Problem Set A

Practice with MATLAB

In this problem set, you will use MATLAB to do some basic calculations, and then to plot, differentiate, and integrate various functions. This problem set is the minimum you should do in order to reach a level of proficiency that will enable you to use MATLAB throughout the course. A solution to Problem 2 appears in Section 4.4.5, and a solution to Problem 4 appears in the *Sample Solutions*.

1. Evaluate:

   (a) $\dfrac{413}{768 + 295}$ (as a decimal),

   (b) 2^{123}, both as an approximate number in scientific notation and as an exact integer,

   (c) π^2 and e to 35 digits,

   (d) the fractions $\frac{61}{88}$, $\frac{13863}{20000}$, and $\frac{253}{365}$, and determine which is the best approximation to $\ln(2)$.

2. Evaluate to 15 digits:

   (a) $\dfrac{\sin(0.1)}{0.1}$,

   (b) $\dfrac{\sin(0.01)}{0.01}$,

   (c) $\dfrac{\sin(0.001)}{0.001}$.

3. Graph the equations:

   (a) $y = x^3 - x$ on the interval $-1.5 \le x \le 1.5$,

   (b) $y = \tan x$ on the interval $-2\pi \le x \le 2\pi$,

   (c) $y^2 = x^3 - x$ on the interval $-2.5 \le x \le 2.5$. (*Hint*: If the first argument to **`ezplot`** is a symbolic expression in two variables, MATLAB plots the locus of points where this expression is equal to 0.)

4. The **factor** command factors an expression containing symbolic variables, and also factors integers into their prime factors.

   (a) Factor $x^3 + 5x^2 - 17x - 21$.

   (b) Find the prime factorization of 987654321.

5. Plot the functions x^8 and 4^x on the same graph, and determine how many times their graphs intersect. (*Hint*: You will probably have to make several plots, using various intervals, in order to find all the intersection points.) Now find the values of the points of intersection, first using the **fzero** command and then using the **solve** command. Do these two commands produce the same answers?

6. Use the command **limit** (type **help limit** for the syntax) to compute the following limits:

   (a) $\displaystyle\lim_{x \to 0} \frac{\tan x}{x}$,

   (b) $\displaystyle\lim_{x \to 0+} \frac{1}{x}$ and $\displaystyle\lim_{x \to 0-} \frac{1}{x}$,

   (c) $\displaystyle\lim_{x \to \infty} x\, e^{-x^2}$ and $\displaystyle\lim_{x \to -\infty} x\, e^{-x}$,

   (d) $\displaystyle\lim_{x \to 0} \frac{\ln(1-x) + x}{x^2}$.

7. Compute the following derivatives using the command **diff**:

   (a) $\displaystyle\frac{d}{dx}\left(\frac{x^3}{x^2+1}\right)$,

   (b) $\displaystyle\frac{d}{dx}(\sin(\sin(\sin x)))$,

   (c) $\displaystyle\frac{d^3}{dx^3}(\arctan x)$,

   (d) $\displaystyle\frac{d}{dx}(\sqrt{1+x^2})$,

   (e) $\displaystyle\frac{d}{dx}(e^{x \ln(x)})$.

8. Compute the following integrals using the command **int**:

   (a) $\int e^{-3x} \sin x \, dx$,

   (b) $\int (x+1) \ln x \, dx$,

   (c) $\displaystyle\int_0^1 \sqrt{\frac{x}{1-x}}\, dx$,

   (d) $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\, dx$,

(e) $\int_0^1 \sqrt{1+x^4}\,dx$. For this example, also compute the numerical value of the integral.

9. Use **solve** to solve the equation

$$x^5 - 3x^2 + x + 1 = 0. \tag{A.1}$$

Find the numerical values of the five roots. Plot the graph of the 5th degree polynomial on the left-hand side of (A.1) on the interval $-2 \le x \le 2$. Now explain your results—in particular, reconcile your five roots with the fact that the graph touches the x-axis only twice. To verify this, you should include the x-axis in your graph and restrict the y-axis appropriately.

10. In one-variable calculus you learned that the local max/min of a differentiable function $y = f(t)$ are found among the *critical points* of f, that is, the points t where $f'(t) = 0$. For example, consider the polynomial function

$$y = f(t) = t^6 - 4t^4 - 2t^3 + 3t^2 + 2t$$

on the interval $[-3/2, 3/2]$.

(a) Graph $f(t)$ on that interval.

(b) How many local max/min points do you see? It's a little hard to determine what is happening for negative values of t. If need be, redraw your graph by restricting the t and/or y axis by using **axis**.

(c) Now use MATLAB to differentiate f and find the points t where $f'(t) = 0$ on the interval $[-3/2, 3/2]$. How many are there? Use **fzero** to hone in on their values.

(d) Now verify that the negative critical point is indeed an *inflection point* by graphing $f''(t)$ on the interval $-1.2 \le t \le -0.8$. How does that graph establish that the point is an inflection point?

11. (a) Use **solve** to simultaneously solve the pair of equations

$$\begin{cases} x^2 - y^2 = 1, \\ 2x + y = 2. \end{cases}$$

(b) Plot the two curves on the same graph and visually corroborate your answer from part (a). (*Hint*: To help you plot the hyperbola $x^2 - y^2 = 1$, recall that its two branches can be parameterized by the parametric equations $x = \cosh t, y = \sinh t$, and $x = -\cosh t, y = \sinh t$.)

12. MATLAB has a command **symsum** that you can use to sum infinite series.

(a) Use it to sum the famous infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(b) Use it to sum the geometric series $\sum_{n=0}^{\infty} x^n$.

(c) Define a function $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$. Use **symsum** to identify the function.

(d) Compute $f'(x)$ by differentiating the function you found in part (c).

(e) Now sum the series $\sum_{n=1}^{\infty} \frac{d}{dx}\left(\frac{x^n}{n}\right)$. What does this suggest about differentiating a function that is defined by a power series?