homework is 4a) q5,6 4b) q1cd, 2

Sorry I did it like this and not on paper like you wanted, I thought it would be easier for you to read and easier for me to revise from.

binomial expansion: $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + ...$

Q5)

$f(x) = \sqrt{1+3x}, \frac{-1}{3} < x < \frac{1}{3}$

a)

$(1+3x)^{\frac{1}{2}}$

$1 + \frac{1}{2}3x + \frac{\frac{1}{2}(\frac{1}{2}-1)(3x)^2}{2!} \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(3x)^3}{3!} + ...$

$1 + \frac{3}{2}x - \frac{9}{8}x^2 + \frac{27}{16}x^3 + ...$

b) show that, when $x = \frac{1}{100}$, the exact value of $f(x)$ is $\frac{\sqrt{103}}{10}$

when $x = \frac{1}{100}, 1 + 3x = \frac{103}{100}$

$(1+3x)^{\frac{1}{2}} = \frac{103}{100}$

$1 + 3x = \sqrt{\frac{103}{100}} = \frac{\sqrt{103}}{\sqrt{100}} = \frac{\sqrt{103}}{10}$

c) Find the percentage error made in using the series expansion in part a to estimate the value of $f(0.01)$. Give your answer to 2sf

$1 + \frac{3}{2}(0.01) - \frac{9}{8}(0.01)^2 + \frac{27}{16}(0.01)^3$

$= 1 + \frac{3}{200} - \frac{9}{80000} + \frac{27}{16}(\frac{1}{1000000})$

$= 1.014889157$

$\frac{1.014889157 - (\frac{\sqrt{103}}{10})}{1.014889157} = 3.1 \times 10^6\%$

Q6) In the expansion of $(1+ax)^{-1/2}$ the coefficient of x^2 is 24

a) Find the possible values of a

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + ...$

$(1+ax)^{\frac{-1}{2}} = 1 - \frac{1}{2}ax + \frac{\frac{-1}{2}(\frac{-1}{2}-1)(ax)^2}{2!} + \frac{\frac{-1}{2}(\frac{-1}{2}-1)(\frac{-1}{2}-2)(ax)^3}{3!} + ...$

$= 1 - \frac{1}{2}ax + \frac{\frac{3}{4}a^2x^2}{2!} + \frac{\frac{-15}{8}a^3x^3}{3!} + ...$

$\frac{\frac{3}{4}a^2}{2!} = 24$

$\frac{3}{4}a^2 = 48$

$a^2 = 64$

$a = 8, or, a = -8$

b)

$\frac{\frac{-15}{8}(8)^3x^3}{3!} = 160x^3$

$\frac{\frac{-15}{8}(-8)^3x^3}{3!} = -160x^3$

Therefore, the coefficient is + or - 160.

4b

Q1) Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

c)

$\frac{1}{(4-x)^2} = (4-x)^{-2} = 4^{-2}[(1-\frac{x}{4})^{-2}] = \frac{1}{16}(1-\frac{x}{4})^{-2}$

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + ...$

$\frac{1}{16}[1 - 2(\frac{x}{4}) + \frac{-2(-3)(\frac{x}{4})^2}{2!} + \frac{-2(-3)(-4)(\frac{x}{4})^3}{3!}]$

$\frac{1}{16} - \frac{1}{32}x + \frac{1}{256}x^2 + \frac{1}{256}x^3$

$|x| < 4$

d)

$\sqrt{9+x} = (9+x)^{\frac{1}{2}} = [9(1+\frac{x}{9})]^{\frac{1}{2}} = 9^{\frac{1}{2}}(1+\frac{x}{9})^{\frac{1}{2}}$

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + ...$

$3[1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{x}{9})^2}{2!} + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{x}{9})^3}{3!}]$

$3 + \frac{x}{6} - \frac{x^2}{216} + \frac{x^3}{3888}$

$|x| < 9$

Q2) $f(x) = (5+4x)^{-2}, |x| < \frac{5}{4}$ Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

$(5+4x)^{-2} = 5^{-2}[(1+\frac{4}{5}x)^{-2}] = \frac{1}{25}(1+\frac{4}{5}x)^{-2}$

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + ...$

$\frac{1}{25}[1 - 2(\frac{4}{5}x) + \frac{-2(-3)(\frac{4}{5}x)^2}{2!} + \frac{-2(-3)(-4)(\frac{4}{5}x)^3}{3!}]$

$\frac{1}{25} - \frac{8}{125}x + \frac{48}{625}x^2 - \frac{256}{3125}x^3$