Statistics and Probability

free and open source book written for educational purposes

Joseph Mehdiyev

A book written for the author's educational purposes

Preface

About the Book

This book is sort of a big notebook to make (or force) the author to self study and understand the field of Probability and Statistics. I have to note that this book is an educational and fun project for the author himself. Through the book, author tries to explain the topics to *himself* .Be careful using the book as the main learning material, since the writer himself is not an expert in the field, there may be mathematical errors in the book.

To learn the field and write this book, I used various books from known authors and wikipedia (duh-duh) articles. These are some of the books I used majority of time:

- Larry Wasserman - All of Statistics - A Concise Course in Statistical Inference.
- Dimitri Bertsekas And John N Tsitsiklis - Introduction To Probability
- Mathematical Statistics with Applications by Dennis Wackerly, William Mendenhall, and Richard L. Scheaffer
- Joseph K. Blitzstein - Introduction to Probability
- Ross, Sheldon - First Course in Probability

I want to note that I did not, by any means, plagiarize any contents, diagrams or other things. I simply wrote whatever I learnt through the the brainstorm.

Book's source

Maybe you may already know this, this book is fully open source with its pictures and tex file shared in author's github. You may use the source code for whatever purposes you want to use it for. If you want to contribute, please send a pull request from the github.Currently the book is in development.

How to use the Book

Without the unnecessary historical chapters and big exercises, you may use the book as a revisit or a secondary material. The book tries to be very explanatory and precise about the mathematical concepts.

Contents

Part I

Probability

Chapter 1

Introduction to Probability

The concept "probability" is used very often in everyday language to describe the chance of something happening. Mathematically, Probability is a language to quantify uncertainty. This chapter will introduce necessary and basic concepts and namely, **Probability Theory**. We will start the chapter with the elementary *Set Theory*.

1.1 Set Theory

Set Theory is a branch of mathematics that studies *sets*, which we will define shortly. This branch is, like other parts of mathematics, very deep and complex. We will learn only the most important concepts , which is in high-school level, needed to understand later sections and chapters.

We will quickly introduce the concepts and briefly explain them. The reader may skip this section if they already know about sets and their basic properties.

Sets

A **Set** is a collection of different objects, which are called *elements* of the set. The sets are notated as capital letters such as S. If x is an element of a set S, we write $x \in S$. Otherwise we write $x \notin S$. A set with no elements is called **empty set** and is notated as $\emptyset$.

If $x_1, x_2, ..., x_n$ are the elements of the set S, we write:

$$S \in \{x_1, x_2, ..., x_n\}$$

We can visualize the sets in 2D with a rectangle, circles and their intersections. The rectangle represent **Sample Space** Ω, or **Universal Set**, which we will explain next section.

If S is set of all even numbers smaller than 12, we can draw the diagram as:

We can specify our set as a selection from a larger set. If we want to write the set of all even integers, we can write (Here the set of integers is the universal set):

$$S = \{n \in \mathbb{Z} : \frac{n}{2} \text{ is an integer}\}$$

If a set A's elements are also the elements of B, we say that A is a **subset** of B. We can notate it as:

$$A \subseteq B$$

If a set A is subset of B, but is not equal to B, we say that A is **proper subset** of B. We can notate it as:

$$A \subsetneq B$$

Set operations

Union of sets A,B is a set that contains the elements of A and B:

$$A \cup B = \{n : n \in A \vee n \in B\}$$

Intersection of sets A,B is a set that contains both the elements of A and B:

$$A \cap B = \{n : n \in A \wedge n \in B\}$$

1.2 Sample Space and Events

The Sample Space, usually denoted as S or Ω, is the *set* of all possible outcomes of an experiment. It is also called **universal set**. Subsets of Ω are called **events**. A sample element of Ω is denoted as ω.

Example 1.2.1. If we toss a six dice once, then $\Omega = \{1, 2, 3, 4, 5, 6\}$, the even that the side is even is $A = \{2, 4, 6\}$ while $\omega \in \{1, 2, 3, 4, 5, 6\}$

With the new definition, we can make another set operation: **complement** of the set A is a set of elements Ω that do not belong to A.

$$A^c = \{n : n \in \Omega \wedge n \notin A\}$$