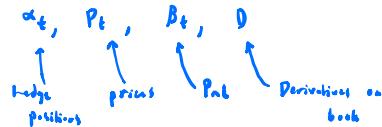


## • Derivatives Pricing and Hedging:

- Incomplete market  $\Rightarrow$  Not all positions can be hedged
- Classical theory of hedging not in course
- Assume we have a position  $D$  in  $m$  derivatives, each of which expire in time  $\leq T$
- State space includes:  $a_t, p_t, b_t, D$ 
  - $a_t$ : # of hedge positions to buy or sell
  - $p_t$ : prices
  - $b_t$ : book
  - $D$ : Derivatives



$$a_t = \# \text{ of hedge positions to buy or sell}$$

① Observe states

② Perform actions (trades)  $a_t$  to produce trading  $P_{t,b} = -a_t \cdot p_t$

③ Trading Transaction costs, example =  $-T p_t \cdot |a_t|$  for some  $T > 0$

... Continue slide 15

## • Pricing and Hedging:

→ Create incremental position  $D'$  with cash flow required to take on  $D'$  should not change optimal value

$$V_0^k((\omega_0, p_0, b_0, x, D \cup D')) = V_0^k((\omega_0, p_0, b_0, D))$$

↑                      ↑                      ↑  
 Price to take  
on new positions    Adding  $D'$  to  
derivatives portfolio    This MDP solved before trades

Now you have a new state and you want to solve for  $x$  such that  $V_0^k$  does not change

→ Want to maximize net present value of utility of accumulated future cash flow

→ Now for  $V_0^k$ , determine hedging strategy with  $T_E^k$

- DRL Approach for practical Trading
  - DRL = Deep Reinforcement Learning is breakthrough for market dynamics
    - ↳ Advantages:
    - ↳ Independent of Market Dynamics
    - ...
- A lot of banks looking at DRL, e.g. JPM

- Review: Finance Examples

- ① Asset Allocation ← Complete
  - ② American Options Pricing,
  - ③ Derivatives Pricing and Hedging
  - ④ Optimal Execution of trades ← Complete
- } Don't worry too much

### Order Book Algorithms:

- People place limit orders on a stock
- Order book has limit sell orders and limit buy orders
- Trading book aggregates limit orders at each unique price

↳ Represent books as two sorted lists:

$$\text{Bids} : \left[ (P_i^{(1)}, N_i^{(1)}) \right] \text{ descending} \rightarrow \text{Best is "bid"}$$

$$\text{Ask} : \left[ (P_i^{(1)}, N_i^{(1)}) \right] \text{ ascending} \rightarrow \text{Best is "ask"}$$

$$\text{midprice} = \frac{\text{bid} + \text{ask}}{2}$$

$$\text{Spread} = \text{ask} - \text{bid}$$

↳ Rehit asks for shares at best possible price = Market order

- ↳ Normally, you don't set sell limit order below highest bid order
- ↳ If you do by mistake, you eat into the bid order book
- ↳ Eat book block is the main function to consider
- How does a market order alter the order book?
  - ↳ Large sell or buy orders leave a gap in the order book
  - ↳ Order book dynamics = How bids/asks change with gap in order book
  - ↳ Change in bid/ask spread is Price Impact
  - ↳ Temporary and Permanent Price Impacts
- Question: How do you break up sell orders?
  - ↳ MDP
  - ↳ Too slow = Uncertainty  $\rightarrow$  Poor utility
  - ↳ Too fast = low average price  $\rightarrow$  Poor utility
- Problem Annotation:
  - $T$  = time step indexed by  $t = 0, 1, \dots, T$
  - $P_t$  = Bid price at start of time step  $t$
  - $N_t$  = number of shares sold at time step  $t$
- See MDP Formulation on slide 11.
- Similar Linear Impact Model w/ No Risk Aversion

$$N_t, N_t, P_t \in \mathbb{R}$$

$$\text{Price Dynamics: } P_{t+1} = P_t - \alpha N_t + \epsilon_t, \text{ where } \alpha \in \mathbb{R}$$

$\epsilon_t$  is i.i.d. ...

$$V_t^{\pi}((P_t, R_t)) = E_{\pi} \left[ \sum_{i=t}^T N_i (P_i - \beta N_i) \mid (P_t, R_t) \right]$$

- Boundary Condition: On last day, sell all remaining shares
  - ↳ Kind of like asset allocation problem
  - ... see slides 12-14
  - ↳ Take partial derivative w.r.t.  $\alpha$  for 2 cases:  $\alpha \geq 2B$  or not
  - ↳  $\alpha \geq 2B \Rightarrow$  Sell everything on one day (work out the math)
    - ↳ Trivial case
  - ↳  $26 < \alpha < 28, \quad N_t^* = \frac{R_t}{T-t}$ 
    - Remaining # Shares
    - # of days remaining
- Price impact and market movement operate additively and don't interact in this basic example.
- Optimal Expected Total Sales Proceeds =  $N P_0 + \underbrace{\frac{N^2}{2} (\alpha + \frac{2B-\alpha}{T})}_{\text{"Slippage" = Implementation Shortfall}}$
- ↳ There is always implementation shortfall unless price snaps back completely
- Slide 17 gives a paper on Optimal Trade Order Execution with more complicated price impact model
  - ↳ We are asked to work out model on slide 18
  - ↳ Limitation: Did not assume a utility function. People are not interested just in maximizing revenue of sales. They want to balance expected returns and variances of returns.
- Slide 19: Almgren - Chriss breakthrough paper "Efficient Frontier"

- Real World Optimal Trade Order Execution

- Lots of time building models of how order book will change (basis of algorithmic trading)
- Incorporating various market factor in states. Models State Space
- Next feature = How best to submit limit orders.