

## homework is 4a) q5,6 4b) q1cd, 2

Sorry I did it like this and not on paper like you wanted, I thought it would be easier for you to read and easier for me to revise from.

binomial expansion:  $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$

Q5)

$$f(x) = \sqrt{1+3x}, \quad \frac{-1}{3} < x < \frac{1}{3}$$

a)

$$(1+3x)^{\frac{1}{2}}$$

$$1 + \frac{1}{2}3x + \frac{\frac{1}{2}(\frac{1}{2}-1)(3x)^2}{2!} + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(3x)^3}{3!} + \dots$$

$$1 + \frac{3}{2}x - \frac{9}{8}x^2 + \frac{27}{16}x^3 + \dots$$

✓

b) show that, when  $x = \frac{1}{100}$ , the exact value of  $f(x)$  is  $\frac{\sqrt{103}}{10}$

$$\text{when } x = \frac{1}{100}, 1+3x = \frac{103}{100}$$

$$(1+3x)^{\frac{1}{2}} = \frac{103}{100}$$

$$1+3x = \sqrt{\frac{103}{100}} = \frac{\sqrt{103}}{\sqrt{100}} = \frac{\sqrt{103}}{10}$$

✓

c) Find the percentage error made in using the series expansion in part a to estimate the value of  $f(0.01)$ . Give your answer to 2sf

$$1 + \frac{3}{2}(0.01) - \frac{9}{8}(0.01)^2 + \frac{27}{16}(0.01)^3$$

$$= 1 + \frac{3}{200} - \frac{9}{80000} + \frac{27}{16} \left( \frac{1}{1000000} \right)$$

$$= 1.014889157$$

$$\frac{1.014889157 - \left( \frac{\sqrt{103}}{10} \right)}{1.014889157} = 3.1 \times 10^6\%$$

✓

Q6) In the expansion of  $(1+ax)^{-1/2}$  the coefficient of  $x^2$  is 24

a) Find the possible values of a

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$$

$$(1+ax)^{-\frac{1}{2}} = 1 - \frac{1}{2}ax + \frac{\frac{-1}{2}(\frac{-1}{2}-1)(ax)^2}{2!} + \frac{\frac{-1}{2}(\frac{-1}{2}-1)(\frac{-1}{2}-2)(ax)^3}{3!} + \dots$$

$$= 1 - \frac{1}{2}ax + \frac{\frac{3}{4}a^2x^2}{2!} + \frac{\frac{-15}{8}a^3x^3}{3!} + \dots$$

$$\frac{\frac{3}{4}a^2}{2!} = 24$$

$$\frac{3}{4}a^2 = 48$$

$$a^2 = 64$$

$$a = 8, \text{ or } a = -8$$

✓

b)

$$\frac{\frac{-15}{8}(8)^3x^3}{3!} = 160x^3$$

$$\frac{\frac{-15}{8}(-8)^3x^3}{3!} = -160x^3$$

Therefore, the coefficient is + or - 160.

✓

**4b**

Q1) Find the binomial expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . Give each coefficient as a simplified fraction.

c)

$$\frac{1}{(4-x)^2} = (4-x)^{-2} = 4^{-2}[(1 - \frac{x}{4})^{-2}] = \frac{1}{16}(1 - \frac{x}{4})^{-2}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$$

$$\frac{1}{16}[1 - 2(\frac{x}{4}) + \frac{-2(-3)(\frac{x}{4})^2}{2!} + \frac{-2(-3)(-4)(\frac{x}{4})^3}{3!}]$$

$$\frac{1}{16} - \frac{1}{32}x + \frac{1}{256}x^2 + \frac{1}{256}x^3$$

$$|x| < 4$$

✓

d)

$$\sqrt{9+x} = (9+x)^{\frac{1}{2}} = [9(1 + \frac{x}{9})]^{\frac{1}{2}} = 9^{\frac{1}{2}}(1 + \frac{x}{9})^{\frac{1}{2}}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$$

$$3[1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{x}{9})^2}{2!} + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{x}{9})^3}{3!}]$$

$$3 + \frac{x}{6} - \frac{x^2}{216} + \frac{x^3}{3888}$$

$$|x| < 9$$

✓

Q2)  $f(x) = (5 + 4x)^{-2}$ ,  $|x| < \frac{5}{4}$  Find the binomial expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . Give each coefficient as a simplified fraction.

$$(5 + 4x)^{-2} = 5^{-2}[(1 + \frac{4}{5}x)^{-2}] = \frac{1}{25}(1 + \frac{4}{5}x)^{-2}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$$

$$\frac{1}{25}[1 - 2(\frac{4}{5}x) + \frac{-2(-3)(\frac{4}{5}x)^2}{2!} + \frac{-2(-3)(-4)(\frac{4}{5}x)^3}{3!}]$$

$$\frac{1}{25} - \frac{8}{125}x + \frac{48}{625}x^2 - \frac{256}{3125}x^3$$

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