.3735
5.373
5.3725
5.372
5.3715
5.371
5.3705
5.37
5.3695
0 10 20 30 40 50 60 70 80 90 100

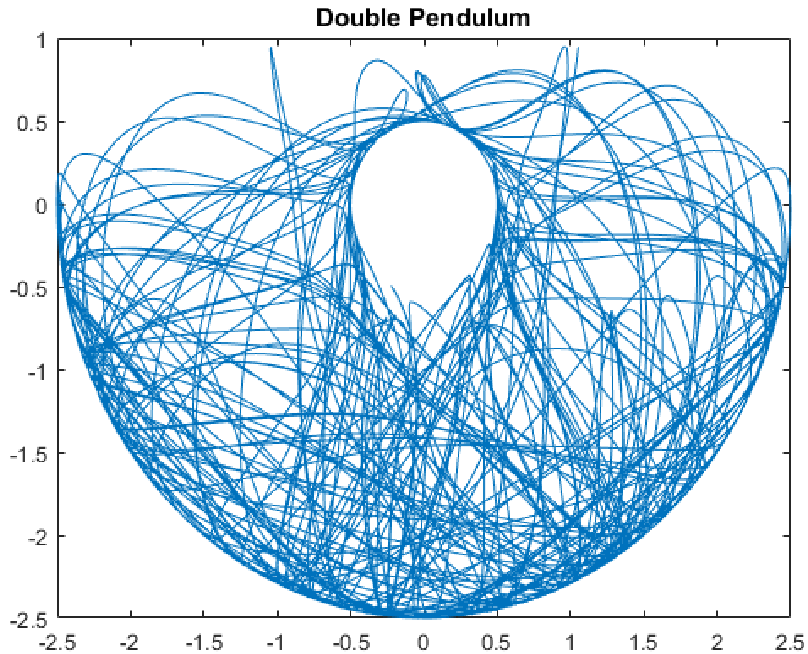


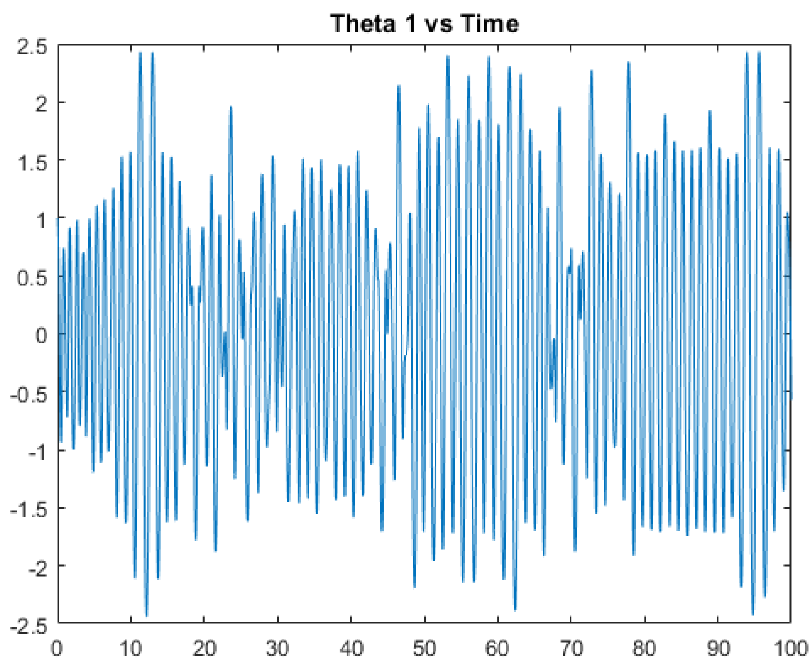
Case 3

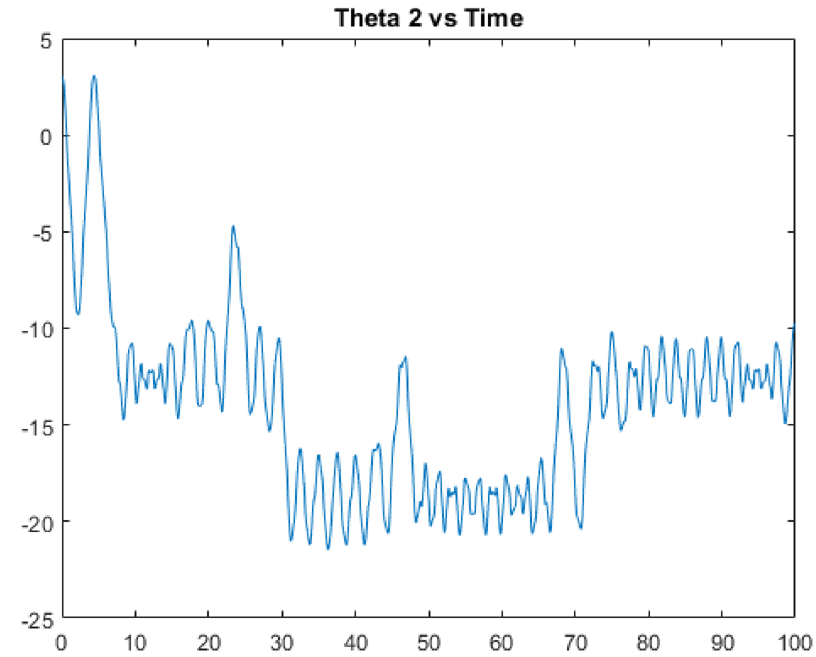
```
clear;
clc;

% Constents
g  = 32;
m1 = 1.5;
m2 = 1;
L1 = 1;
L2 = 1.5;

fprintf('For case 3, the mass and length differ from Case 1.\n')
fprintf('The following inital values are used\nm1 = 1.5\n')
fprintf('m2 = 1\nL1 = 1\nL2 = 1.5\nu1 = 1\nu1prime = 0\nv1 = 3\nv1prime = 0')

u1       = 1;
u1_prime = 0;
v1       = 3;
v1_prime = 0;

% Solve Differential Equation
options = odeset('AbsTol',1e-6,'RelTol',1e-6);
[t,y]= ode45('pend_func' , [ 0 100], [ u1 ; u1_prime ; v1 ; v1_prime ],options);

% Potential Energy
PE = -(m1+m2)*g*L1*cos(y(:,1))-m2*g*L2*cos(y(:,3));
% Kinetic Energy
KE = 1/2*m1*L1.^2*y(:,2).^2 + 0.5*m2*(L1.^2*y(:,2).^2+L2.^2*y(:,4).^2 + 2* L1*L2*y(:,2).*y(:,4).*cos(y(:,1)-y(:,3)));

% Total Energy
E=PE+KE;

% Position Values
x = L1* sin( y(:,1)) + L2*sin(y(:,3));
yplot = -L1* cos(y(: ,1)) - L2*cos(y(:,3));

% Plots
plot(x, yplot);
title('Double Pendulum')
figure;
plot(t,y(:,1));
title('Theta 1 vs Time')
figure;
```

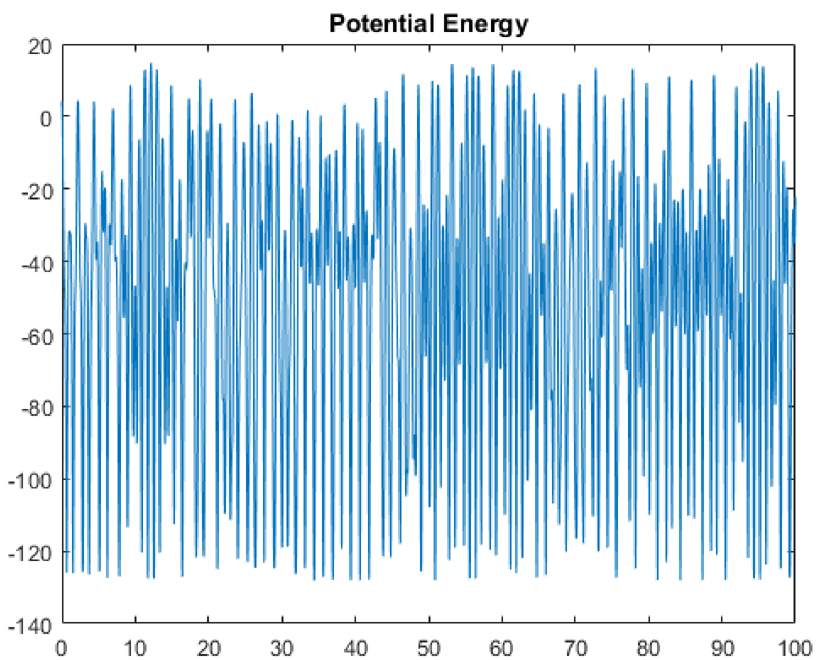
```
plot(t,y(:,3));
title('Theta 2 vs Time')
figure;
plot(t,PE);
title('Potential Energy')
figure;
plot(t,KE);
title('Kinetic Energy')
```

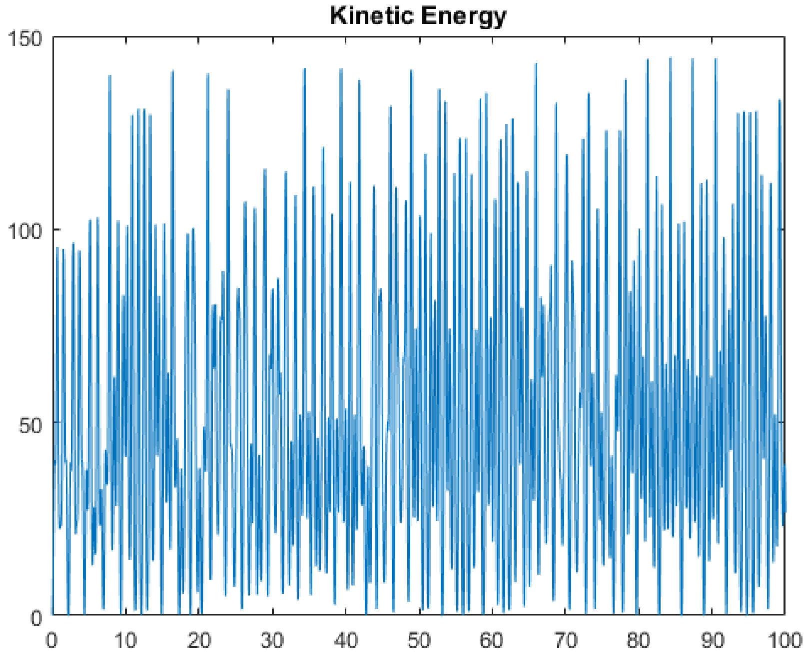
```
For case 3, the mass and length differ from Case 1.
The following inital values are used
m1 = 1.5
m2 = 1
L1 = 1
L2 = 1.5
u1 = 1
u1prime = 0
v1 = 3
v1prime = 0
```


Double Pendulum
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5


Theta 1 vs Time
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
10
20
30
40
50
60
70
80
90
100


Theta 2 vs Time
5
0
-5
-10
-15
-20
-25
0
10
20
30
40
50
60
70
80
90
100


Potential Energy
20
0
-20
-40
-60
-80
-100
-120
-140
0
10
20
30
40
50
60
70
80
90
100



Conclution

In the words of Edward Lorenz, "Chaos [is] when the present determines the future, but the approximate present does not approximately determine the future."[1] Even though the double pendelum is chaotic and appears to be random, the amazing thing about math and physics is that with enough initial conditions, the path the pendulum will travel can be determined and graphed and simulated.

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1 | From http://fouriestseries.tumblr.com/post/86253333743/chaos-and-the-double-pendulum
Published with MATLAB® R2016b