Project 1

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Math 308-510

STATEMENT OF PROBLEM: Consider the following differential equations:

I. $\dfrac{dy}{dx} = xy$

II. $\dfrac{dy}{dx} = y^2$

III. $x\dfrac{dy}{dx} + \sin(x)y = x^2$

1. Graph the direction field in the neighborhood of the origin
2. Graph approximate integral curves for 10 different initial conditions (you can use dfield8.m)
3. Solve the differential equation using Euler's method (e.g. with euler.m), for three initial conditions and two values of the stepsize, h
4. If possible compute the error, either using the exact solution (if available) or a very accurate numerical solution (from Matlab)

I. Using Matlab, we investigate the following differential equation

$$\frac{dy}{dx} = xy$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

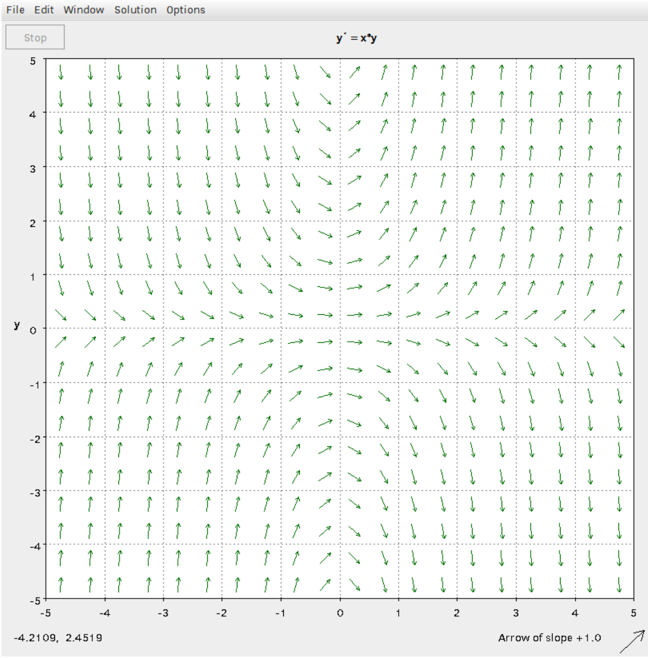


Figure 1: Direction field of $y' = xy$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

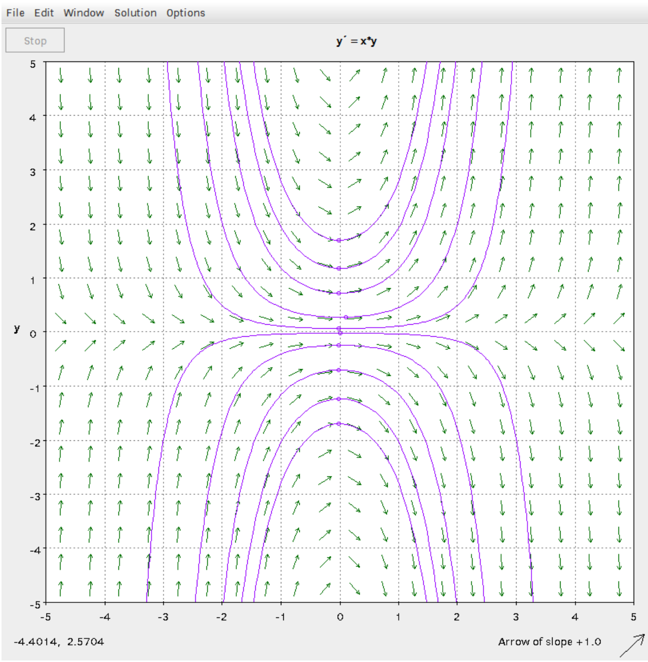


Figure 2: Integral curves of $y' = xy$

c) **Numerical solution.**

The exact solution for $y(0) = 1$ is using Matlab is given by

$$y = e^{x^2/2}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$ is compared to the solution before.

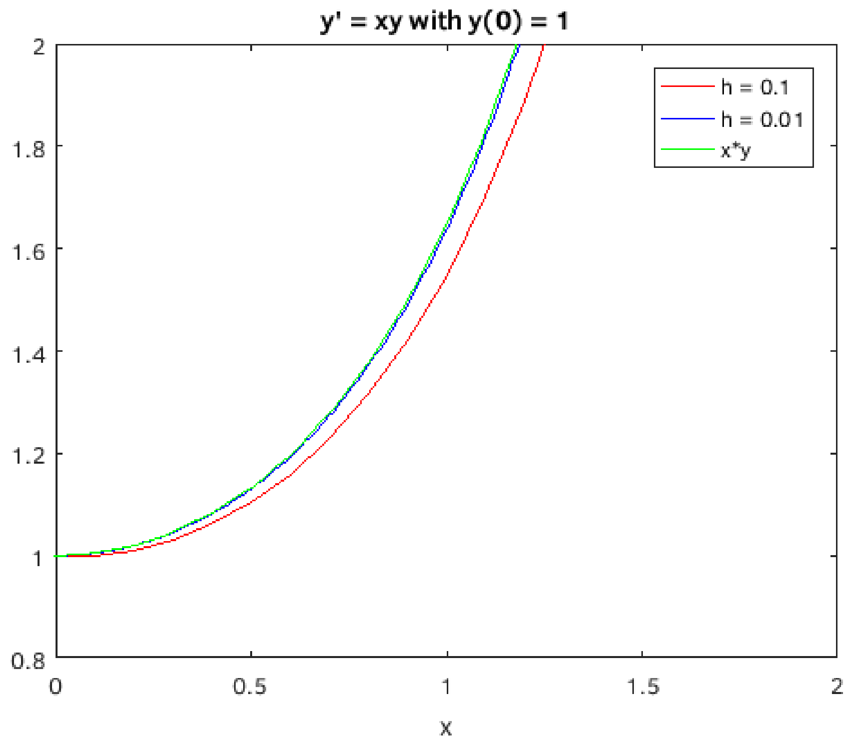


Figure 3: With $y(0) = 1$

The exact solution for $y(0) = 2$ found using Matlab is given by

$$y = 2e^{x^2/2}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

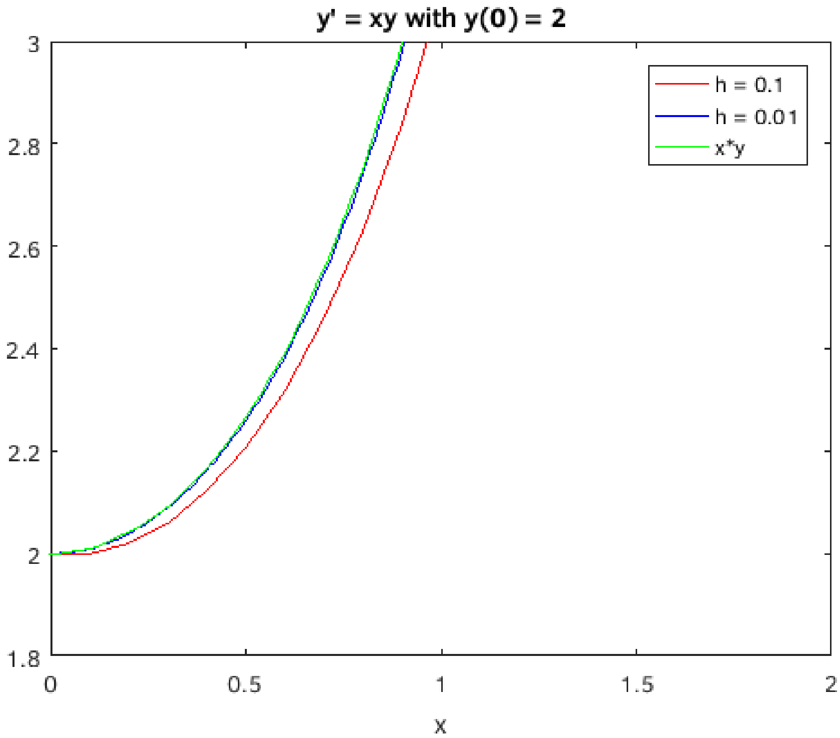


Figure 4: With $y(0) = 2$

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -e^{x^2/x}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

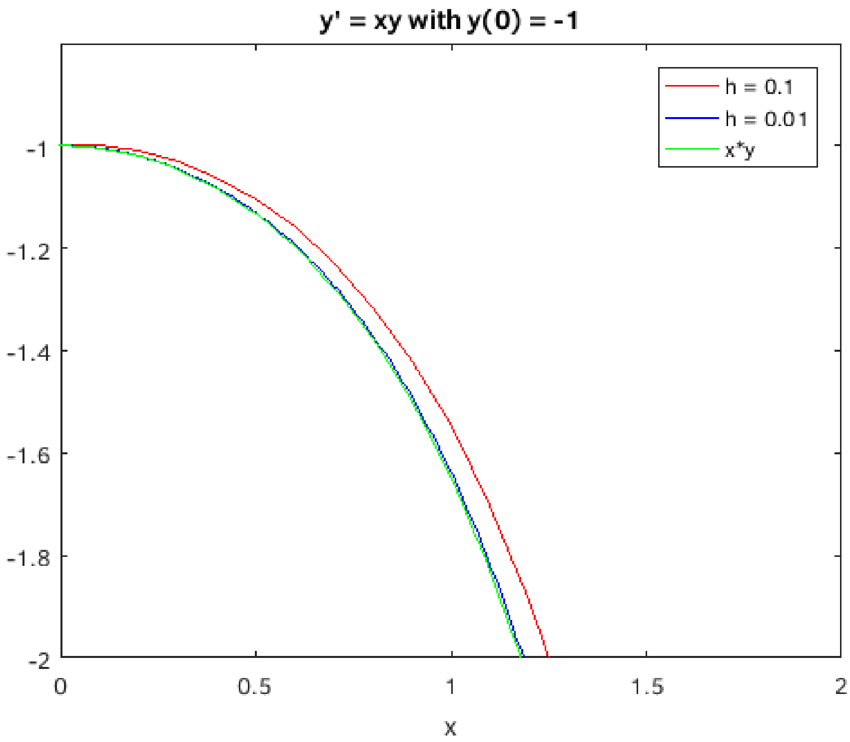


Figure 5: With $y(0) = -1$

d) **Error.** Error, as a function of h. We fix the initial condition, and the interval [0,1], and then compute the error (defined as the maximum of the absolute value of the pointwise error over the interval) versus the step size h. The initial condition being analyzed is for $y(0) = 2$

Table 1: Error Analysis of $y = 2e^{x^2/2}$

| Step | Max Abs Difference |
|---|---|
| .1 | 0.20322 |
| .05 | 0.10556 |
| .01 | 0.00218 |
| .005 | 0.10946 |
| .001 | 0.00220 |

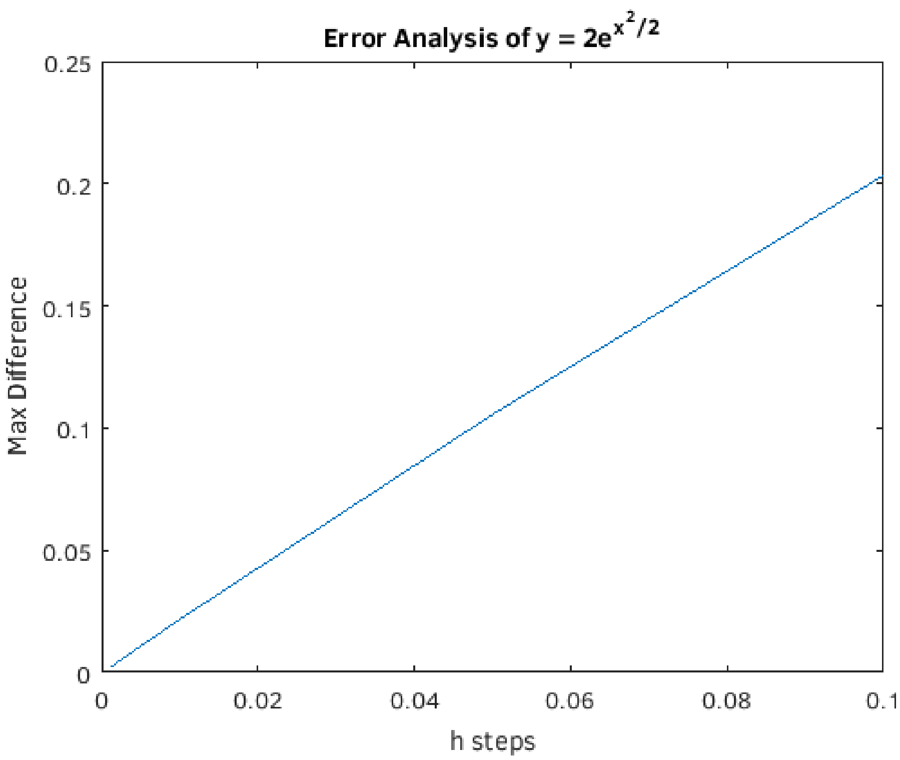


Figure 6: Error Analysis of $y = 2e^{x^2/2}$

II. Using Matlab, we investigate the following differential equation

$$\frac{dy}{dx} = y^2$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

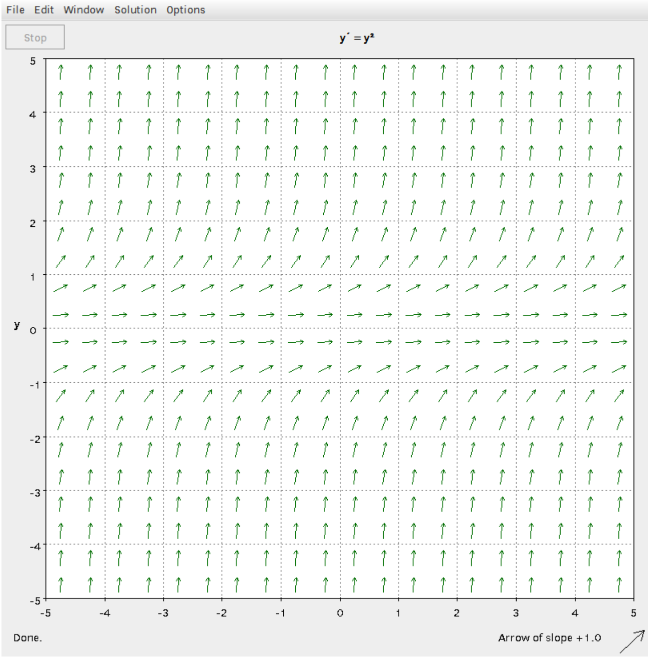


Figure 7: Direction field of $y\,' = y^2$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

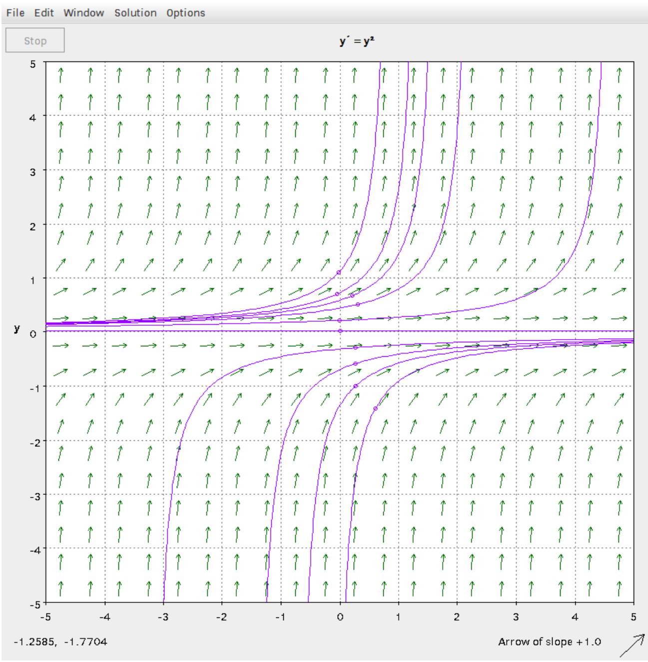


Figure 8: Integral curves of $y\,' = y^2$

c) **Numerical solution.**

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -\frac{1}{x-1}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -\frac{1}{x-\frac{1}{2}}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -\frac{1}{x+1}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

d)

III. Using Matlab, we investigate the following differential equation

$$xy\,' + \sin(x)y = x^2$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

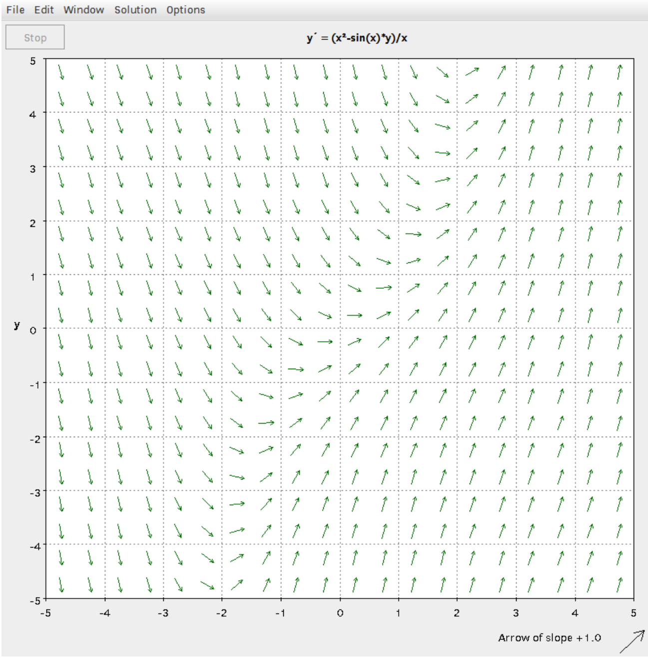


Figure 9: Direction field of $xy' + \sin(x)y = x^2$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

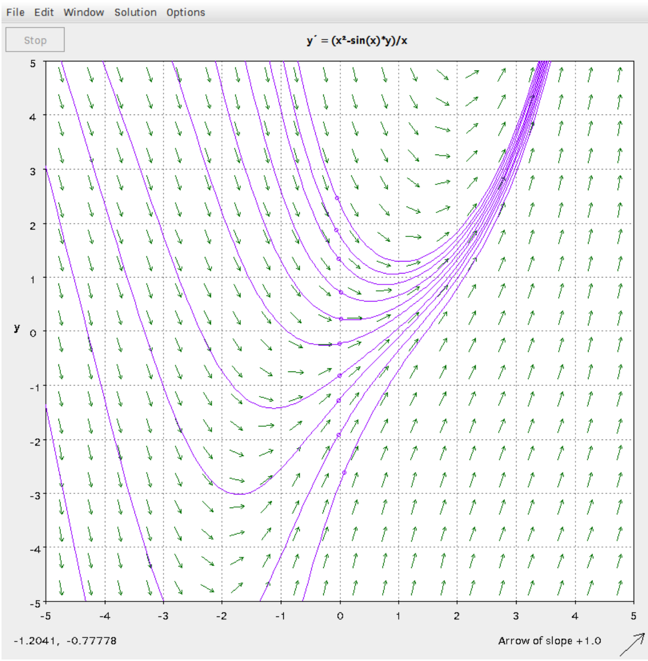


Figure 10: Integral curves of $xy' + \sin(x)y = x^2$

c)

d)