

hw q5,q6,q7 ex 4c

5) a) Express $\frac{2x^2+7x-6}{(x+5)(x-4)}$ in partial fractions

$$\frac{2x^2+7x-6}{(x+5)(x-4)} = \frac{A}{(x+5)} + \frac{B}{(x-4)}$$

$$2x^2 + 7x - 6 \equiv A(x - 4) + B(x + 5)$$

$$\text{sub } x = 4$$

$$2(-5)^2 + 7(-5) - 6 = -9A$$

$$A = -1$$

$$\text{sub } x = -5$$

$$2(4)^2 + 7(4) - 6 \equiv 9B$$

$$B = 6$$

(get 2 from poly long div)

$$\text{therefore } \frac{2x^2+7x-6}{(x+5)(x-4)} = 2 + \frac{-1}{(x+5)} + \frac{6}{(x-4)}$$

✓

b) Hence, or otherwise, expand $\frac{2x^2+7x-6}{(x+5)(x-4)}$ in ascending powers of x as far as the term in x^2

$$\frac{-1}{(x+5)} = -1(x+5)^{-1} = -1(5+x)^{-1} = -5(1+x)^{-1}$$

$$\frac{6}{(x-4)} = 6(x-4)^{-1} = 6(-4+x)^{-1} = -24(1-x)^{-1}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!}$$

$$-5[(1+x)^{-1} = 1 - x + x^2] = -5 + 5x - 5x^2$$

$$-24[(1-x)^{-1} = 1 + x + x^2] = -24 - 24x - 24x^2$$

$$\text{so } \frac{2x^2+7x-6}{(x+5)(x-4)} = -5 + 5x - 5x^2 - 24 - 24x - 24x^2$$

$$\frac{2x^2+7x-6}{(x+5)(x-4)} = -29 - 19x - 29x^2$$

X

c)

$$|x| < 4$$

✓

$$6) \frac{3x^2+4x-5}{(x-3)(x-2)} = A + \frac{B}{x+3} + \frac{C}{x-2}$$

a) Find the values of the constants A, B and C

$$\frac{3x^2+4x-5}{(x-3)(x-2)} = 3 + \frac{19x-23}{(x+3)(x-2)}$$

$$\frac{3x^2+4x-5}{(x-3)(x-2)} = 3 + \frac{B}{x+3} + \frac{C}{x-2}$$

$$3x^2 + 4x - 5 = 3(x-2)(x+3) + B(x-2) + C(x+3)$$

$$\text{sub } x = 2$$

$$3(2)^2 + 4(2) - 5 = 3(0)(5) + 0B + 5C$$

$$C = 3$$

$$\text{sub } x = -3$$

$$3(-3)^2 + 4(-3) - 5 = 3(-5)(0) - 5B + 0C$$

$$B = -2$$

$$\text{Therefore } A = 3, B = -2, C = 3$$

✓

B) Hence or otherwise expand $\frac{3x^2+4x-5}{(x-3)(x-2)}$ in ascending powers of x, as far as the term x^2 .

$$\frac{3x^2+4x-5}{(x-3)(x-2)} = 3 + \frac{-2}{x+3} + \frac{3}{x-2}$$

$$\frac{3x^2+4x-5}{(x-3)(x-2)} = 3 + -2(x+3)^{-1} + 3(x-2)^{-1}$$

$$-2(x+3)^{-1} = -6(1+x)^{-1}$$

$$-6[(1+x)^{-1} = 1+x+x^2]$$

$$3(x-2)^{-1} = -6(1+x)^{-1}$$

$$-6[(1+x)^{-1} = 1+x+x^2]$$

$$-6[(1+x)^{-1} = 1+x+x^2] - 6[(1+x)^{-1} = 1+x+x^2] = -12 - 12x - 12x^2$$

X

$$7) \frac{2x^2+5x+11}{(2x-1)^2(x+1)}, |x| < \frac{1}{2}$$

f(x) can be expressed in the form

$$f(x) = \frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{x+1}$$

a) Find the values of A, B and C

$$\frac{2x^2+5x+11}{(2x-1)^2(x+1)} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{x+1}$$

$$2x^2 + 5x + 11 \equiv A(2x-1)(x+1) + B(x+1) + C(2x-1)^2$$

$$\text{sub } x = -1$$

$$2(-1)^2 + 5(-1) + 11 = 9C$$

$$8 = 9C$$

$$C = \frac{8}{9}$$

$$\text{sub } x = \frac{1}{2}$$

$$2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + 11 = 1.5B$$

$$B = \frac{28}{3}$$

To get A sub both b and c into the main formulae and solve

$$A = \frac{-7}{9}$$

✓

B) Hence or otherwise find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 Simplifying each term

$$\frac{2x^2+5x+11}{(2x-1)^2(x+1)} \equiv \frac{\frac{-7}{9}}{2x-1} + \frac{\frac{28}{3}}{(2x-1)^2} + \frac{\frac{8}{9}}{x+1}$$

$$\frac{-7}{9}(2x-1)^{-1} + \frac{28}{3}(2x-1)^{-2} + \frac{8}{9}(x+1)^{-1}$$

$$\frac{-7}{9}(2x-1)^{-1} = \frac{-7}{9}(-1+2x)^{-1} = \frac{7}{9}(1-2x)^{-1}$$

$$= \frac{7}{9}[1+2x+4x^2]$$

$$\frac{28}{3}(2x-1)^{-2} = \frac{28}{3}(-1+2x)^{-2} = \frac{-28}{3}(1-2x)^{-2}$$

$$= \frac{-28}{3}[1+4x+12x^2]$$

$$\frac{8}{9}(x+1)^{-1} = \frac{8}{9}(1+x)^{-1}$$

$$= \frac{8}{9}[1+x+x^2]$$

$$\text{therefore } \frac{2x^2+5x+11}{(2x-1)^2(x+1)} \equiv \frac{7}{9}[1+2x+4x^2] + \frac{-28}{3}[1+4x+12x^2] + \frac{8}{9}[1+x+x^2]$$

$$\frac{2x^2+5x+11}{(2x-1)^2(x+1)} = \frac{-23}{3} + \frac{-314}{9}x - 108x^2$$

X

C) Find the percentage error made in using the series expansion in part B to estimate the value of $f(0,05)$. Give your answer to 2sf

I got the wrong answer for the prior part so it I cant get it right

X