Project 1

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Math 308-510

STATEMENT OF PROBLEM: Consider the following differential equations:

I. $\dfrac{dy}{dx} = xy$

II. $\dfrac{dy}{dx} = y^2$

III. $x\dfrac{dy}{dx} + \sin(x)y = x^2$

1. Graph the direction field in the neighborhood of the origin
2. Graph approximate integral curves for 10 different initial conditions (you can use dfield8.m)
3. Solve the differential equation using Euler's method (e.g. with euler.m), for three initial conditions and two values of the stepsize, h
4. If possible compute the error, either using the exact solution (if available) or a very accurate numerical solution (from Matlab)

I. Using Matlab, we investigate the following differential equation

$$\frac{dy}{dx} = xy$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

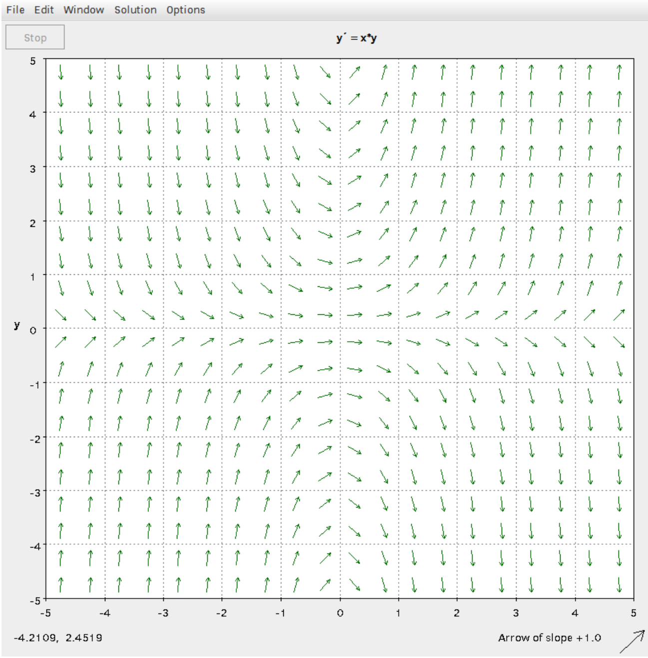

Figure 1: Direction field of $y' = xy$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

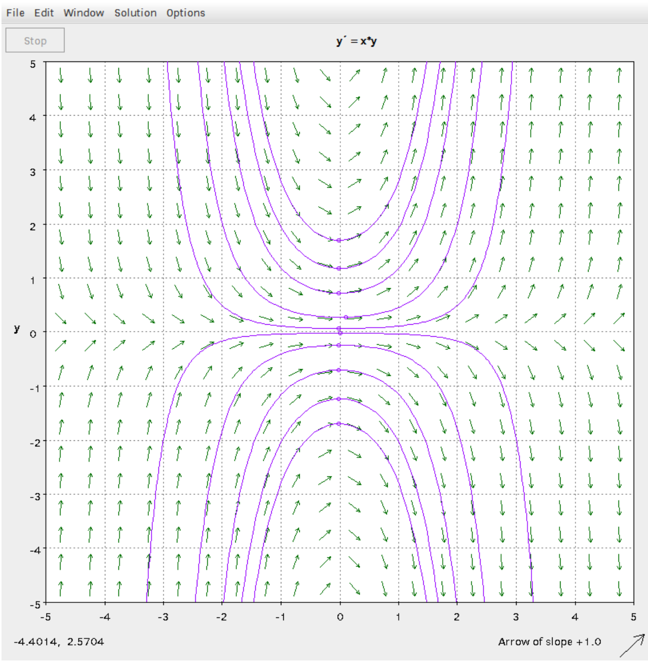

Figure 2: Integral curves of $y' = xy$

c) **Numerical solution.**

The exact solution for $y(0) = 1$ is using Matlab is given by

$$y = e^{x^2/2}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$ is compared to the solution before.

The exact solution for $y(0) = 2$ found using Matlab is given by

$$y = 2e^{x^2/2}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -e^{x^2/x}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

d)

II. Using Matlab, we investigate the following differential equation

$$\frac{dy}{dx} = y^2$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

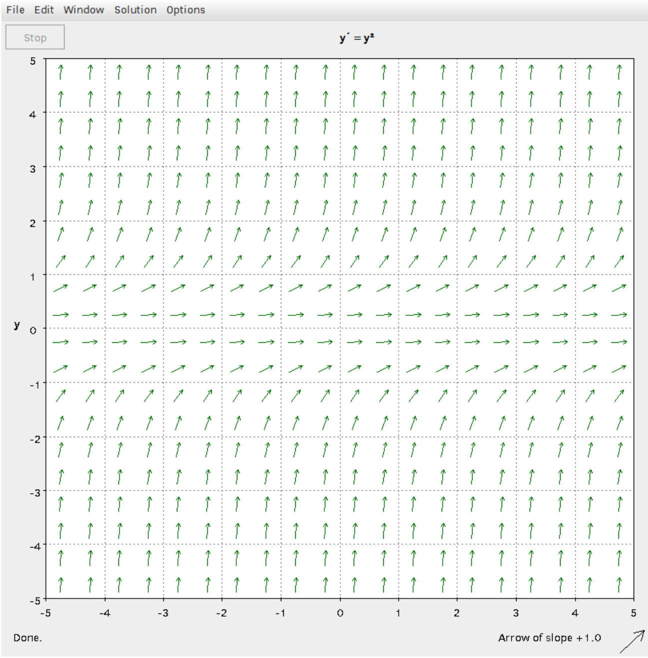

Figure 3: Direction field of $y\,' = y^2$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

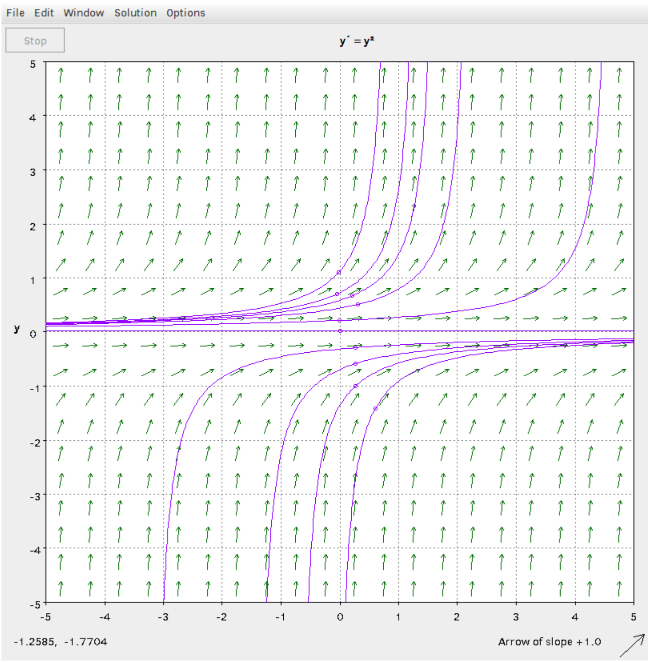

Figure 4: Integral curves of $y\,' = y^2$

c) **Numerical solution.**

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -\frac{1}{x-1}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -\frac{1}{x-\frac{1}{2}}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -\frac{1}{x+1}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

d)

III. Using Matlab, we investigate the following differential equation

$$xy' + \sin(x)y = x^2$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

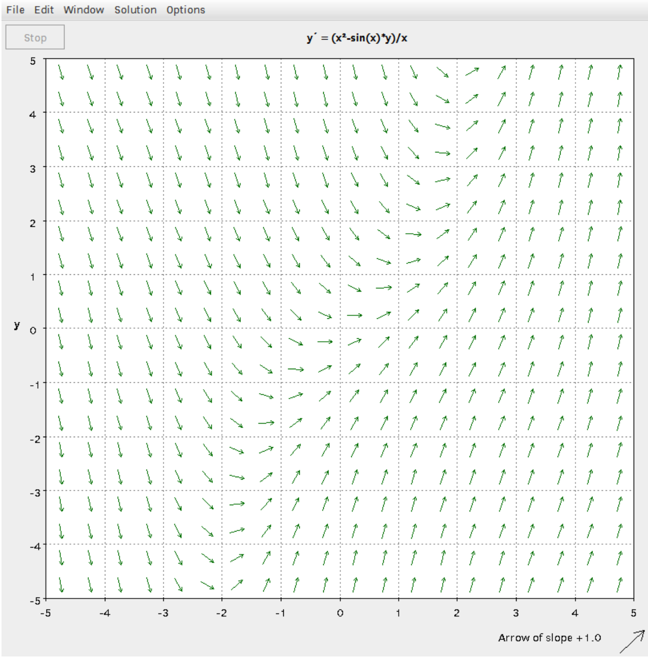

Figure 5: Direction field of $xy' + \sin(x)y = x^2$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

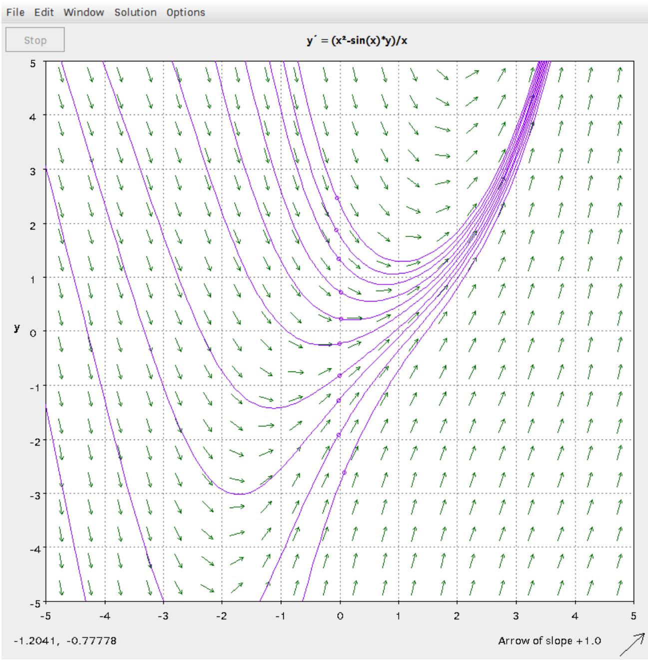

Figure 6: Integral curves of $xy' + \sin(x)y = x^2$

c)

d)