Project 1

Joseph Martinsen

Math 308-510

STATEMENT OF PROBLEM: Consider the following differential equations:

I. $\frac{dy}{dx} = xy$

II. $\frac{dy}{dx} = y^2$

III. $x\frac{dy}{dx} + \sin(x)y = x^2$

1. Graph the direction field in the neighborhood of the origin
2. Graph approximate integral curves for 10 different initial conditions (you can use dfield8.m)
3. Solve the differential equation using Euler's method (e.g. with euler.m), for three initial conditions and two values of the stepsize, h
4. If possible compute the error, either using the exact solution (if available) or a very accurate numerical solution (from Matlab)

I. Using Matlab, we investigate the following differential equation

$$\frac{dy}{dx} = xy$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

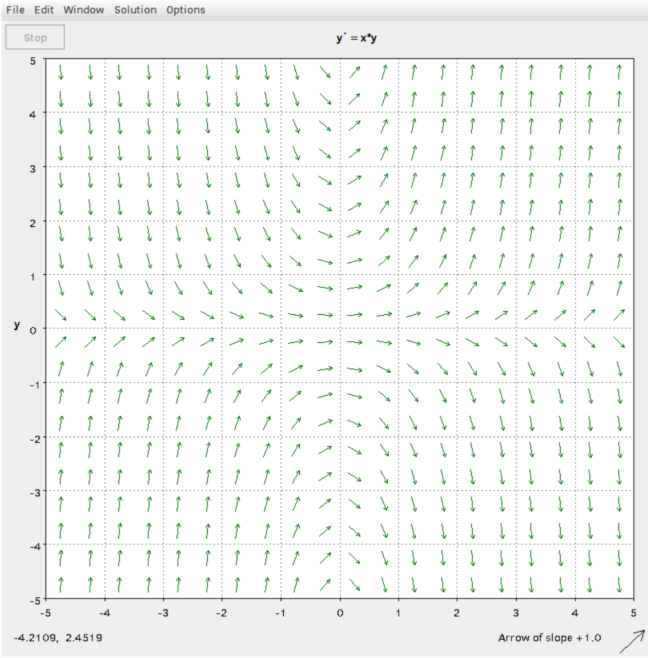

Figure 1: Direction field of $y\,' = xy$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

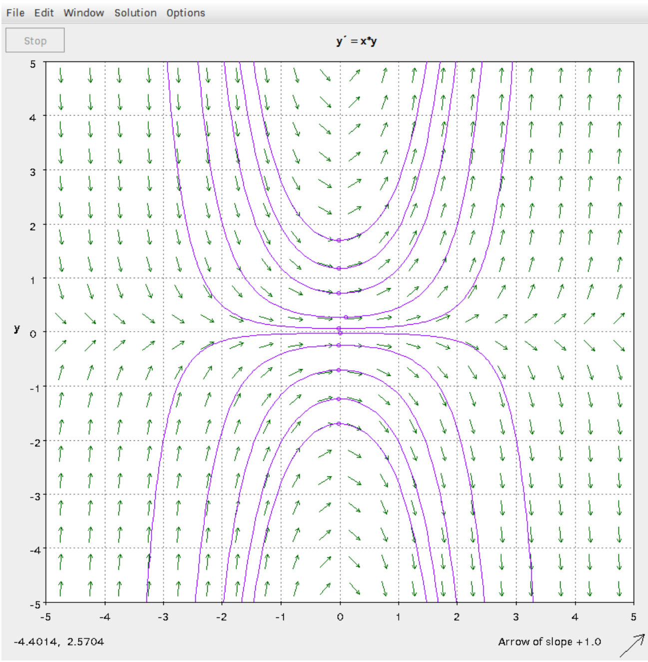

Figure 2: Integral curves of $y\,' = xy$

c) **Numerical solution.**

The exact solution for $y(0) = 1$ is using Matlab is given by

$$y = e^{x^2/2}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$ is compared to the solution before.

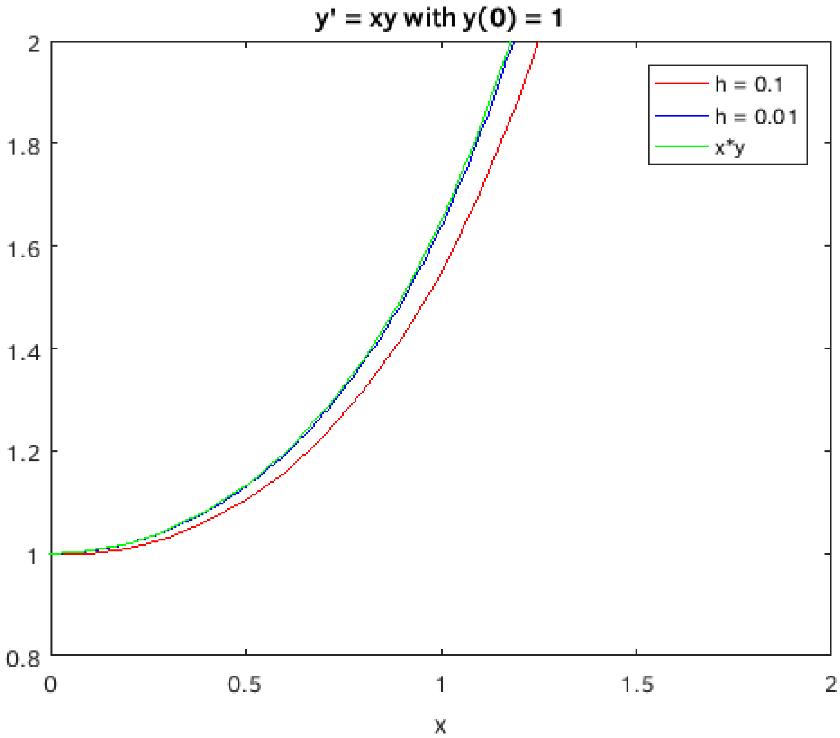

Figure 3: With $y(0) = 1$

The exact solution for $y(0) = 2$ found using Matlab is given by

$$y = 2e^{x^2/2}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

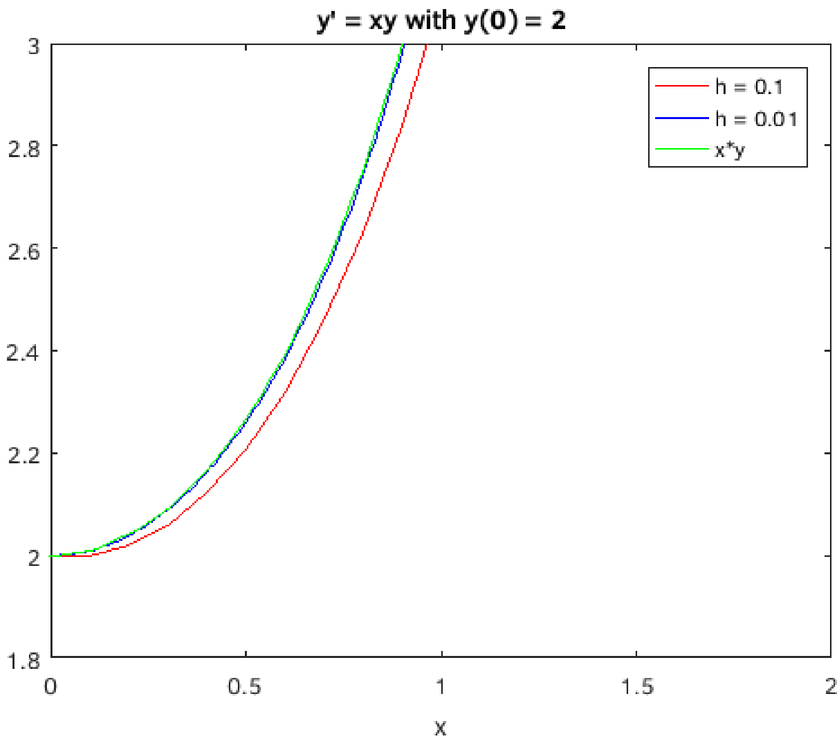

Figure 4: With $y(0) = 2$

The exact solution for $y(0) = -1$ found using Matlab is given by

$$y = -e^{x^2/x}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

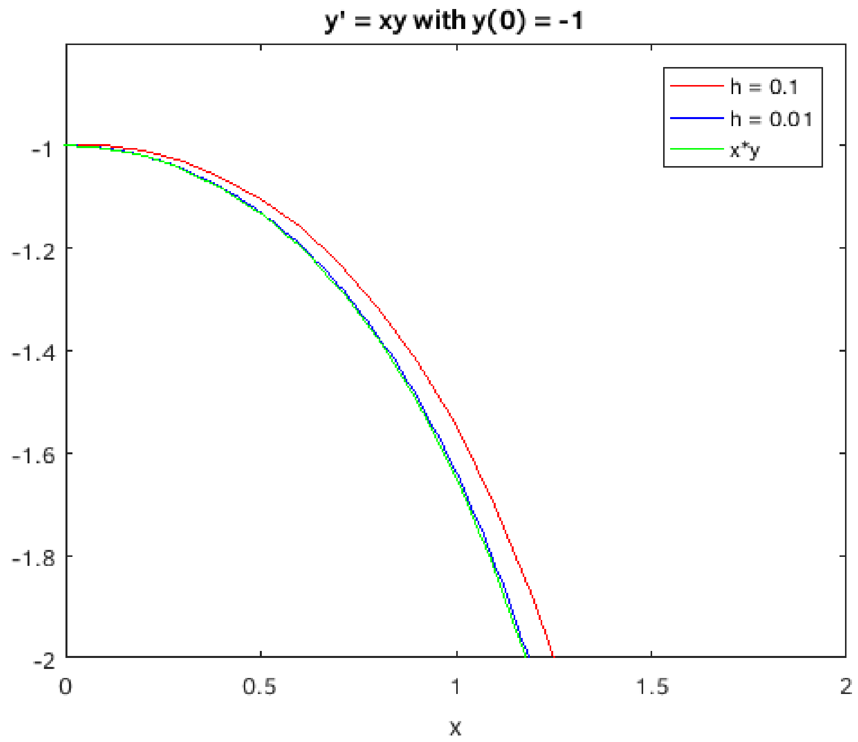

Figure 5: With $y(0) = -1$

d) **Error.** Error, as a function of h. We fix the initial condition, and the interval [0,1], and then compute the error (defined as the maximum of the absolute value of the pointwise error over the interval) versus the step size h. The initial condition being analyzed is for $y(0) = 2$

Table 1: Error Analysis of $y = 2e^{x^2/2}$

Step	Max Abs Difference
.1	0.20322
.05	0.10556
.01	0.00218
.005	0.10946
.001	0.00220

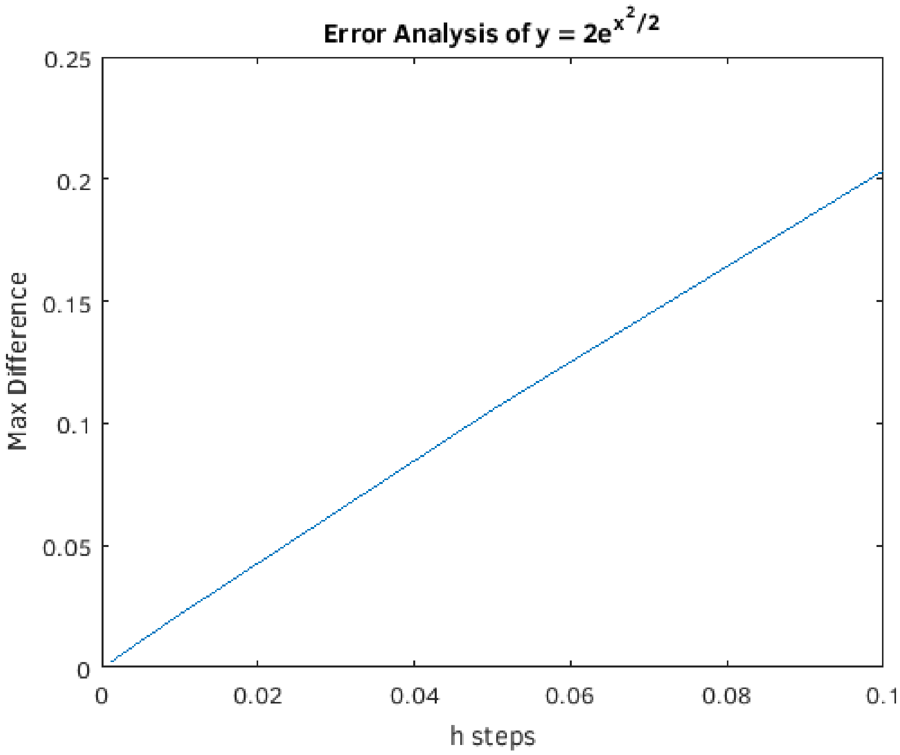

Figure 6: Error Analysis of $y = 2e^{x^2/2}$

II. Using Matlab, we investigate the following differential equation

$$\frac{dy}{dx} = y^2$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

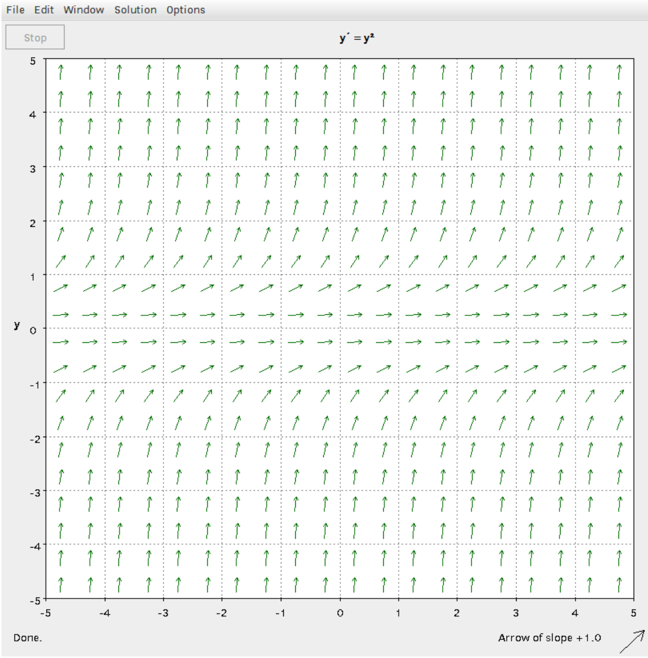

Figure 7: Direction field of $y\,' = y^2$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

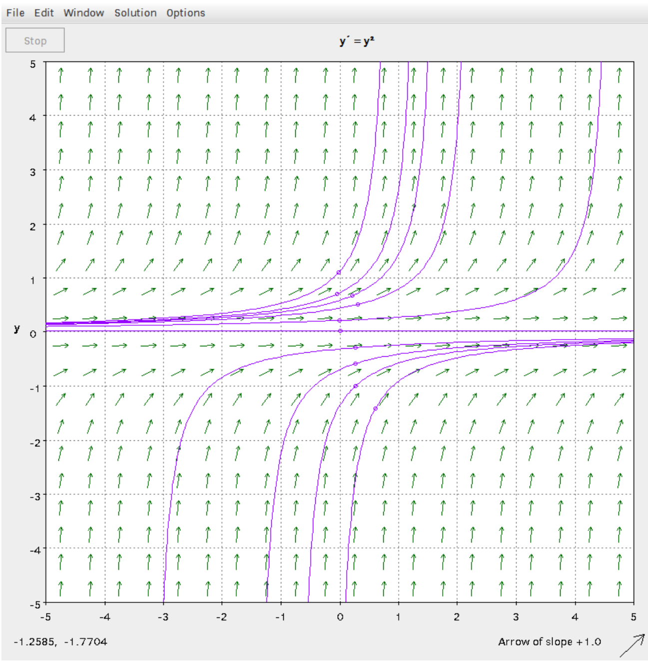

Figure 8: Integral curves of $y\,' = y^2$

c) **Numerical solution.**

The exact solution for $y(0) = 1$ found using Matlab is given by

$$y = -\frac{1}{x-1}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

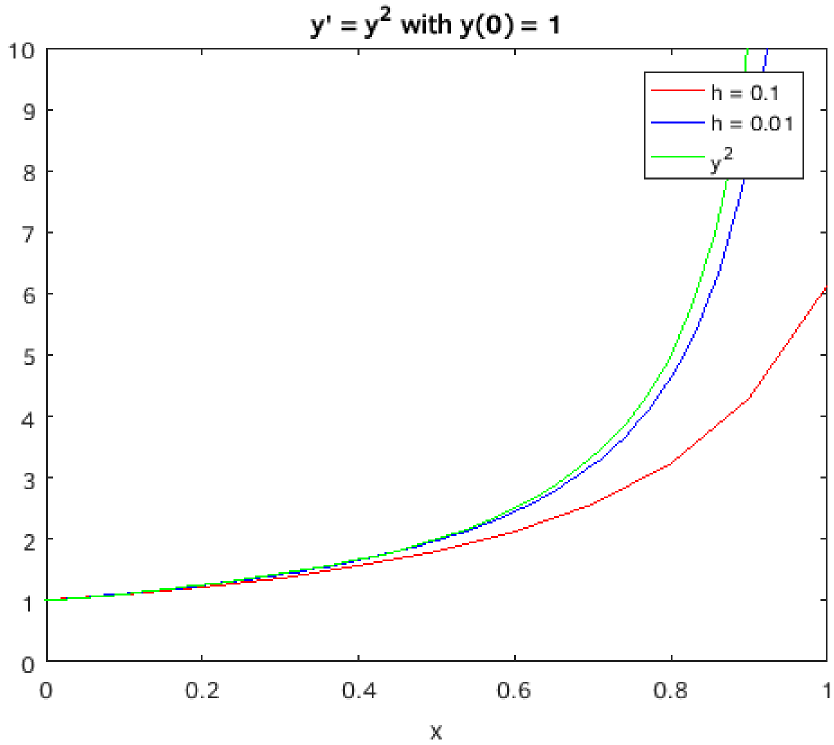

Figure 9: With $y(0) = 1$

The exact solution for $y(0) = 2$ found using Matlab is given by

$$y = -\frac{1}{x-\frac{1}{2}}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

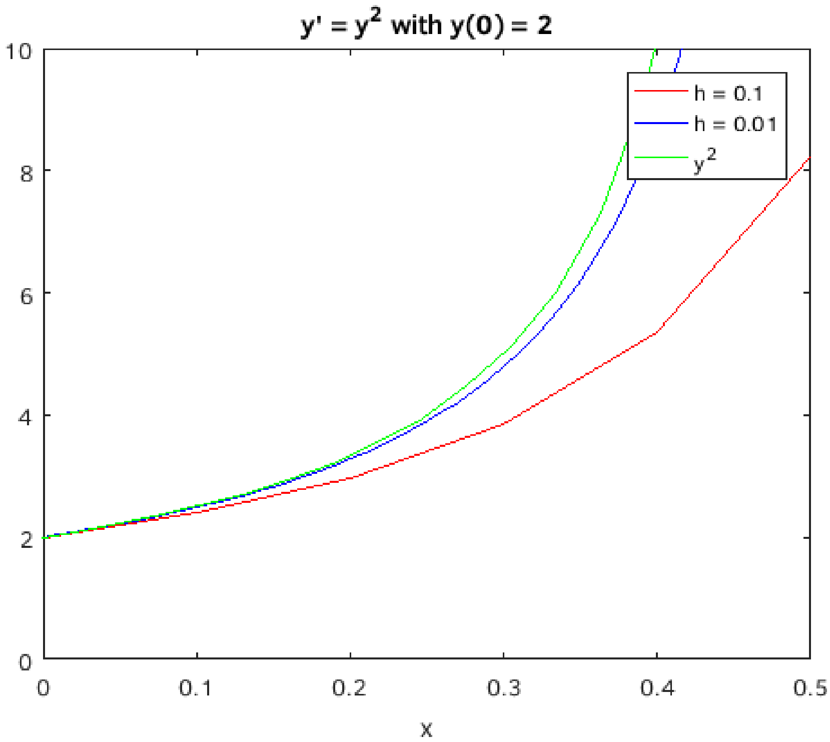

Figure 10: With $y(0) = 2$

The exact solution for $y(0) = -1.5$ found using Matlab is given by

$$y = -\frac{1}{x + \frac{2}{3}}$$

The graph of the results of Euler's method, with $h = 0.1$ and $h = 0.01$, is compared to the exact solution below

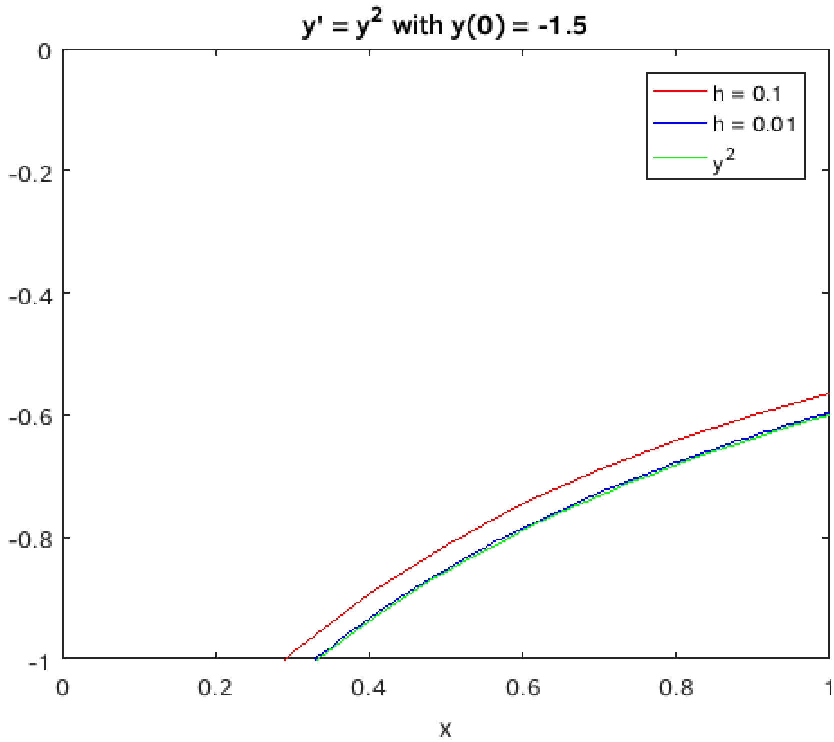

Figure 11: With $y(0) = -1.5$

d) **Error.** Error, as a function of h. We fix the initial condition, and the interval [0,1], and then compute the error (defined as the maximum of the absolute value of the pointwise error over the interval) versus the step size h. The initial condition being analyzed is for $y(0) = -1.5$

Table 2: Error Analysis of $y = -\dfrac{1}{x + \frac{2}{3}}$

Step	Max Abs Difference
.1	0.04656
.05	0.02187
.01	0.00418
.005	0.00208
.001	0.00041

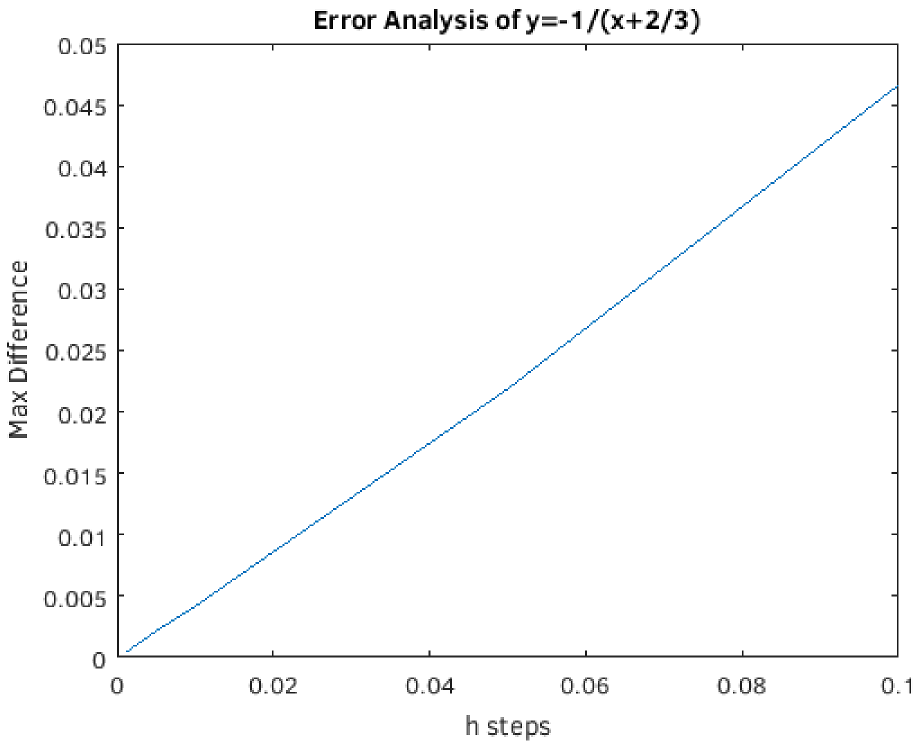

Figure 12: Error analysis with $y(0) = -1.5$

III. Using Matlab, we investigate the following differential equation

$$xy' + \sin(x)y = x^2$$

a) **Direction Field.** The direction field near the origin, $-5 \leq x \leq 5. -5 \leq y \leq 5$ is shown below

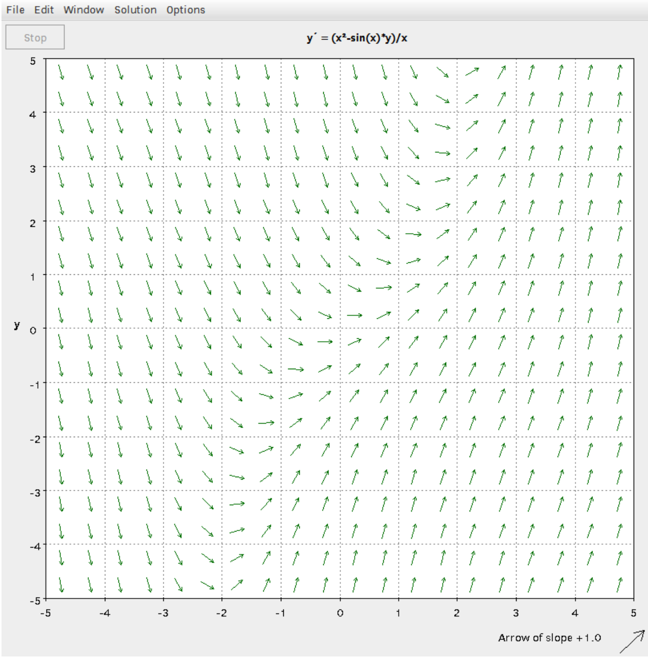

Figure 13: Direction field of $xy' + \sin(x)y = x^2$

b) **Integral curves.** 10 different initial conditions are shown in the figure below

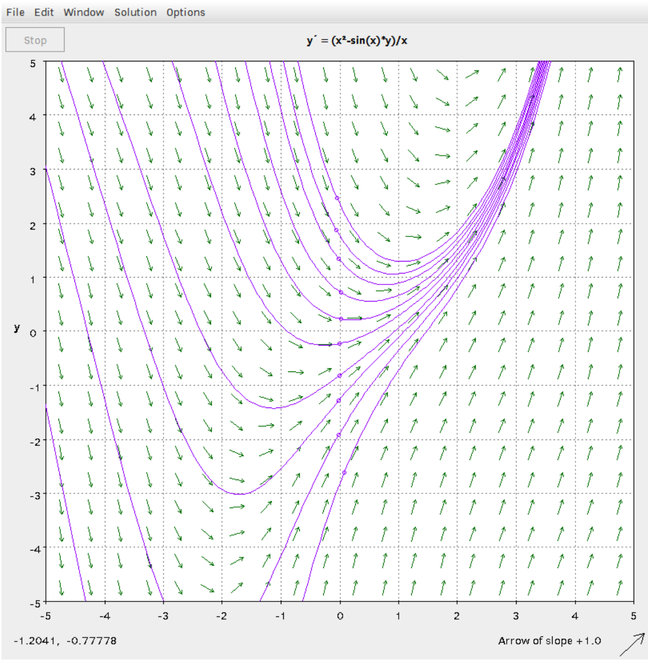

Figure 14: Integral curves of $xy' + \sin(x)y = x^2$

c)

d)

Conclusions

Appendix I: Program Listings

Code Block 1: euler.m

```
function [tout, yout] = euler(rhs, t0, y0, h, N)

% Usage: euler('rhs',t0,y0,h,N)
% Note: error will occur of you omit quotes, e.g.
% euler(rhs,t0,y0,h,N)
%
% integrates a single ordinary differential equation
% of the form
%       y'=f(t,y)
% where
%   the initial value of t = t0
%   and y(t0) = y0
%   the stepsize = h
%   and the number of steps = N
% Note: the right hand side must be defined in the file rhs.m

% allocate space and initialize to vectors to zero
tout = zeros(N+1,1);
yout = zeros(N+1,1);

% set initial values
tout(1) = t0;
yout(1) = y0;

% apply Euler's method
for i=1:N
```

```
  tout(i+1) = tout(i) + h;
  slope = feval(rhs,tout(i),yout(i));
  yout(i+1) = yout(i) + slope*h;
end
```

Code Block 2: rhs.m

```
function yprime = rhs(x, y)

% rhs returns a function which is the right hand side
% of a first order differential equation
% dy/dx = y' = f(x,y)

yprime = x.*y;
```

Code Block 3: rhs2.m

```
function yprime = rhs(x, y)

% rhs returns a function which is the right hand side
% of a first order differential equation
% dy/dx = y' = f(x,y)

yprime = y.^2;
```

Code Block 4: main.m

```
%% PROJECT 01
% written by Joseph Martinsen
% Math 308-510
clear;clc;

%% Problem 1
clear;clc;
```

```
syms y(x)
eqn = diff(y,x) == x*y;
cond = y(0) == 1;
cond1 = y(0) == 2;
cond2 = y(0) == -1;
ySol1 = char(dsolve(eqn,cond));
ySol2 = char(dsolve(eqn,cond1));
ySol3 = char(dsolve(eqn,cond2));

fprintf('Problem 1: dy/dx = xy\n\n')
fprintf('Y solution for y(0) = 1: %s\n',ySol1)
fprintf('Y solution for y(0) = 2: %s\n',ySol2)
fprintf('Y solution for y(0) = -1: %s\n',ySol3)

% Plot for y(0) = 1
[x1, y1] = euler('rhs',0,1,0.1,100);
[x2, y2] = euler('rhs',0,1,0.01,200);

plot(x1,y1,'r')
hold on
plot(x2,y2,'b')
p = ezplot(ySol1);

set(p,'Color','g')
title('y'' = xy with y(0) = 1')
axis([0 2 0.8 2])
legend('h = 0.1','h = 0.01', 'x*y')

% Plot for y(0) = 1
figure
[x3, y3] = euler('rhs',0,2,0.1,100);
```

```
[x4, y4] = euler('rhs',0,2,0.01,200);

plot(x3,y3,'r')
hold on
plot(x4,y4,'b')
p = ezplot(ySol2);

set(p,'Color','g')
title('y'' = xy with y(0) = 2')
axis([0 2 1.8 3])
legend('h = 0.1','h = 0.01', 'x*y')


% Plot for y(0) = -1
figure
[x3, y3] = euler('rhs',0,-1,0.1,100);
[x4, y4] = euler('rhs',0,-1,0.01,200);

plot(x3,y3,'r')
hold on
plot(x4,y4,'b')
p = ezplot(ySol3);

set(p,'Color','g')
title('y'' = xy with y(0) = -1')
axis([0 2 -2 -.8])
legend('h = 0.1','h = 0.01', 'x*y')

% Error Analysis fixing y(0) = 2
[x5, y5] = euler('rhs',0,2,0.1,10);
[x6, y6] = euler('rhs',0,2,0.05,20);
```

```
[x7, y7] = euler('rhs',0,2,0.01,100);
[x8, y8] = euler('rhs',0,2,0.005,200);
[x9, y9] = euler('rhs',0,2,0.001,1000);

eq5 = 2 .* exp(x5.^2/2);
eq6 = 2 .* exp(x6.^2/2);
eq7 = 2 .* exp(x7.^2/2);
eq8 = 2 .* exp(x8.^2/2);
eq9 = 2 .* exp(x9.^2/2);

maxdiff(1) = max(abs(y5-eq5));
maxdiff(2) = max(abs(y6-eq6));
maxdiff(3) = max(abs(y7-eq7));
maxdiff(4) = max(abs(y8-eq8));
maxdiff(5) = max(abs(y9-eq9));

fprintf('\nStep \t Difference\n')
fprintf('.1 \t %d\n', maxdiff(1))
fprintf('.05 \t %d\n',maxdiff(2))
fprintf('.01 \t %d\n',maxdiff(3))
fprintf('.005 \t %d\n',maxdiff(4))
fprintf('.001 \t %d\n',maxdiff(5))

step = [.1 .05 .01 .005 .001];

figure
plot(step,maxdiff)
title('Error Analysis of y = 2e^{x^2/2}')
xlabel('h steps')
ylabel('Max Difference')

```

```

%% Problem 2
syms y(x)
eqn = diff(y,x) == y^2;
cond = y(0) == 1;
cond1 = y(0) == 2;
cond2 = y(0) == -1.5;
ySol1 = dsolve(eqn,cond);
ySol2 = dsolve(eqn,cond1);
ySol3 = dsolve(eqn,cond2);

fprintf('\n\nProblem 2: dy/dx = y^2\n\n')
fprintf('Y solution for y(0) = 1: %s\n',ySol1)
fprintf('Y solution for y(0) = 2: %s\n',ySol2)
fprintf('Y solution for y(0) = -1.5: %s\n',ySol3)


% Plot for y(0) = 1
[x1, y1] = euler('rhs2',0,1,0.1,200);
[x2, y2] = euler('rhs2',0,1,0.01,200);

figure
plot(x1,y1,'r')
hold on
plot(x2,y2,'b')
p = ezplot(ySol1);

set(p,'Color','g')
title('y'' = y^2 with y(0) = 1')
axis([0 1 0 10])
legend('h = 0.1','h = 0.01', 'y^2')
```

```
% Plot for y(0) = 2
[x1, y1] = euler('rhs2',0,2,0.1,200);
[x2, y2] = euler('rhs2',0,2,0.01,200);

figure
plot(x1,y1,'r')
hold on
plot(x2,y2,'b')
p = ezplot(ySol2);

set(p,'Color','g')
title('y'' = y^2 with y(0) = 2')
axis([0 .5 0 10])
legend('h = 0.1','h = 0.01', 'y^2')

% Plot for y(0) = 1
[x1, y1] = euler('rhs2',0,-1.5,0.1,200);
[x2, y2] = euler('rhs2',0,-1.5,0.01,200);

figure
plot(x1,y1,'r')
hold on
plot(x2,y2,'b')
p = ezplot(ySol3);

set(p,'Color','g')
title('y'' = y^2 with y(0) = -1.5')
axis([0 1 -1 0])
legend('h = 0.1','h = 0.01', 'y^2')
```

```
% Error Analysis fixing y(0) = 2
[x5, y5] = euler('rhs2',0,-1.5,0.1,10);
[x6, y6] = euler('rhs2',0,-1.5,.05,20);
[x7, y7] = euler('rhs2',0,-1.5,0.01,100);
[x8, y8] = euler('rhs2',0,-1.5,0.005,200);
[x9, y9] = euler('rhs2',0,-1.5,0.001,1000);

eq5 = -1./(x5+2/3);
eq6 = -1./(x6+2/3);
eq7 = -1./(x7+2/3);
eq8 = -1./(x8+2/3);
eq9 = -1./(x9+2/3);

maxdiff(1) = max(abs(y5-eq5));
maxdiff(2) = max(abs(y6-eq6));
maxdiff(3) = max(abs(y7-eq7));
maxdiff(4) = max(abs(y8-eq8));
maxdiff(5) = max(abs(y9-eq9));

fprintf('\nStep \t Difference\n')
fprintf('.1 \t %d\n', maxdiff(1))
fprintf('.05 \t %d\n',maxdiff(2))
fprintf('.01 \t %d\n',maxdiff(3))
fprintf('.005 \t %d\n',maxdiff(4))
fprintf('.001 \t %d\n',maxdiff(5))

step = [.1 .05 .01 .005 .001];

figure
plot(step,maxdiff)
title('Error Analysis of y=-1/(x+2/3)')
```

```
xlabel('h steps')
ylabel('Max Difference')

%% Problem 3
% syms y(x)
% eqn = x*diff(y,x) + sin(x)*y == x^2;
% cond = y(0) == 1;
% cond1 = y(0) == 2;
% cond2 = y(0) == -1;
% ySol(x) = dsolve(eqn,cond)
% ySol(x) = dsolve(eqn,cond1)
% ySol(x) = dsolve(eqn,cond2)
```