Problem Set B
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Math 308-510
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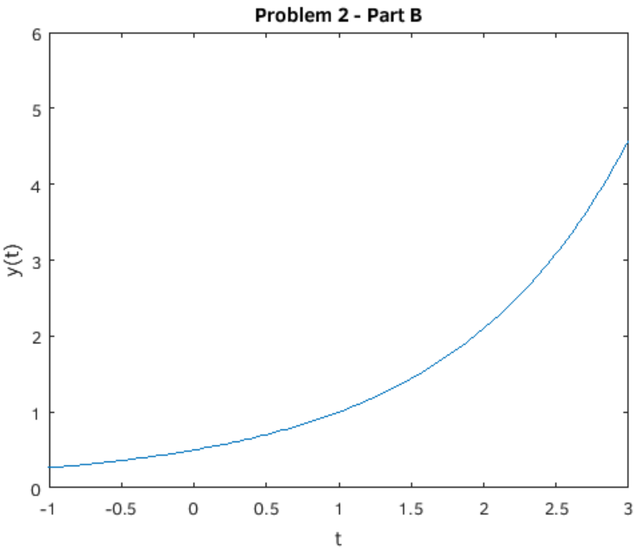
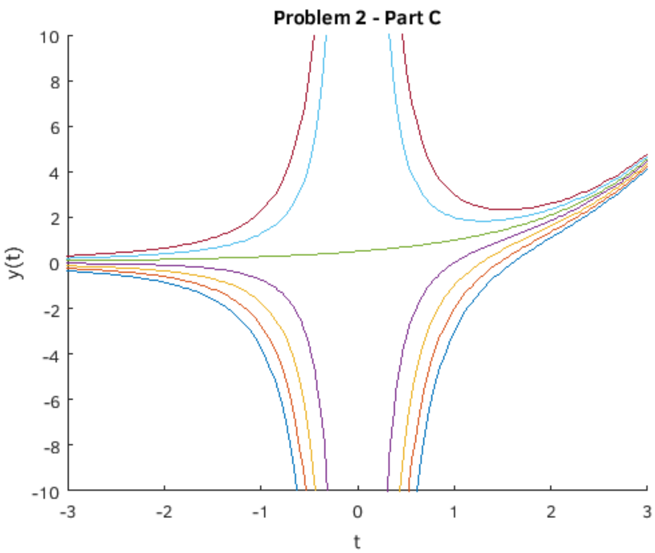
Contents

Problem 2

```
fprintf('Problem 2\n')

clear;
syms y(t)
eqn = t*diff(y,t) + 2*y == exp(t);

% Part A
ySol(t) = dsolve(eqn,y(1) == 1);
yp = (exp(t)*(t-1)+1)/t^2;
yprime = diff((exp(t)*(t-1)+1)/t^2,t);
a = t*yprime +2*y;
an = simplify(a);
fprintf('Part A\n\n')
fprintf('y = %s\ny'' = %s\n',char(yp),char(yprime))
fprintf('Subs into the %s results in %s\n',char(ySol(t)),char(a))
fprintf('Using MATLAB''s simplify command results in %s which is exactly what is should be\n\n',char(an))

% Part B
fprintf('Part B\n\n')
figure
ezplot(yp)
axis([-1 3 0 6])
title('Problem 2 - Part B')
ylabel('y(t)')
fprintf('The solution graph for t=0, y(t) approaches 0. The solution graph for large t''s, y(t) approaches Inf\n\n')


% Part C
fprintf('Part C\n\n')
clear ySol(t)
figure
hold on
for i= -3:3
    cond = y(1) == i;
    ySol(t) = dsolve(eqn,cond);
    ezplot(ySol(t))
end
axis([-3 3 -10 10])
title('Problem 2 - Part C')
ylabel('y(t)')
fprintf('See graphs for C\n\n')

% Part D
fprintf('Part D\n\n')
fprintf('As the graphs approach t=0, the graphs diverge. For larg t''s, the graphs begin to approach Inf\n')
fprintf('Yes there is a singularity, for the condition y(t) = 1, the graph does not diverge at t=0\n')
```

```
Problem 2
Part A

y = (exp(t)*(t - 1) + 1)/t^2
y' = (exp(t) + exp(t)*(t - 1))/t^2 - (2*(exp(t)*(t - 1) + 1))/t^3
Subs into the 1/t^2 + (exp(t)*(t - 1))/t^2 results in 2*y(t) + t*((exp(t) + exp(t)*(t - 1))/t^2 - (2*(exp(t)*(t - 1) + 1))/t^3)
Using MATLAB's simplify command results in 2*y(t) + t*((exp(t) + exp(t)*(t - 1))/t^2 - (2*(exp(t)*(t - 1) + 1))/t^3) which is exactly
what is should be

Part B

The solution graph for t=0, y(t) approaches 0. The solution graph for large t's, y(t) approaches Inf

Part C

See graphs for C

Part D

As the graphs approach t=0, the graphs diverge. For larg t's, the graphs begin to approach Inf
Yes there is a singularity, for the condition y(t) = 1, the graph does not diverge at t=0
```

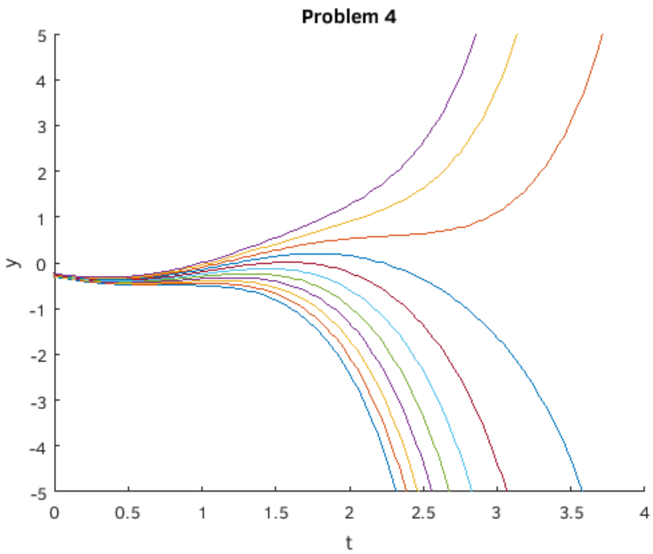




Problem 4

```
clear;
fprintf('Problem 4\n\n')
syms y(t)
eqn = diff(y,t) - 2*y == sin(2*t);
figure
hold on

for i = -0.5:.05:0
    cond = y(1) == i;
    ySol(t) = dsolve(eqn,cond);
    ezplot(ySol(t))
end
axis([0 4 -5 5])
title('Problem 4')
ylabel('y')

fprintf('As t increases, the slope increaces. ')
fprintf('The 3 different behaviors are, slope is always negative (c = -.5,-.45,-.4,-.35), the slope begins positive then becomes negative. (c = -.3,-
fprintf('and the third is the slope is always positive (c = -.1, -.05, 0)\n\n')
```

```
Problem 4

As t increases, the slope increaces. The 3 different behaviors are, slope is always negative (c = -.5,-.45,-.4,-.35), the slope
begins positive then becomes negative. (c = -.3,-.25,-.2,-.15) and the third is the slope is always positive (c = -.1, -.05, 0)
```



Problem 9

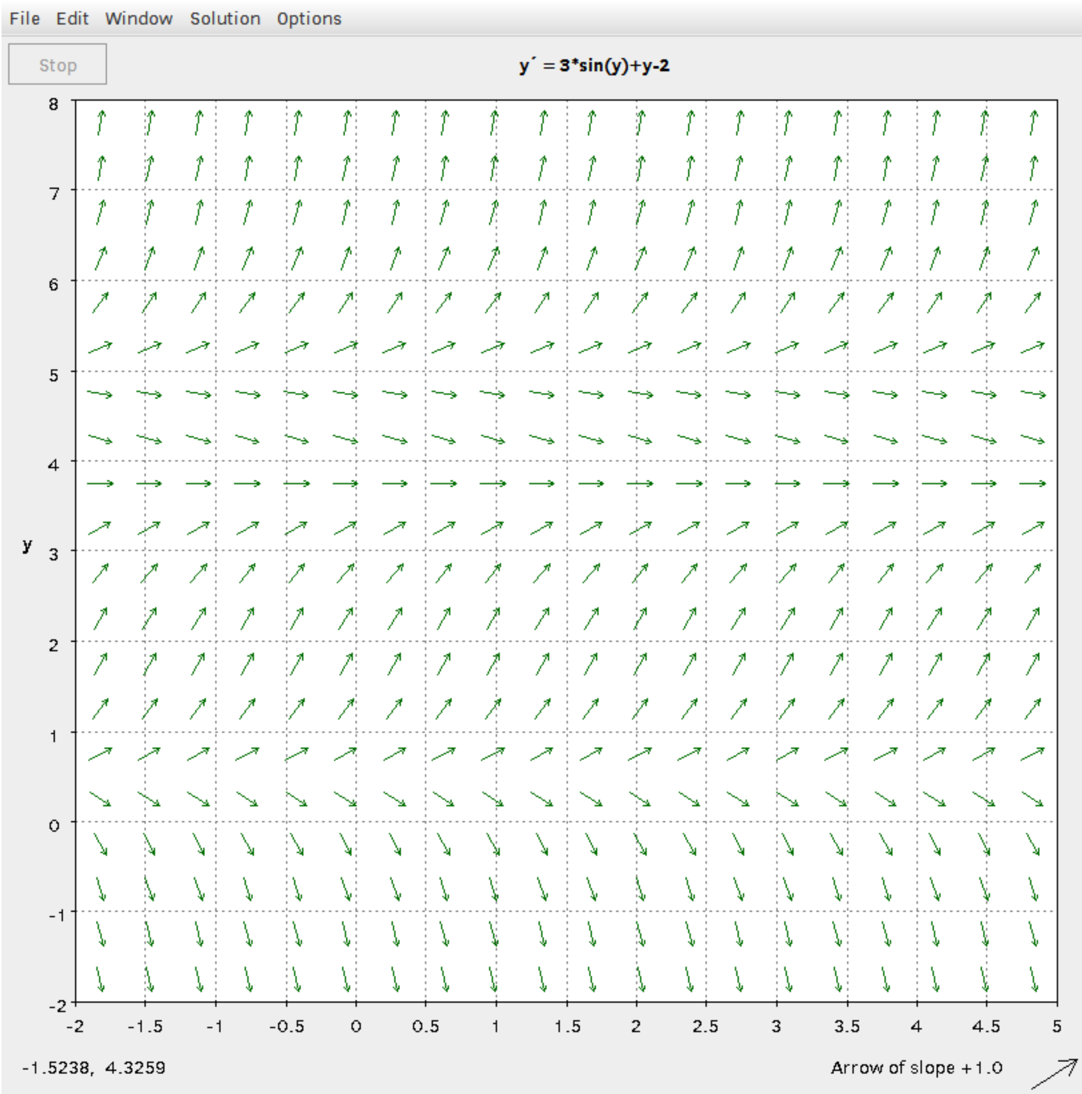
```
clear;
fprintf('Problem 9\n\n')
syms r k y(t) a(t)

% Part A
fprintf('Part A\n')
a = diff(y,t) == r*y - k * y^2;
aANS(t) = dsolve(a);
fprintf('The solutions for %s are:\n', char(a))
display(aANS(t))

% Part B
fprintf('\nPart B\n')
b = diff(y,t) == t*(t^2+1)/(4*y^3);
bANS = dsolve(b, y(0) == -1/sqrt(2));
fprintf('The solution to %s with y(0) = -1/sqrt(2) is\n %s\n\n',char(b), char(bANS))

% Part C
c = (exp(t)*sin(y)+3*y)/(3*t-exp(t)*sin(y)) == diff(y,t);
% dsolve(c)

% Part D
d = diff(y,t) == (2*y-y)/(2*t-y);
dsolve(d)

% Part E
fprintf('Part E\n')
e = diff(y,t) == (2*t+y)/(3+3*y^2-t);
eANS = dsolve(e,y(0)==0);
fprintf('The solution for %s is:\n %s\n\n',char(e),char(eANS))
```

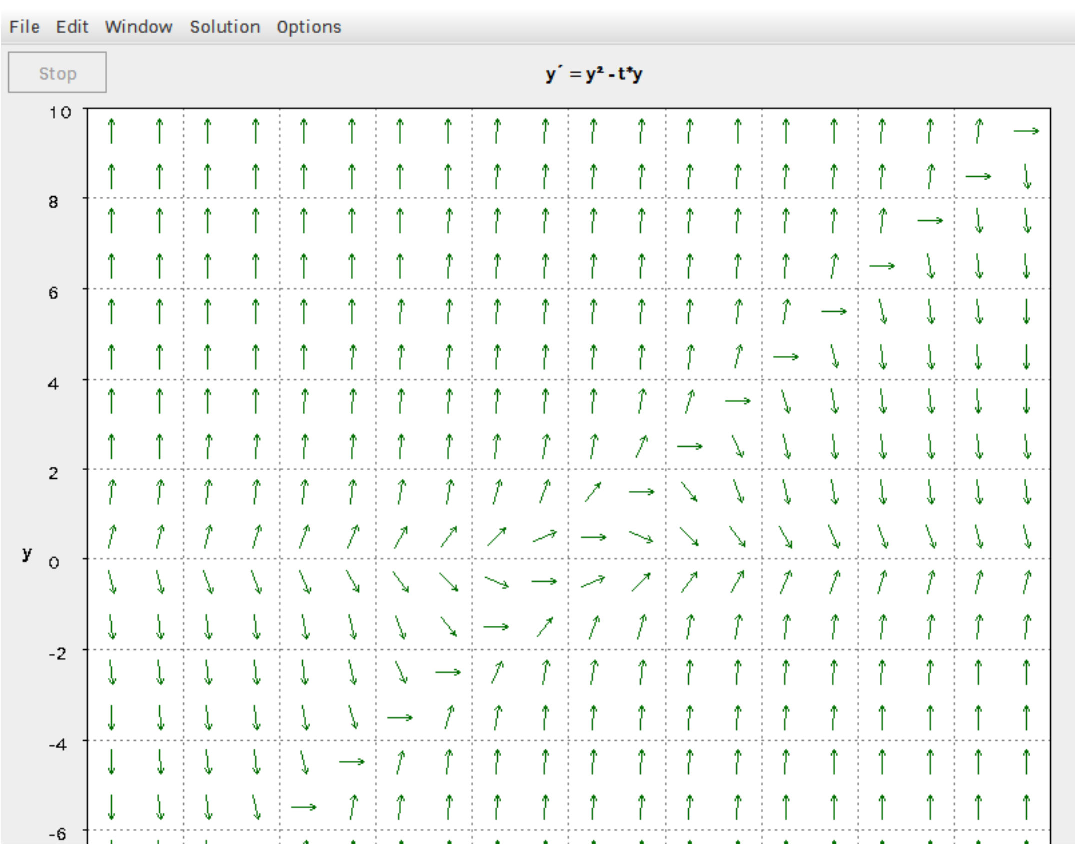
```
Problem 9

Part A
The solutions for diff(y(t), t) == r*y(t) - k*y(t)^2 are:

ans =

                                     0
                                   r/k
 (r*(tanh((r*(C34 + t))/2) + 1))/(2*k)


Part B
The solution to diff(y(t), t) == (t*(t^2 + 1))/(4*y(t)^3) with y(0) = -1/sqrt(2) is
 -4^(1/4)*((t^2*(t^2 + 2))/16 + 1/16)^(1/4)


ans =

                                                              t
                                                              0
 exp(C40)/2 + (t^(1/2)*exp(C40/2)*(-(4*t - exp(C40))/t)^(1/2))/2
 exp(C40)/2 - (t^(1/2)*exp(C40/2)*(-(4*t - exp(C40))/t)^(1/2))/2

Part E
The solution for diff(y(t), t) == (2*t + y(t))/(3*y(t)^2 - t + 3) is:
 (t/3 - 1)/(t^2/2 + (t^4/4 - (t/3 - 1)^3)^(1/2))^(1/3) + (t^2/2 + (t^4/4 - (t/3 - 1)^3)^(1/2))^(1/3)
```

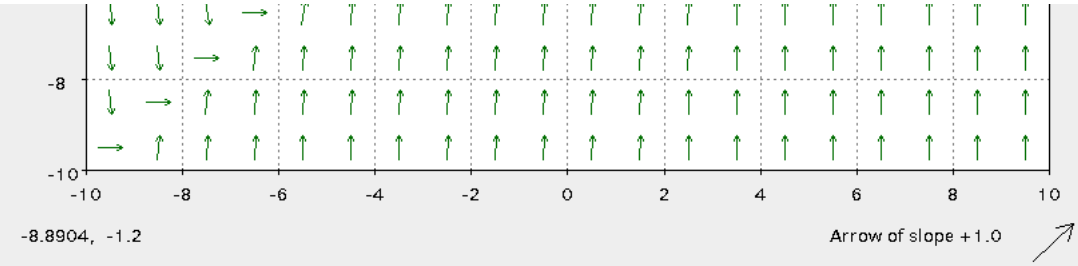
Problem 12

```
clear;
fprintf('Problem 12\n\n')
fzero(@f12,1)
fzero(@f12,5)

% Part B - It is obvious that a solution is y = t because lines approach
% that line
fprintf(['The solution is evident because ',...
        'because vector fields point at t axis'])
% Part C
fprintf('Part C\n\n')
syms y(t) c
eq = dsolve(diff(y,t) == y^2 - t*y,y(0) ==c);
fprintf('Solving y(0) = c\ny(t) = %s\n',char(eq))
for i = -5:5
    eq = dsolve(diff(y,t) == y^2 - t*y,y(0) ==i);
    fprintf('Solving y(0) = %d\ny(t) = %s\n',i,char(eq))
end
```

```
Problem 12


ans =

    0.5170


ans =

    4.9295

The solution is evident because because vector fields point at t axisPart C

Solving y(0) = c
y(t) = exp(-t^2/2)/(1/c - (2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2)
Solving y(0) = -5
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 + 1/5)
Solving y(0) = -4
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 + 1/4)
Solving y(0) = -3
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 + 1/3)
Solving y(0) = -2
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 + 1/2)
Solving y(0) = -1
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 + 1)
Solving y(0) = 0
y(t) = 0
Solving y(0) = 1
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 - 1)
Solving y(0) = 2
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 - 1/2)
Solving y(0) = 3
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 - 1/3)
Solving y(0) = 4
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 - 1/4)
Solving y(0) = 5
y(t) = -exp(-t^2/2)/((2^(1/2)*pi^(1/2)*erf((2^(1/2)*t)/2))/2 - 1/5)
```


File Edit Window Solution Options
Stop
y´ = 3*sin(y)+y-2
8
7
6
5
4
y 3
2
1
0
-1
-2
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
-1.5238, 4.3259
Arrow of slope +1.0

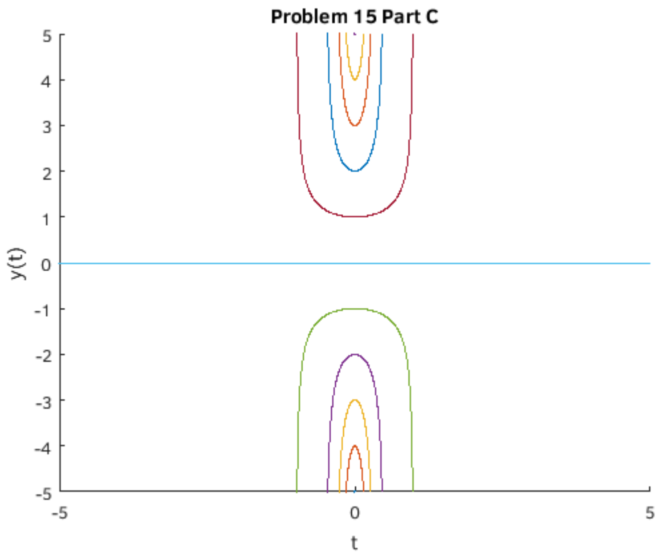

File Edit Window Solution Options
Stop
y´ = y² - t*y
10
8
6
4
2
y 0
-2
-4
-6



Problem 15

```
clear;
fprintf('Problem 15\n\n')
syms y(t) c

% Part A
fprintf('Part A\n\n')
eqn = diff(y,t) == t * y^3;
ansA = dsolve(eqn,y(0)==c);
fprintf(['Using dsolve to solve %s with initial condition y(0) = c ', ...
         'the solutions are\ny(t) = %s\ny(t) = %s\nI think Matlab g', ...
         'ives two solutions because it gives the solution for posi', ...
         'tive or negative values of c\n\n'] ...
         , char(eqn),char(ansA(1)),char(ansA(2)))

% Part B
fprintf('Part B\n\n')
fprintf('Substituting for c = 0\n')
% Can't sub because of division by 0
try
    subs(ansA(1),c,0)
    subs(ansA(2),c,0)
catch
    err = lasterror;
    msg = err.message;
    warning(msg)
end

l1 = limit(ansA(1),c,0);
l2 = limit(ansA(2),c,0);
fprintf('Taking the limit as c approaches c results in %s and %s\n\n' ...
        ,l1,l2)

% Part C
fprintf('Part C\n\n')
figure
hold on
for i = -5:5
   ezplot(dsolve(eqn,y(0)==i))
end

axis([-5 5 -5 5])
title('Problem 15 Part C')
ylabel('y(t)')
fprintf(['From looking at the plot, it appears that the solution is ', ...
        'unstable because the plots diverge above\nand below t = 0\n'])
```

```
Problem 15

Part A

Using dsolve to solve diff(y(t), t) == t*y(t)^3 with initial condition y(0) = c the solutions are
y(t) = (2^(1/2)*(1/(1/(2*c^2) - t^2/2))^(1/2))/2
y(t) = -(2^(1/2)*(1/(1/(2*c^2) - t^2/2))^(1/2))/2
I think Matlab gives two solutions because it gives the solution for positive or negative values of c

Part B

Substituting for c = 0
Warning: Error using symengine
Division by zero.
Taking the limit as c approaches c results in 0 and 0

Part C

From looking at the plot, it appears that the solution is unstable because the plots diverge above
and below t = 0
```


Problem 15 Part C
y(t)
t
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
0
5