Efficient Quasi-Dynamic Simulations of Earthquakes and Aseismic Slip Including Off-Fault Viscoelastic Deformation Using Hierarchical Matrices

Joseph Wick[1]; Valère Lambert[1]

[1]University of California, Santa Cruz

Abstract

The interplay between localized fault slip and distributed viscoelastic deformation within the crust and upper mantle has substantial implications for the loading of stress on faults, the depth-extent and recurrence of earthquake ruptures, and the overall state of stress within the lithosphere. However, explicitly incorporating inelastic off-fault deformation into long-term numerical simulations of sequences of earthquakes and aseismic slip (SEAS) remains challenging. Analytic solutions for the stress field due to distributed deformation allow for the efficient inclusion of bulk deformation in SEAS simulation using the integral method, which involves taking the product between a matrix of Green's functions and a deformation vector (Lambert & Barbot, GRL 2016; Barbot GRL 2018). In standard boundary integral methods (BIM), the number of calculations necessary scales quadratically with the number of computational elements, limiting the feasibility of larger-scale simulations. A number of studies have examined avenues for improving the efficiency of the boundary integral method, including the use of hierarchical matrices as sparse approximations of the stress kernel. Here, we compare a hierarchical matrix SEAS simulation that includes viscoelastic deformation to a dense BIM implementation considering the same physical problem. Our ultimate goal is to facilitate the consideration of bulk viscoelastic deformation in larger-scale studies of long-term fault processes, including 3D SEAS simulations with 2D fault geometries and models of fault networks.

Simulations of Earthquake Sequences Including Off Fault Viscoelastic Effects

The boundary integral method uses elastic Green's functions (e.g. Okada, BSSA 1992) to simulate the evolution of slip and the corresponding stress interactions along a discretized fault. Lambert & Barbot (GRL, 2016) introduced a method to incorporate quasi-static stress interactions from viscoelastic deformation in discretized strain volumes. Using the integral method, the quasi-dynamic evolution of stress within each fault patch and strain volume is determined by the sum of the matrix-vector products between the rate of deformation (slip or strain rate) vector and the corresponding stress kernel.

For the 2D antiplane problem discussed in Lambert & Barbot (GRL, 2016), the stressing rate in the strain volume elements is given by:

$$\sigma_\alpha = K_\alpha(V-V^0)+\Sigma_\beta G_{\alpha\beta}(\varepsilon_\beta-\varepsilon_\beta^0)$$

Where alpha and beta take values of 12 or 13 for the corresponding stress/strain components. *K* is the stress kernel due to fault slip and *G* relates the change in stress component alpha due to strain of component beta.

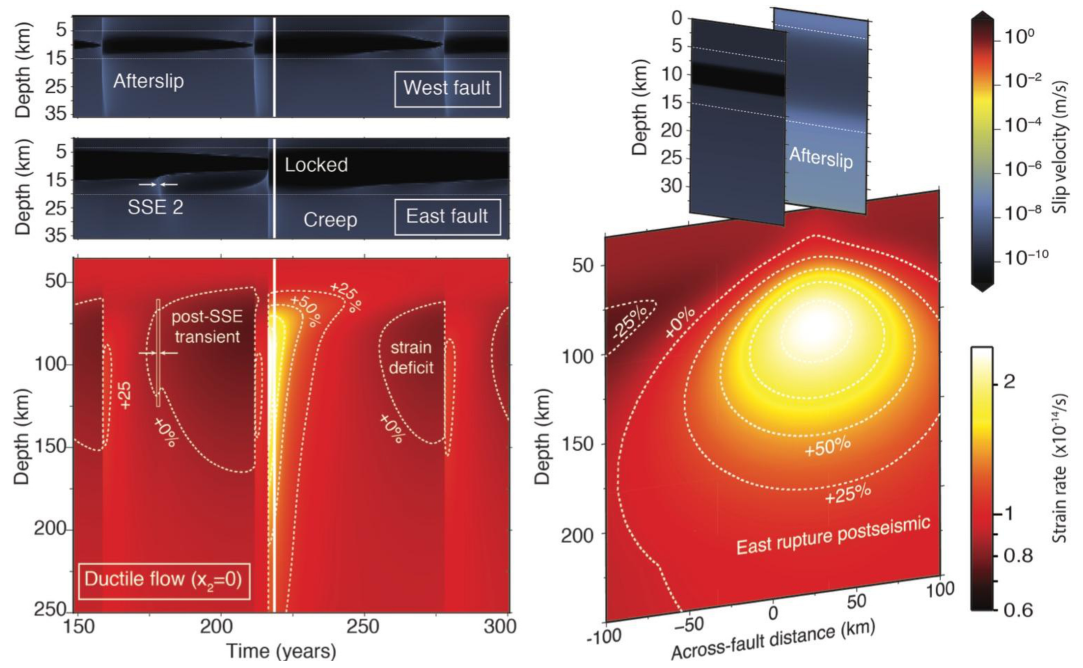

Figure 1: Example of quasi-dynamic 2D antiplane SEAS simulation including viscoelastic deformation. This simulation considers interactions between two parallel faults above a ductile region. Qualitatively, shear strain accelerates post-seismically.

Use of Hierarchical Matrices to Optimize Earthquake Sequence Simulations

SEAS simulations involve repeated calculations of the matrix-vector product

$$\tau = G\varepsilon$$

where G is the Green's function matrix, epsilon is the strain vector (or slip, for fault patches) and tau is the resultant stress. The matrix G is NxN, where N is the number of elements in discretized problem space, so the runtime of a SEAS simulation scales quadratically with the number of elements considered.

Model Schematic

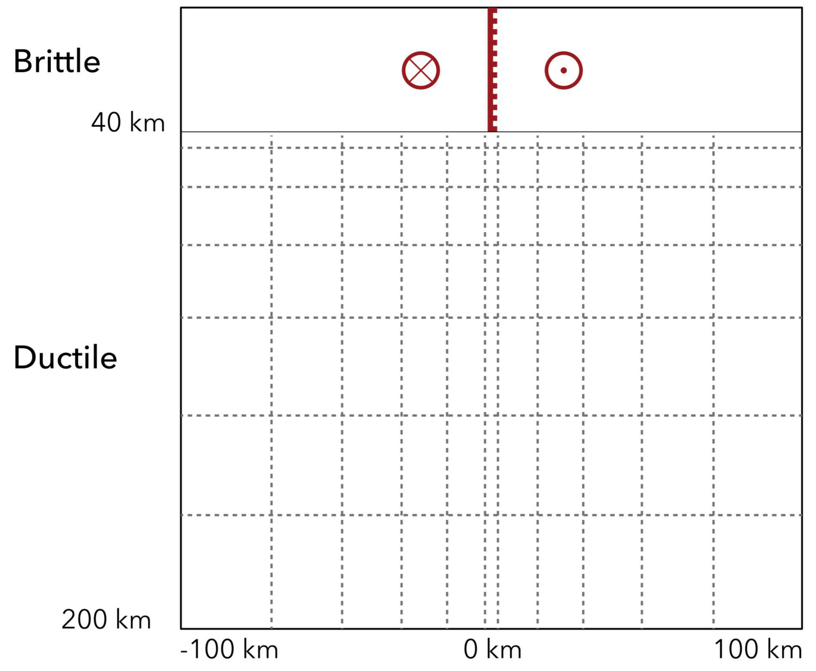

Figure 2: We consider a physical problem consisting of a single fault above a strain volume.

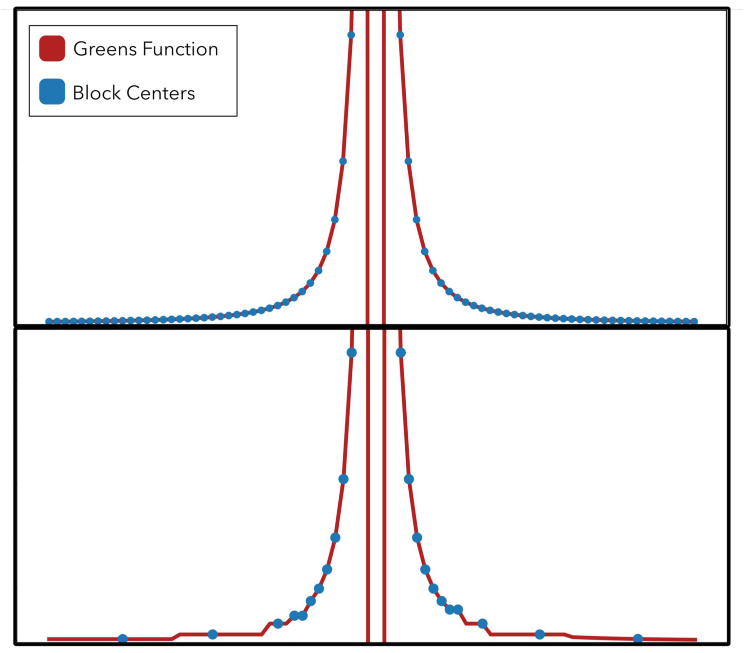

Figure 3: *Top*: A dense Green's function, where the function at each block is evaluated independently. *Bottom:* A hierarchical matrix style function, where blocks farther from the block receiving strain are grouped together, decreasing the number of calculations.

Hierarchical matrices make low rank approximations for blocks of a dense matrix, decreasing the number of elements present for MVP. Blocks farther from the diagonal require less accuracy, and are approximated with more error. In this work, we use a hierarchical matrix implementation developed by Andrew Bradley and known as *hmmvp.*

Hierarchical Matrix Error

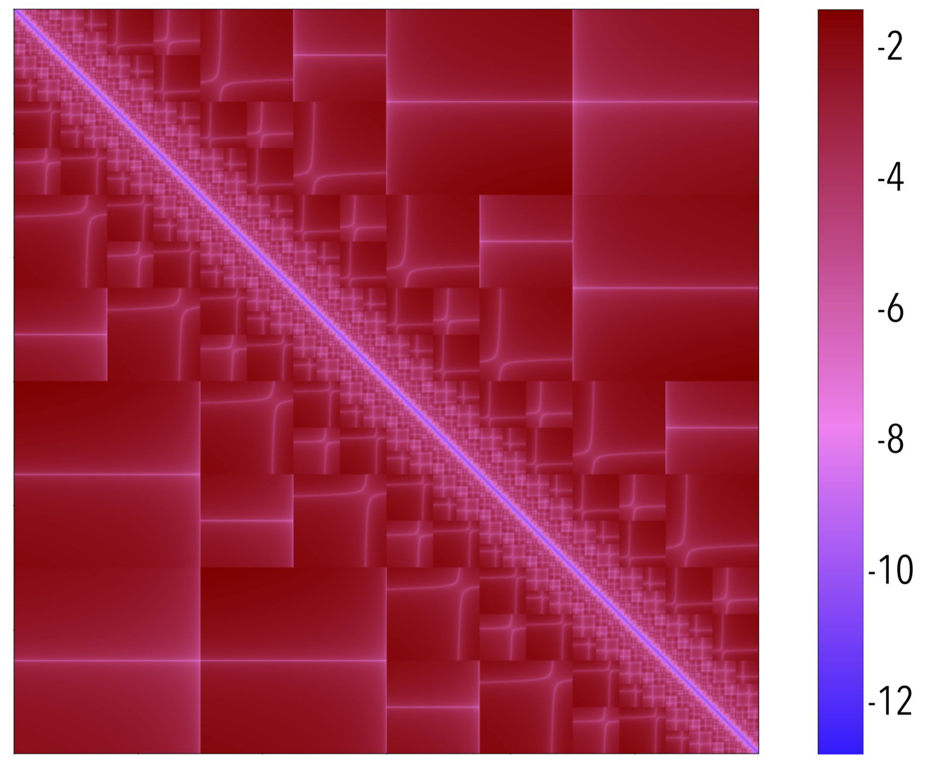

Figure 3: Log-error of a hierarchical matrix stress kernel.

Runtime of a Single MVP Operation

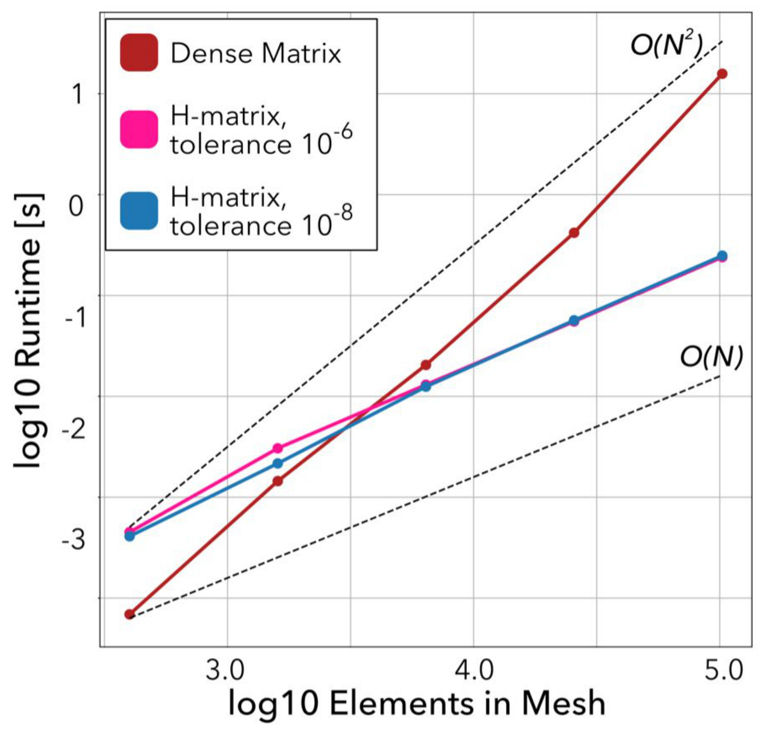

Figure 4: Hierarchical matrices produce faster results for meshes with more than a few thousand elements, and require less memory at all sizes.

Comparing Dense and Hierarchical Simulations

Slip [m] From a Single Rupture

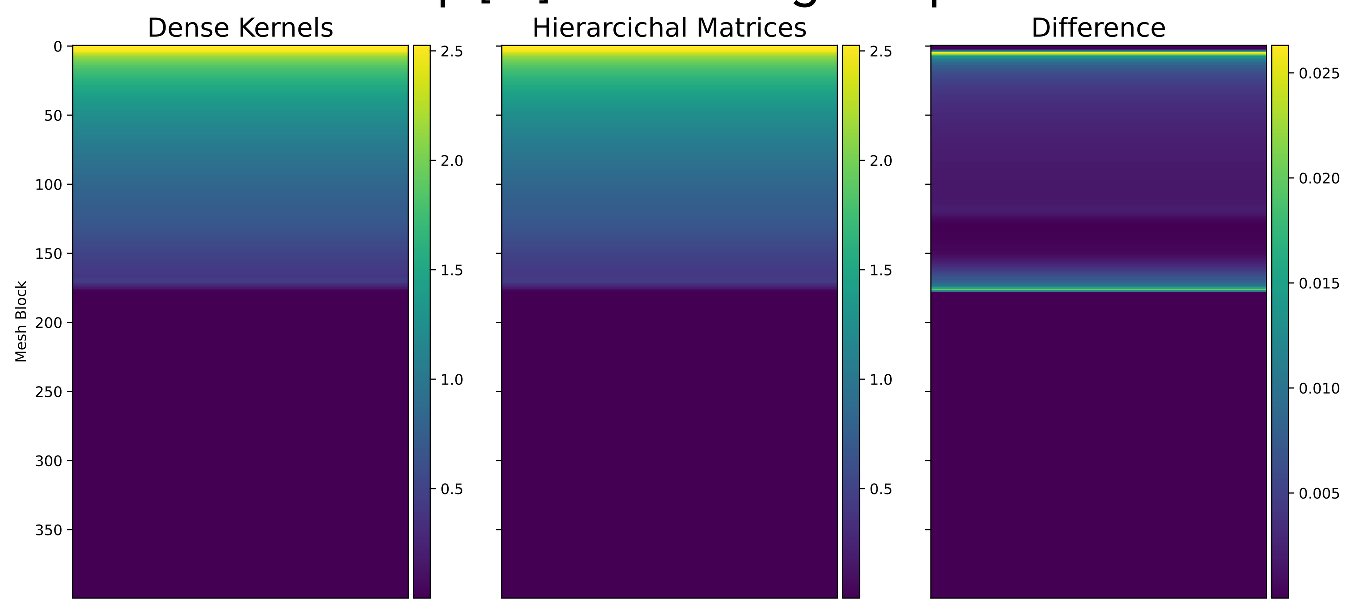

Figure 6: The resolved co-seismic slip from ruptures is comparable between simulations using dense and hierarchical matrices.

Strain Acquired Between Ruptures

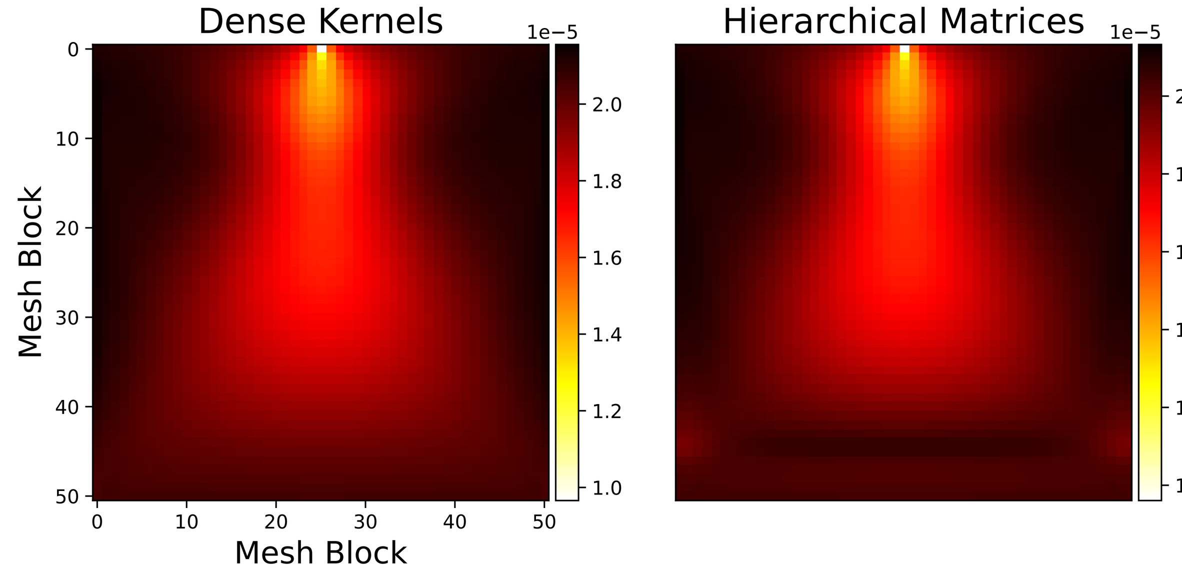

Figure 7: Distribution of interseismic distributed viscoelastic strain between two simulated earthquakes. The resolved cumulative strain is comparable for simulations using dense and hierarchical matrices, with slightly higher error in resolved strain at greater distances from the fault for the simulation using h-matrices.

Fault Error

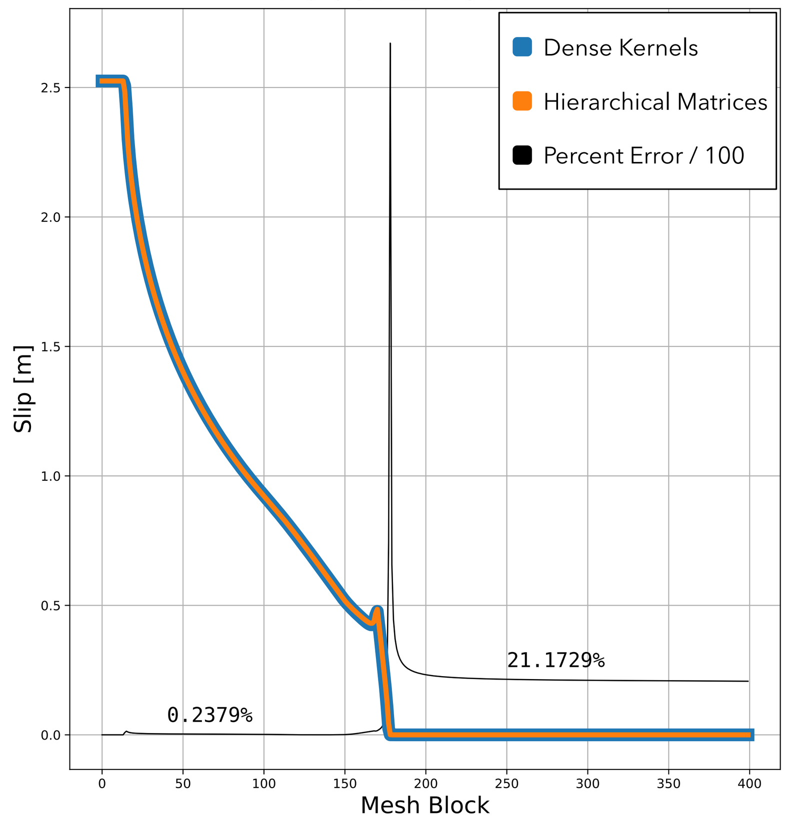

Figure 8: Spatial distribution of coseismic slip and relative error between simulations using dense and hierarchical matrices. The relative error in resolved slip is generally low (<1%), with slightly higher error in regions with sharp gradients to negligible slip.

Conclusion

1. Simulating interactions between brittle and ductile layers in the Earth's crust can provide important insight into the loading of faults, the recurrence and depth-extent of earthquake ruptures, and the general state of stress within the lithosphere.
2. Hierarchical matrices provide an augmentation to current methods that dramatically speed up SEAS simulations and allow for the consideration of large scale problems, without sacrificing accuracy of fault mechanics.
3. Future work can take the form of applying hierarchical matrices to larger scale 2D problems and to 3D problems of faults and viscoelastic effects.

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