- NO Pushed L19 Notes due to Privacy Concerns (Guest talk)

- March 19, 2021 Notes: Applications to physical stores and retail

  - Mathematical economics:

    ↪ Real world retail business involves stochastic optimization problems

    1. How do we supply products optimally to a store or warehouse given stochastic demand and known cost structure

    2. How do you price optimally given supply and demand

    ↪ Retail also has problems like forecasting demand (supervised learning), which serves as a model inside the MDP

  - Inventory Control

    - How to optimally move inventory from suppliers, to warehouses, to stores, to shoppers.

    - Represent this as a supply ⇒ demand optimization problem

      ↪ Best case scenario: supply appears just in time to satisfy demand

    - Start with two classical inventory control problems:

      1. Economic Order Quantity:

         ↪ Assume constant rate of demand μ units/year (deterministic)

         ↪ Assume new order delivered in full when inventory goes to zero.

         ↪ Fixed cost K for each non-zero order

         ↪ Holding cost h/unit/year for storage in store

→ Objective : What is the optimal number of units to order given costs ?

→ EOQ problem goes beyond retail

Annual total cost : $\frac{\mu K}{Q} + \frac{hQ}{2}$

→ $Q/2$ is average inventory in year

→ Take derivative wrt Q and set equal to zero to get optimal order quantity Q.

→ There is a trade-off between order cost and holding cost.

② Newsvendor Problem :

- Daily demand for newspapers is a random variable x
- At the end of the day, newspapers are useless
- Low inventory incurs stock-out cost : Potential loss of future customers.
  - → Rather risk having extra newspapers than incurring stockout cost.
- Problem : How many extra newspapers should we order to minimize costs given a distribution of demand
- To solve this, review slide 8 (basic calculus)
  - → Yields popular "Newsvendor" formula :

$$\text{Optimal Supply } S^* = \underbrace{F^{-1}}_{\text{Inverse CDF of demand distribution}}\left(\frac{\overbrace{p}^{\text{stockout cost}}}{p + \underbrace{h}_{\text{holdout cost}}}\right)$$

→ $\left(\frac{p}{p+h}\right)$ is fraction of days you expect to go out of stock

- Practicality of Inventory Control :

  - Simple Model : Single store, single item inventory control
    - ↳ Order arrives L days after being ordered
    - ↳ Holding cost h
    - ↳ Stockout cost p
    - ↳ Ordering cost

    * This can be modelled as an MDP
      - ↳ State is current inventory level and inventory on-order.

$$S = \{(i, o) : i \in \mathbb{Z}_{\geq 0}, o \in \mathbb{Z}_{\geq 0}\}$$

      - ↳ Action to order each day
      - ↳ Reward given h, p, order cost
      - ↳ Model given by random demand

    * Optimal, intuitive closed-form solution :
      - ↳ Order point : Below which, order as much as is required to arrive at "order up to" level.
      - ↳ Order up to level : Level up to which each order is placed.

  - At the start of the day, balance under capacity (visual detergent, sparse shelves) and overcapacity cost (bathroom space)

- ↳ Convex cost curves:
  - ↳ Under capacity cost has to do with customer psychology
    - ↳ Big cost when customers are unhappy
    - ↳ Retail mantra: "Stack it high and watch it fly."
      - ↳ There is a feeling of abundance in produce sections in stores
    - ↳ Visual emptiness is a known sales deterrent
    - ↳ Undercapacity cost curve varies from item to item, season to season.
    - ↳ Modeling cost curve takes human judgement
  - ↳ Overcapacity cost: Backrooms have limited space
    - ↳ Ideal setup is all inventory on shelves
    - ↳ These days, backroom is used to stage online orders.

- <u>Other costs</u>: Interest on inventory (optimal cost), damage costs, staff costs, electricity costs, labor costs, food spoilage costs (acute perishability), end-of-season/obsolescence costs (think christmas trees)

- Practical inventory control is an MDP.
  - ↳ Some logistics:
    1. Replenishment orders in casepack multiples

↳ State : Inventory + on-order

↳ Action : Number of casepacks

↳ Rewards : Costs

↳ State transitions given demand distribution

↳ Solve with model-based DP or RL (if large state space)

↳ Generate sampling traces rather than using models together

- **Multinode Inventory Control** : Represented as a graph - Multiple locations to sell from
  - ↳ Inventory / State is joint inventory across stores and items
  - ↳ Actions are all movements between all nodes.
  - ↳ Reward is aggregated cost across network

- **Clearance Pricing** : What is the optimal sequence of price markdowns ?
  - ↳ Build a model of price elasticity of demand
  - ↳ Goal is maximize total profit
  - ↳ Performing markdown has a cost associated with it.
  - ↳ If price markdown is small, risk surplus
  - ↳ If price reductions are large, risk stockout cost, reduced profits
  - ↳ MDP representation

    State = [Days remaining, Current inventory, current price, market info]

    Action : Price markdown

    Reward : Sales revenue - markdown cost - stockout cost + salvage costs

    State transitions governed by price elasticity of demand

- **Ambitious Goal:** Blend inventory and Price Control into one MDP

- **How to solve real-world MDPs: General advice**

  1. Talk to business experts to develop intuition
     ↳ Start by creating a simplified version of problem
  2. DP for simple version of problem just to build intuition
  3. Move onto approximate DP
  4. As problem gets more large and curse of dimensionality hits, move to RL

- **Effective RL Approaches:**

  1. Deep Q networks
  2. When possible, use linear Function Approx, LSPI
  3. Without more mainstream algorithms, when hit by deadly triad, use exact gradient TD.

  ... Other methods and links on Slide 21.

- Separate Model Estimation from Policy Optimization
  ↳ Take data, build model with data, add your own intuition to build a simulator.
  ↳ With Simulator, you can then run RL for policy optimization
  ↳ Customize RL algorithms to take advantage of:
    1. Known transition probabilities
    2. Knowledge of reward function

③ Use any problem-specific structure that simplifies algorithms

↳ Perform feature engineering based on closed-form approximations

↳ In real world, RL is laborious with DNN architecture and hyperparameter tuning.

- Forecasting : Slide 22

- Some Planning problems :

  ① Assortment Selection

  ② Shelf-Size Selection

  ③ Casepack Sizing

  ④ Network planning

  ⑤ Labor planning

- Slide 25 : Fixed point planning and control

  ↳ Area of ongoing research