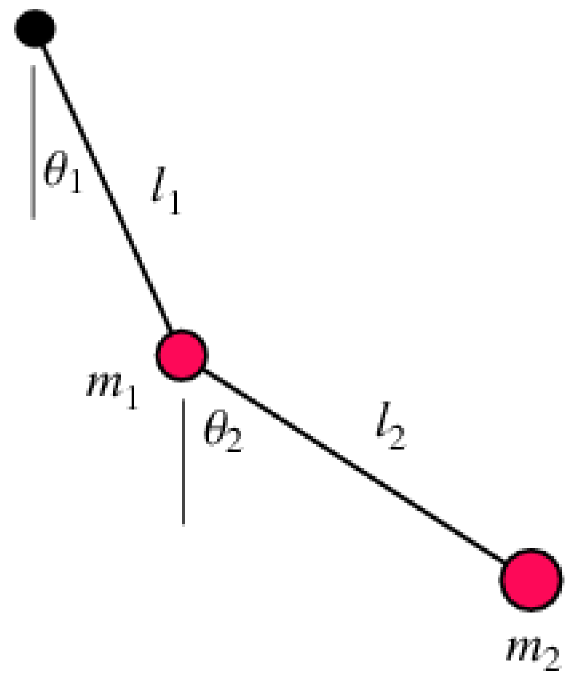
Double Pendulum and Chaos



The following derivation is from `http://scienceworld.wolfram.com/physics/DoublePendulum.html`

The equations of motion for the double pendulum are given by

$$
\begin{gathered}
(m_1 + m_2)L_1 \frac{d^2\theta_1}{dt^2} + m_2 L_2 \frac{d^2\theta_2}{dt^2} \cos(\theta_1 - \theta_2) \\
+ m_2 L_2 \left(\frac{d\theta_2}{dt}\right)^2 \sin(\theta_1 - \theta_2) + g(m_1 + m_2)\sin(\theta_1) = 0 \\
m_2 L_2 \frac{d^2\theta_2}{dt^2} + m_2 L_1 \frac{d^2\theta_1}{dt^2} \cos(\theta_1 - \theta_2) \\
- m_2 L_1 \left(\frac{d\theta_1}{dt}\right)^2 \sin(\theta_1 - \theta_2) + m_2 g \sin(\theta_2) = 0
\end{gathered}
$$

If we use the variables

$$
\begin{aligned}
u_1 &= \theta_1(t) \\
u_2 &= \theta_1'(t) \\
v_1 &= \theta_2(t) \\
v_2 &= \theta_2'(t)
\end{aligned}
$$

we get the system

$$
\begin{aligned}
(m_1+m_2)L_1\frac{du_2}{dt} + m_2L_2\frac{dv_2}{dt}\cos(u_1-v_1) \\
+m_2L_2\left(v_2\right)^2\sin(u_1-v_1) + g(m_1+m_2)\sin(u_1) = 0 \\
m_2L_2\frac{dv_2}{dt} + m_2L_1\frac{du_2}{dt}\cos(u_1-v_1) \\
-m_2L_1\left(u_2\right)^2\sin(u_1-v_1) + m_2g\sin(v_1) = 0 \\
\frac{du_1}{dt} = u_2(t) \\
\frac{dv_1}{dt} = v_2(t)
\end{aligned}
$$

Using the substitutions $a = (m_1+m_2)L_1$, $b = m_2L_2\cos(u_1-v_1)$ $c = m_2L_1\cos(u_1-v_1)$, $d = m_2L_2$, $e = -m_2L_2\left(v_2\right)^2\sin(u_1-v_1) - g(m_1+m_2)\sin(u_1)$, and $f = m_2L_1\left(u_2\right)^2\sin(u_1-v_1) - m_2g\sin(v_1)$ we can write this system as

$$
\begin{aligned}
a\frac{du_2}{dt} + b\frac{dv_2}{dt} &= e && (1) \\
c\frac{du_2}{dt} + d\frac{dv_2}{dt} &= f && (2) \\
\frac{du_1}{dt} &= u_2(t) && (3) \\
\frac{dv_1}{dt} &= v_2(t) && (4)
\end{aligned}
$$

Equations (1) and (2) can be solved for du_2/dt and dv_2/dt by

$$\frac{du_2}{dt} = \frac{ed - bf}{ad - cb} \tag{5}$$

$$\frac{dv_2}{dt} = \frac{af - ce}{ad - cb} \tag{6}$$

Equations (3)-(6) are now in the form that Matlab can use.

Initial Conditions The initial conditions given in the reference are (angles given in terms of radians)

$$\begin{aligned} u_1(0) &= 1.5 \\ u_2(0) &= 0.0 \\ v_1(0) &= 3.0 \\ v_2(0) &= 0.0 \end{aligned}$$

The physical parameters are given by

$$\begin{aligned} L_1 &= 1 \\ L_2 &= 2 \\ m_1 &= 2 \\ m_2 &= 1 \\ g &= 9.8 \end{aligned}$$

Matlab Project 3: Using the system of equations given above, and the parameters given above, reproduce the animated graphic contained in the reference.

Find the component plots $\theta_1(t) = u_1(t)$ vs t and $\theta_2(t) = v_1(t)$ vs t.

Examine several other initial conditions, and check for chaotic orbits.