2022. S34 2022J003A1EL



Coimisiún na Scrúduithe Stáit State Examinations Commission

Junior Cycle Final Examination 2022

Mathematics

Higher Level

Friday 10 June Afternoon 1:30 - 3:30

270 marks

Examination Number	
Day and Month of Birth	For example, 3rd February is entered as 0302

For Superintendent
Centre Stamp

For Examiner							
Running total							
Grade							

	For Examiner												
Q.	Ex.	Adv. Ex.	Q.	Ex.	Adv. Ex.								
1			11										
2			12										
3													
4													
5													
6													
7													
8													
9													
10			Total										

Instructions

There are 12 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

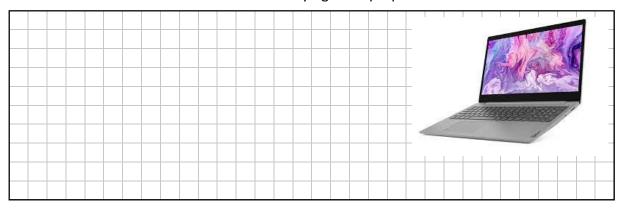
You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:	
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(Suggested maximum time: 10 minutes)

(a) Jane buys a laptop online for \$699, plus a shipping cost of \$30. The exchange rate is \$1 = €0.90.

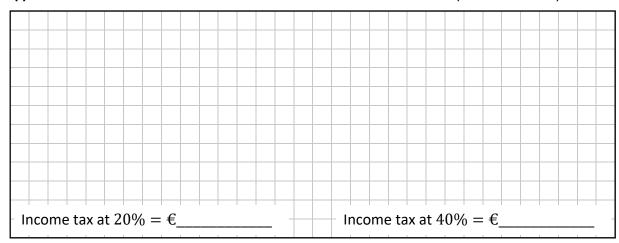
Work out in euro the total cost to Jane of buying the laptop online.



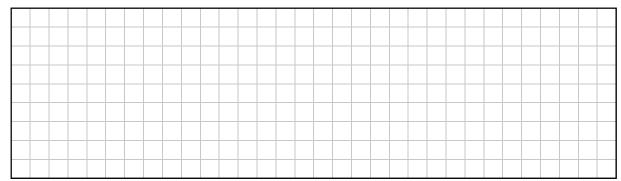
(b) Jane has a gross annual income of €56000.

Jane pays income tax on her gross income at a rate of 20% on the first \le 44 300, and 40% on the balance.

(i) Work out Jane's annual income tax at each of these two rates (20% and 40%).



(ii) Jane has annual tax credits of €3300.Work out Jane's annual take-home pay.



When Maeve's team play a match, they can win (W), draw (D), or lose (L).

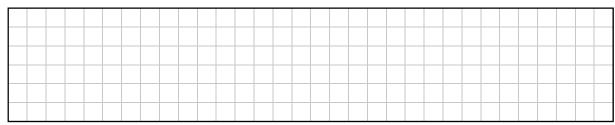
(a) Fill in the table below to show the 9 possible outcomes when Maeve's team play **two** matches. One is already done. **W D** means they win Match 1 and draw Match 2.

		Match 2									
		W	D	L							
_	w		W D								
Match 1	D										
~	L										



(b) Maeve thinks that each outcome in the table is equally likely.

Based on this, find the **probability** that, when Maeve's team play two matches, they win at least one match. Give your answer as a fraction.



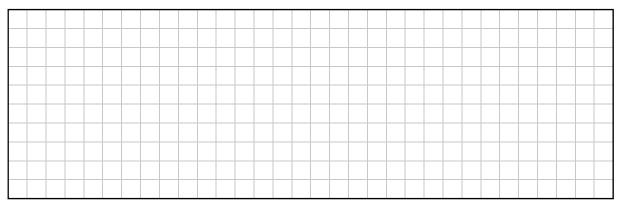
(c) Maeve's team play 5 matches in a competition.

Work out the total number of different possible outcomes for Maeve's team for these 5 matches. For example, one possible outcome would be **W W L D W**.

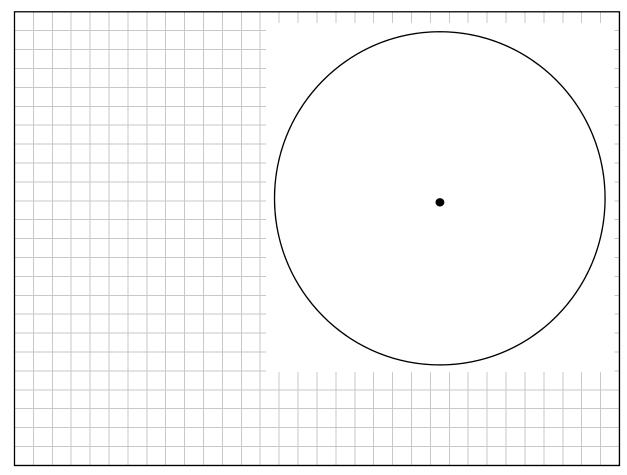


Maeve's team plays 11 matches in a league. The table below shows the number of goals that Maeve's team score in each of these 11 matches.

(d) Work out the **mean** number of goals that Maeve's team score per match. Give your answer correct to 1 decimal place.

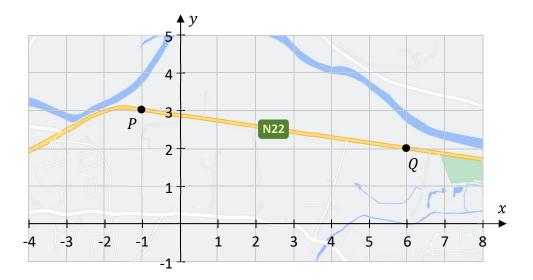


(e) Complete the pie chart below, to summarise the data above, showing the proportion of their games in which Maeve's team scored 0 goals, 1 goal, and so on. Label each sector and the size of the angle clearly. Show any working out and construction lines.



(Suggested maximum time: 10 minutes)

The co-ordinate diagram below shows part of the N22 road in County Cork. Two points on the road, P and Q, are marked on the diagram.

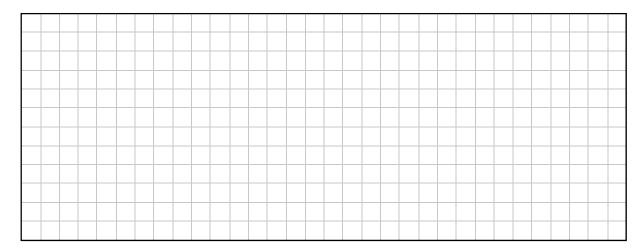


(a) The point Q has co-ordinates (6, 2). Write down the co-ordinates of the point P.

(b) The equation of the line PQ is:

$$x + 7y = 20$$

Using this, or otherwise, find the co-ordinates of the point where the line $\it PQ$ crosses the $\it y$ -axis.



(c) A new road is being built through the point Q(6,2).

On the co-ordinate diagram, it will be a straight line segment which is **perpendicular** to PQ.

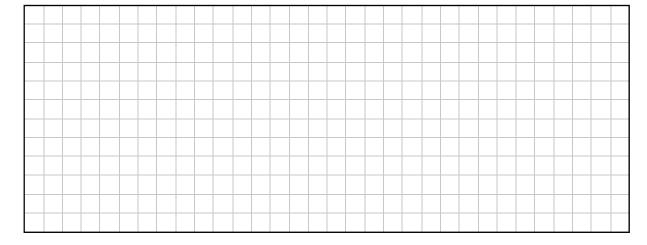
Work out the equation of this new road.

Give your answer in the form ax + by + c = 0, where $a, b, c \in \mathbb{R}$.



(d) The distance |PQ| on the diagram is $7\cdot 1$ cm, correct to 1 decimal place. 5 mm on the diagram represents 100 m.

Use this to work out the **actual** distance from P to Q. Give your answer in km.



(Suggested maximum time: 10 minutes)

The three triangles **A**, **B**, and **C** are shown below.

The given lengths of the sides of each triangle are in centimetres, where $x, y \in \mathbb{N}$.

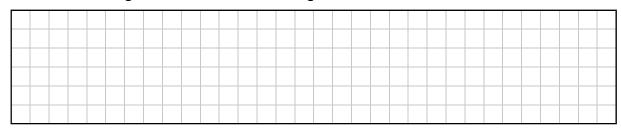
In this question, take "the perimeter" to mean "the length of the perimeter".

Triangle A	Triangle B	Triangle C
2 3.5	$ \begin{array}{c c} 3 & 2x \\ \hline 2x + 1 \end{array} $	$ \begin{array}{c c} & y^2 \\ & y^2 + 3 \end{array} $

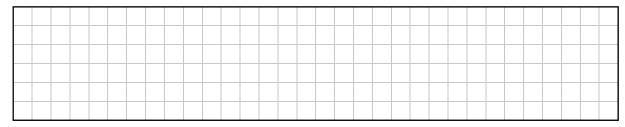
(a) The perimeter of Triangle ${\bf A}$ is $8~{\rm cm}.$

Two of the sides have length 2 cm and 3.5 cm, respectively, as shown.

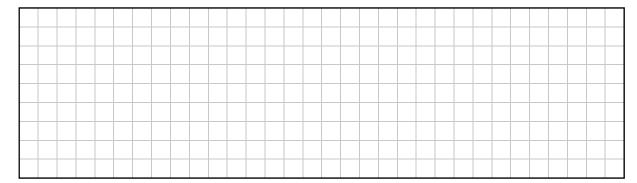
Work out the length of the third side of Triangle A.



(b) (i) Write down the perimeter of Triangle **B**, in terms of x.



(ii) The perimeter of Triangle **B** is 24 cm. Use this to work out the value of x.

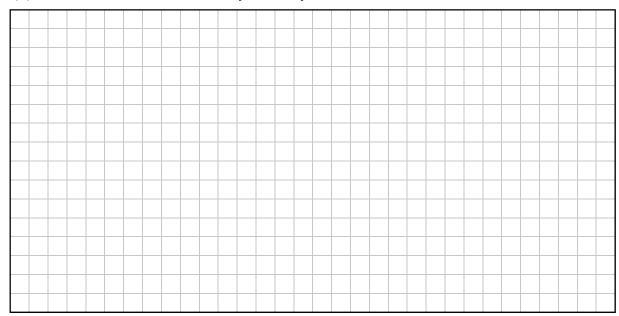


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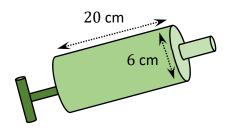
- (c) The perimeters of the three triangles A, B, and C form a linear sequence. Triangle C has the largest perimeter.
 - (i) The perimeter of Triangle C is k cm, where $k \in \mathbb{N}$. Find the value of k.



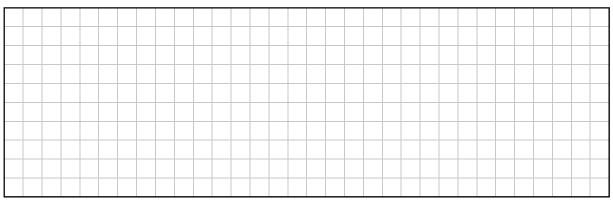
(ii) Hence work out the value of y, where $y \in \mathbb{N}$.



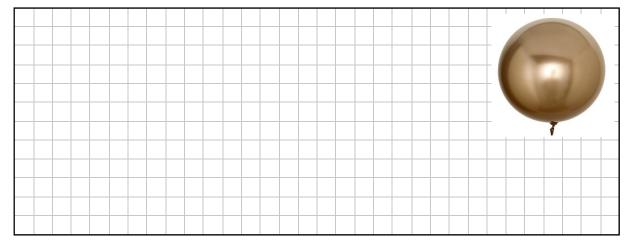
A balloon pump is made from a cylinder with an internal diameter of 6 cm and a height of 20 cm, as shown. Each time the pump is pumped, it passes one full cylinder of air into a balloon.



(a) Show that the **volume** of one full cylinder of air is 180π cm³.

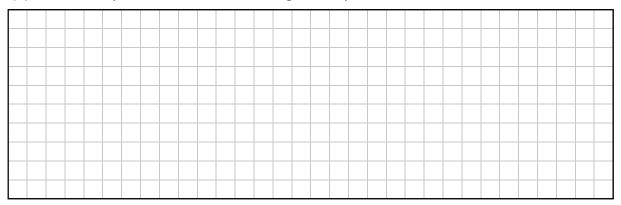


- (b) Darragh is inflating a balloon in the shape of a **sphere**. When fully inflated, the balloon has a radius of 15 cm.
 - (i) Find the **volume** of Darragh's balloon when it is fully inflated. Give your answer in cm³, in terms of π .



Darragh pumps the pump once every second.

(ii) How many seconds will it take Darragh to fully inflate his balloon?



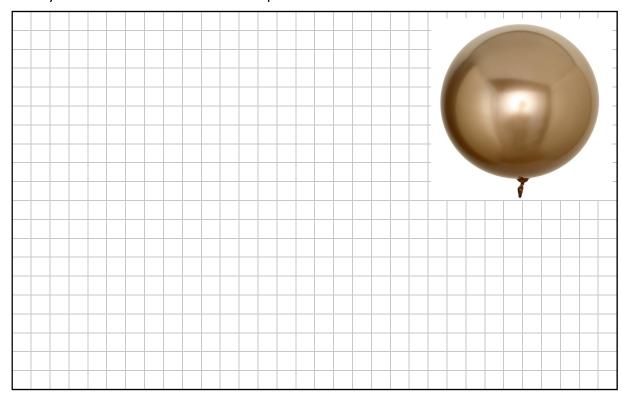
(c) Gustav is inflating a bigger balloon in the shape of a sphere.

He also pumps the pump once every second.

His balloon is fully inflated after 50 seconds.

Find the radius of Gustav's balloon when it is fully inflated.

Give your answer correct to 1 decimal place.



(Suggested maximum time: 5 minutes)

80 students in a group were asked what they had done during their summer holidays.

Some of the students got a job (J), some went on holidays (H), some did both, and some did neither.

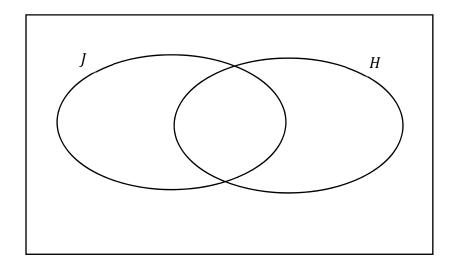
 $\frac{1}{5}$ of the students in the group did neither.

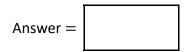
25% of the students got a job.

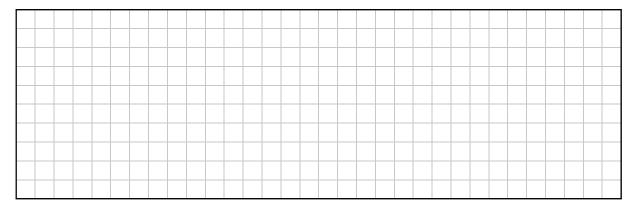
Of those students who got a job, half also went on holidays.



Work out the **total** number of students in the group who went on **holidays**. You may use the Venn diagram below to help answer the question.







(Suggested maximum time: 5 minutes)

(a) Amie and Joe are asked to pick values for the numbers p, q, and r so that the following is true for all $a \in \mathbb{R}$:

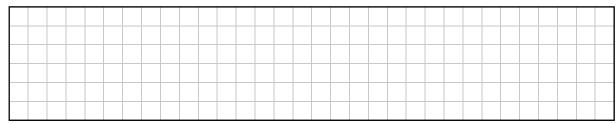
$$a^p \times a^q \times a^r = a^{12}$$

(i) Amie picked three values that were all the same, so p=q=r. Write down the values of p,q, and r that Amie picked.





$$r =$$

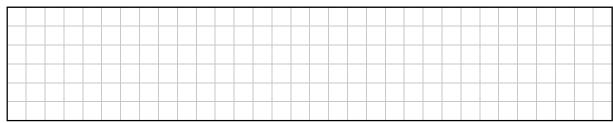


(ii) Joe picked three values that were all different. Write down possible values of p, q, and r that Joe might have picked.

$$p =$$



$$r =$$



(b) Find the value of m so that the following is true for all $b \in \mathbb{R}$:

$$\frac{b^m \times b^{-2}}{b} = b^{10}$$

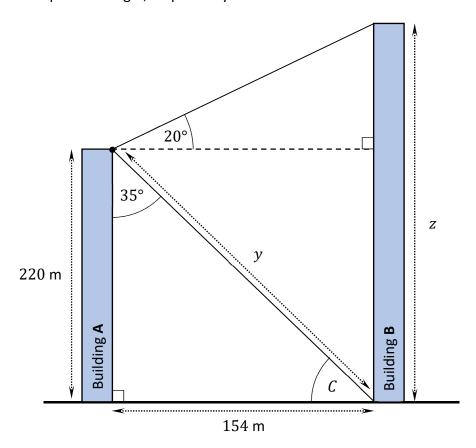


The diagram below shows two vertical buildings, **A** and **B** (diagram not to scale).

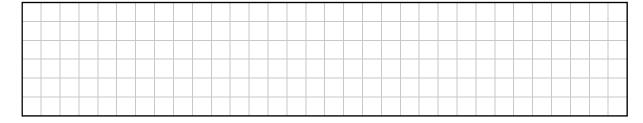
Mary stands at the top of Building A. She is 220 m above the ground.

She wants to work out the distances marked y and z in the diagram – that is, the distance from the top of Building **A** to the bottom of Building **B**, and the height of Building **B**, respectively.

Mary measures the two angles that are marked 35° and 20° in the diagram, to the bottom of Building **B** and the top of Building **B**, respectively. The broken line is horizontal.

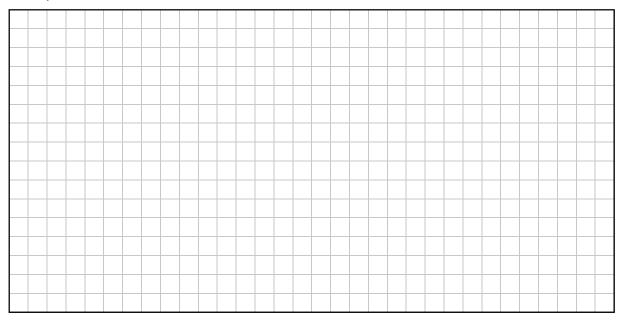


(a) Work out the size of the angle C.

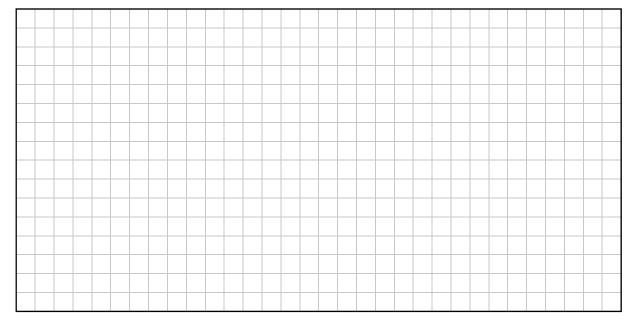


(b) Mary works out that the horizontal distance between the two buildings is 154 m, correct to the nearest metre, as shown.

Use the **Theorem of Pythagoras** to work out the distance marked y on the diagram. Give your answer correct to the nearest metre.

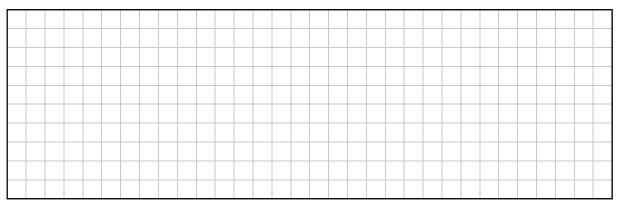


(c) Use **trigonometry** to work out the value of z, the height of Building **B**. Give your answer correct to the nearest metre.

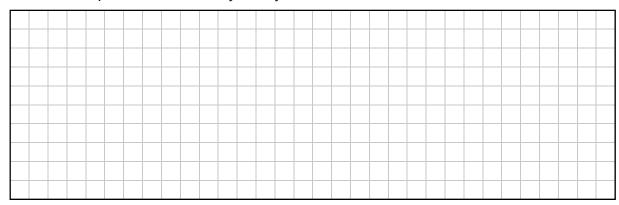


(Suggested maximum time: 10 minutes)

(a) k = 7 and m - k = 4. Work out the value of 9k - 6m.



(b) Factorise fully 8ax - 14bx + 4ay - 7by.



(c) Write the following as a single fraction in its simplest form:

$$\frac{2}{2x+1} - \frac{3}{3x+5}$$



(d) Solve the equation $2x^2 - 7x - 3 = 0$.

Give each answer correct to 2 decimal places.

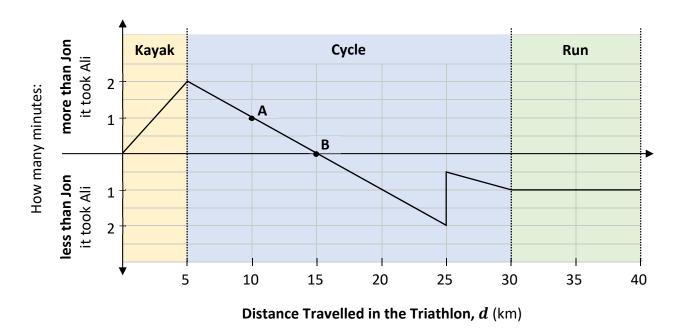
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Ali and Jon took part in a triathlon.

In the triathlon they had to complete a 5 km kayak, then a 25 km cycle, and then a 10 km run.

The diagram below was drawn after both of them had finished the race.

It shows how many minutes more than Jon (or less than Jon) it took Ali to travel d km in the triathlon, for $0 \le d \le 40$. For example, the point **A** shows that it took Ali 1 minute more than Jon to travel the first 10 km. In total, it took Ali 1 minute less than Jon to finish the triathlon.



(a)	(Tick one (\checkmark) box only.)	anead of Jon, ben	ind Jon, or at the	same time as Jon?
	Ali finished the kayak section:	ahead of Jon	behind Jon	at the same time as Jon
(b)	Ali had to stop briefly during the State what distance Ali had tra		opped, and for ho	ow long he was stopped.

Length of time Ali was stopped (in minutes):

(c) What was happening Jon and Ali at the point marked **B** on the diagram?



(d) The table below shows the time it took Jon to complete each of the three sections in the triathlon, as well as his total time for the triathlon.

Using the diagram, fill in the four missing times for Ali.

	Kayak	Cycle	Run	Total
Jon's time (minutes)	32	38	36	106
Ali's time (minutes)				



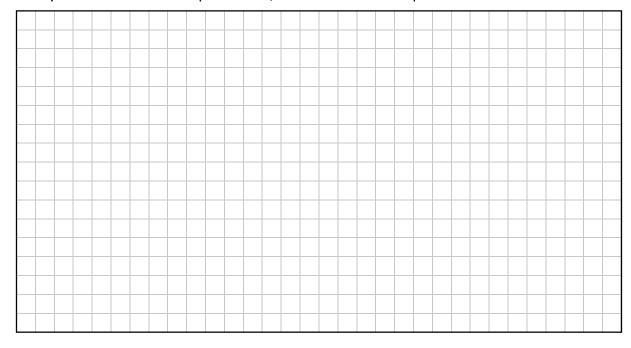
(e) Jon and Ali also ran a 400 m race.

Jon's average speed for the 400 m was 7.8 metres per second.

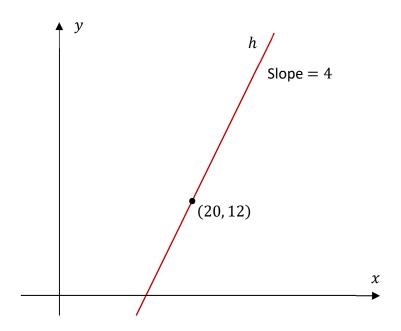
It took Ali 2 seconds more than Jon to run the 400 m.

Work out Ali's average speed for the 400 m race.

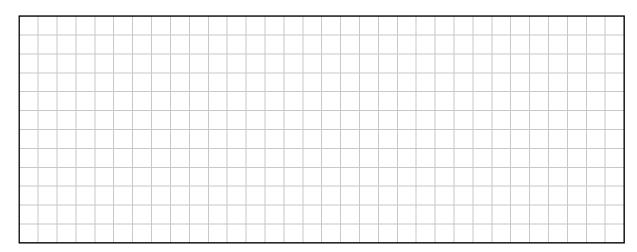
Give your answer in metres per second, correct to 1 decimal place.



The line h has a **slope of 4** and passes through the point (20, 12).



Find the co-ordinates of another point on the line h, other than the point (20,12). Show your working out.

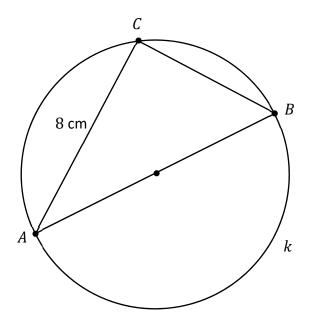


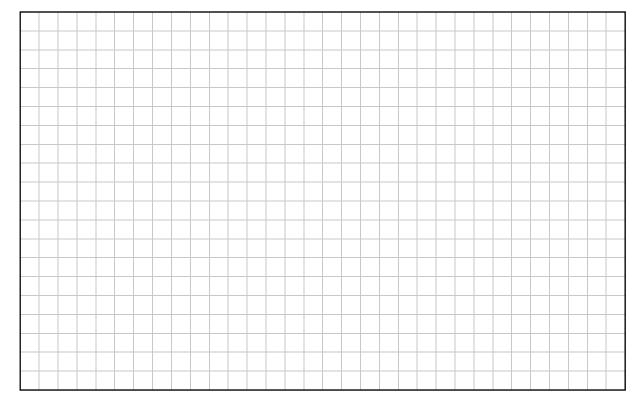
(Suggested maximum time: 5 minutes)

The diagram below shows the circle k (not to scale). The points A, B, and C lie on the circle. $\lceil AB \rceil$ is a diameter of the circle, and |AC| = 8 cm.

The **area** of the circle k is 25π cm².

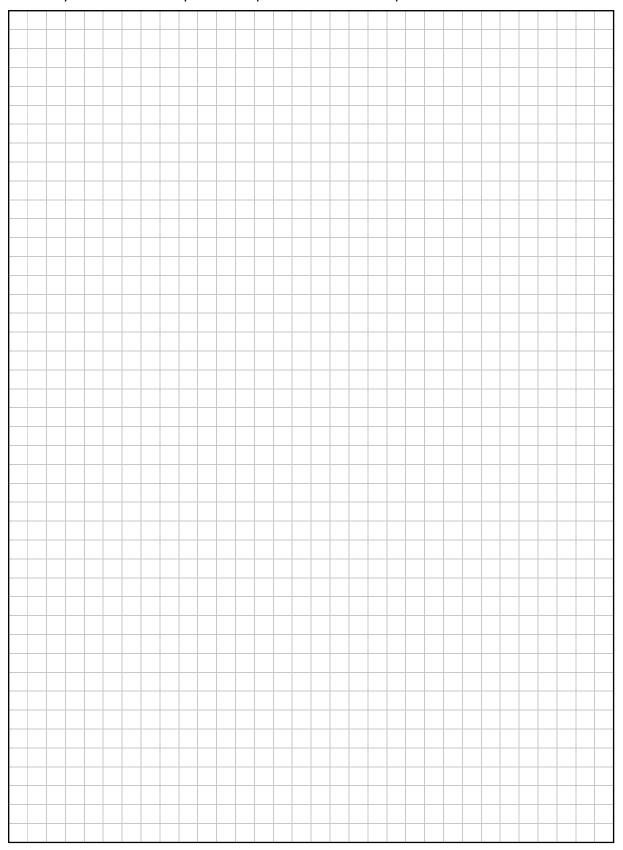
Work out the size of the **smallest** angle in the triangle *ABC*.





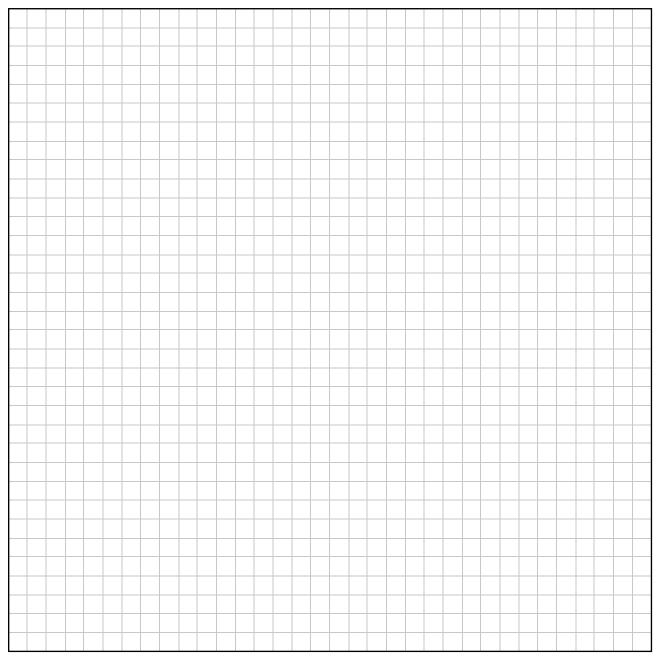
Page for extra work.

Label any extra work clearly with the question number and part.



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Acknowledgements

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Junior Cycle Final Examination - Higher Level

Mathematics

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