UNITED STATES MILITARY ACADEMY

FINAL PROJECT: PROBLEM #4

MA461: GRAPH THEORY & NETWORKS

SECTION B1

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SIGNATURES: _____

Army/Navy Weekend: Finding a "Best" Route

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Abstract

The purpose of this report is to provide cadets from the United States Military Academy with the "best" route home from the Army Navy Game in Philadelphia to Thayer Gate at West Point if they depart the city of brotherly love following the game as late as possible. The solution to this problem is found using mathematical techniques involving graph theory and networks.

The Problem

The Army Navy Game is one of the most storied rivalries in the history of college sports. For the past few years the game has been played at Lincoln Financial Field in Philadelphia, Pennsylvania. Every cadet is required to attend the game. Therefore, cadets want to know what is the "best" route home from the Army Navy Game in Philadelphia to Thayer Gate at West Point if they depart the city of brotherly love following the game as late as possible. The travel cost for each team member must be at most $45.00, which means that a two person team can spend no more than $90.00 in their quest to return to West Point. Cadets can choose to travel by rental car or travel by public transportation. Every cadet must be back at West Point no later than Sunday, December 15th, at 1800 ET.

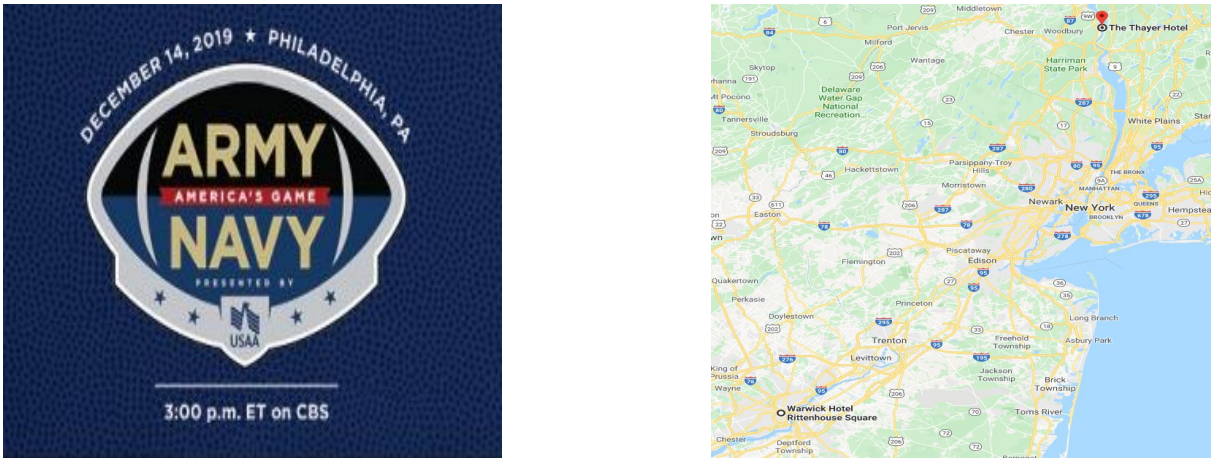


Figure 1: Army Navy Game 2019

Parameters and Assumptions

The problem may appear as if it has a fairly simple solution, but in fact it is one that involves a pair of decisions, multiple constraints, and a large amount of time variance and time considerations that could impact the final solution. The time spent analyzing the problem now will ultimately gain more time for us to enjoy the freedoms of being away from West Point and the hospitality found in the city of brotherly love.

Decisions

As two cadets without a car, the journey back to West Point ultimately requires us to make two key decisions. The first decision is how we will travel. We can choose to travel by rental car or travel by public transportation. Each of these options has its advantages and disadvantages, which will be discussed later, but once we make this initial decision, we then must decide on the "best" route to travel in order to make it back in time for recall formation at 1800 ET.

Constraints

There are two major constraints that limit travel options. The first is the time constraint. Every Sunday at West Point there is a corps wide formation to help gain accountability of every cadet at the academy. The goal for cadets is to be ready for this formation with as little time remaining without being considered an FTR (Failure to Report). If this can be done successfully, then it will ensure that time was maximized in Philadelphia during Army/Navy weekend and avoid the horrible punishment of walking hours in North Area. The time constraint is to be back no later than 1800 ET. The second constraint for this problem is the means available to travel from Philadelphia to West Point. Combined, the two of us are restricted to spending a total of $90.00 on the trip back to West Point. This limits some of the methods of public transportation that could be used, such as flying or even certain train routes.

Assumptions

Before beginning the planning phase of the trip, there are a few guidelines and assumptions that must be established:

1. Together, the two cadets will be staying at the Warwick Hotel in downtown Philadelphia. The time of departure begins at check out of the hotel room and ends as soon as they are ready for recall formation.
2. The hotel allows late checkout. The cadets may checkout no later than 1500.
3. Before recall, there needs to be some time to change from civilian clothes into uniform, so cadets should really arrive back on campus no later than 1745.
4. If there is a faster, more efficient route to get on campus other than through Thayer Gate, this will be authorized.
5. Public transportation is being defined as any means that can be used by the public to get from one place to another. This includes trains, buses, ferrys, and even more capacity constrained options such as Uber and Lyft.
6. Since it is a short weekend, luggage is light and walking anything less than two miles is not a problem.

Description of the Data

The decisions to be made for this problem can only be done properly by thoroughly researching the many transportation options back to West Point. Using online resources such as Google Maps, the Uber and Lyft apps, Enterprise.com, etc., each variable may be carefully considered in helping find the "best" route back.

Public Transportation

Data Sources Used

For public transportation, all relevant information for each variable to be considered is not found in the same place and requires multiple resources. To find feasible routes that may be used in getting back to West Point, Rome2rio.com is a source that provides multiple route options, including routes with combinations of more than one form of public transportation. This site links to other websites such as Greyhound.com, New.mta.info, and Web.coachusa.com to find all relevant scheduling, pricing, and instructional information for the buses and trains. Lastly, the Uber and Lyft apps may be used to find the cost of traveling short distances that are a little too far for walking. From each of these resources, the pertinent data for the variables that must be considered during this trip can be found.

Impacting Variables

Several variables should be examined when it comes to using public transportation including schedules, costs, potential delays, etc. For each form of public transportation these variables should be considered.

The simplest form of transportation that may be used during this trip is the taxi-esque service provided through Uber and Lyft. This service is very useful for traveling distances between 2-10 miles because it is cheap, reliable, and allows for instant transportation to any location. When using this service, especially in a city like Philadelphia, traffic can significantly impact arrival times.

Looking at trains, this should be considered the most efficient and least risky form of transportation. As long as the trains are on time, the travel departure and arrival times provided by the stations are fairly precise. Their schedules are fairly consistent by traveling every hour to every two hours depending on the day. There is no opportunity for traffic on the railways, but there always remains the risk of a breakdown while traveling in mechanical machinery.

For buses, there is considerably more variation when it comes to travel times and scheduling. Some buses that travel mainly in the city make frequent pick-ups at their assigned stops. For other buses that travel longer distances, the schedules are more sporadic. The convenience of a consistent travel schedule is not comparable to that of the trains. In terms of traveling, large buses are slower moving and therefore become more susceptible to traffic along main highways and roads. However, one of the benefits of taking the bus is that prices tend to be fairly low in comparison to some train costs.

For both trains and buses, the user is not given the same schedule freedom as someone riding in a car. The train and bus run on tight schedules and will leave if you are not on time. Given this constraint, time considerations become of paramount importance.

Time Considerations

Traveling and managing time can be difficult, especially when using public transportation because these publicly available means do not necessarily provide a single efficient route to the desired location. These routes may not be direct; they may require stops along the way to pick more people up, a change in drivers, or even a change in bus or train to continue along the route. For example, there is not a train that travels directly from Philadelphia to West Point, so this forces travelers to switch from one form to the next in order to reach their final destination. This is different from using a personal operating vehicle such as a car, as this enables one to drive directly to the location that is desired.

With a car there is less variation when it comes to time, but with public transportation there are many more things to consider. If traveling a route that we have never traveled before, we must factor in some buffer time in case we cannot find a specific location or stop and need to ask around. It should be understood that between each change over between forms of transportation, there should be between 30-45 minutes of buffer time to allow for a stress free trip and to reduce the risk of missing a bus or a train. Traveling on a Sunday should not produce heavy traffic, but traffic is a dynamic concept that should also be built into this buffer time. The distance from West Point to Philadelphia is not terribly far, so traffic should not greatly change the expected travel time. As travelers, we can not expect the transitions to be flawless.

Even for a service like Uber or Lyft, both which pick you up fairly quickly and at a location convenient for the traveler, there still needs to be a time consideration for placing the order for a driver, waiting for the driver to come to the pickup site, and the time it takes to drive to the destination with traffic.

Rental Car

Data Sources Used

For travel by rental car, most relevant information concerning each impacting variable can be found through the two most popular rental car services in the area–Enterprise and Hertz. The websites for these two companies provide accurate price estimates for any car rental. Google maps is another important information source that provides directions from one location to another along with mileage and travel time estimates. Lastly, the Uber and Lyft apps provide prices for traveling short distances, such as to and from the rental car site prior to picking up the car and after dropping it off. From each of these resources, the pertinent data for the variables that must be considered during this trip can be found.

Impacting Variables

Traveling by rental car requires consideration of several variables such as rental prices, traffic patterns, the availability of tax-services, etc. All these variables will be accounted for in the following paragraphs.

Once again, the simplest form of transportation that may be used during this trip is the taxi-esque service provided through Uber and Lyft. This service provides a cheap, reliable, and efficient way to get to the rental car location to pick up the car and return back to West Point after dropping off the car. Finding a driver using these apps typically takes anywhere from 5-15 minutes, yet after linking up with the driver only traffic can impact the arrival time.

In terms of selecting a rental car, there are two main companies in the area that provide this service–Enterprise and Hertz. Both companies maintain numerous rental locations throughout New York and Pennsylvania; therefore, there are a few different options as to where we may pick up and drop off the rental car. The two main fees that we will encounter in renting from both companies is a drop off fee and a young driver fee. The drop off fee is due to the fact that we must drop off the rental car at a different location than we picked it up at. The young driver fee is due to the fact that we both are young and relatively inexperienced drivers.

While on the road driving the rental car, the main variables to consider are toll prices and gas prices. The trip from Philadelphia to West Point requires driving on several toll roads, yet both Enterprise and Hertz offer toll passes for an additional fee. This fee costs less than paying for each individual toll out of pocket. In terms of gas, the trip should not require more than one full tank of gas. Gas stations are conveniently located right next to each drop off site; therefore, it will be easy to fill up the rental car's tank prior to returning it.

Time Considerations

Compared to using public transportation, traveling by rental car can be much more efficient in terms of time since there are less required stops during the trip. Once one arrives at the rental car location, the wait to receive the rental car should be no longer than 10-15 minutes. Then it is up to the driver to select a route to the drop off site. Google Maps provides drivers with directions for the fastest route possible based on up to the minute traffic updates. Traveling on a Sunday afternoon should not produce very heavy traffic, yet Google Maps will still factor any expected delays into its estimated travel time. After reaching the drop off site, the only things left to do are fill up on gas and turn in the car, which should take no longer than 10-15 minutes.

The only other time consideration when traveling by rental car is the trip to the rental location to pick up the car and the trip from the drop off site to West Point. The most efficient way to travel these relatively short distances is through the Uber and Lyft apps. However, even for Uber and Lyft, there needs to be a

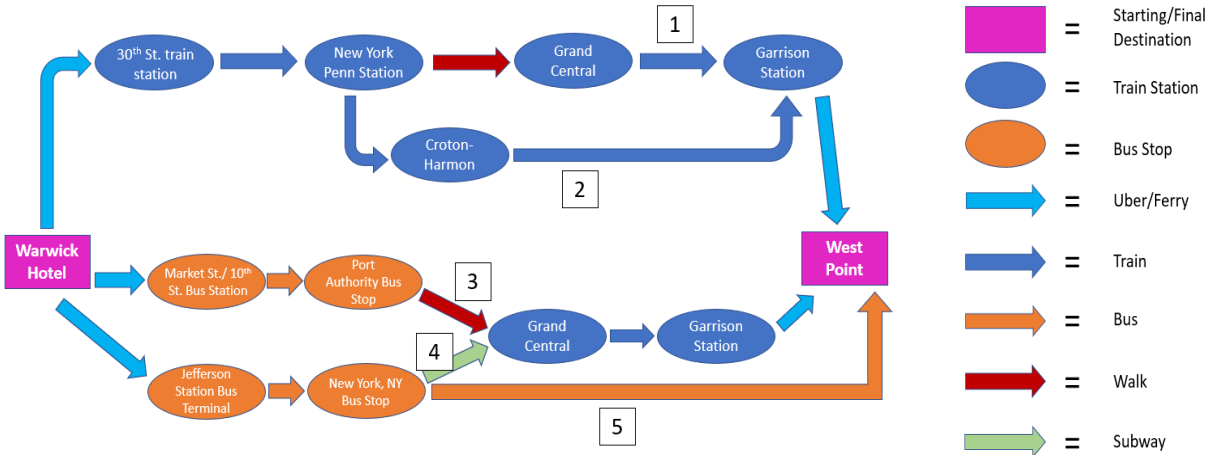
time consideration for placing the order for a driver, waiting for the diver to arrive at the pickup site, and the time it takes the driver to reach the destination with local traffic.

Methods

This problem can be solved in many different ways. In this report, the problem is broken down into two sub-problems. First, we determine the available routes back to West Point using public transportation and the available routes back to West Point using a rental car. Then, we decide which route allows us to get back to school in the least amount of time and meets our time and monetary constraints.

Public Transportation

Considering the fact that public transportation often does not permit travel directly from point A to the desired point B, it becomes important to find all prevalent transshipment locations that will help find the "best" route back to West Point. Finding these routes required use of the website Rome2rio.com. This site found every reasonable route that could be taken using any form of public transportation to include flights. Due to the high cost of flights and our cost constraint, this options was immediately eliminated from the realm of possibility. After decreasing the number of options available, only five remained feasible without considering any of the constraints. These five routes are graphically displayed as a directed network in Figure 2 and can be viewed on an actual map in Appendix A(see Figures 8, 9, and 10).



(a) Potential Routes

(b) Legend

Figure 2: A visual representation of every feasible public transportation route without constraints

Route 1 consists of taking an uber to 30^{th} St. Train Station, taking a train to New York Penn Station, walking a couple blocks to Grand Central, buying a ticket to Garrison Train Station, then ordering a driver through Uber to get back to West Point.

Route 2 begins similarly, but instead of walking to Grand Central from New York Penn Station, it requires taking another train from New York Penn Station to Croton-Harmon Train Station and then from there to Garrison and finally, taking an Uber back to West Point.

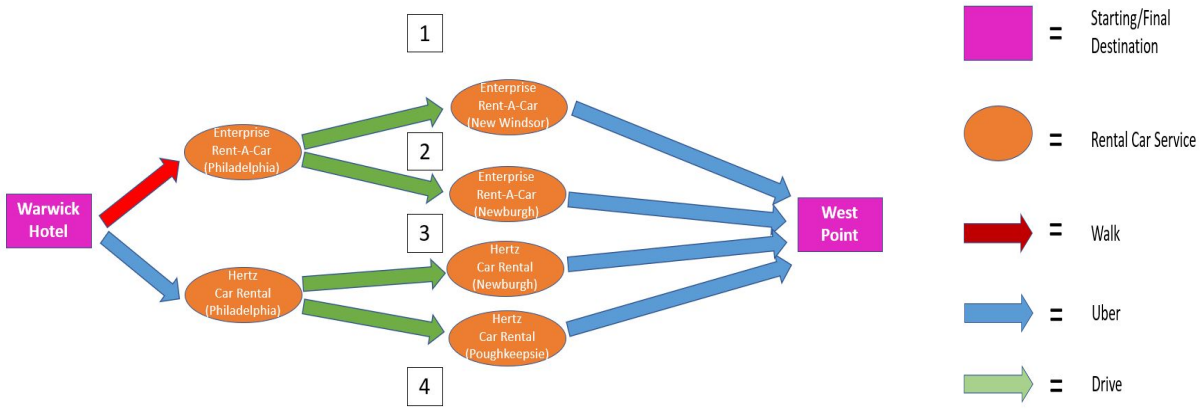
Route 3 is the first route using buses. After taking an Uber to the bus station on Market Street and 10^{th} Street, it requires taking a bus to Port Authority Bus Stop in New York. From there it then involves walking to Grand Central, taking the train to Garrison and taking an Uber back to West Point.

Route 4 begins with an Uber to Jefferson Station Bus Terminal. It then requires riding a bus to another bus stop in New York City, taking the subway to Grand Central, taking a train to Garrison, and then finally taking an Uber back to West Point.

Finally, Route 5 begins the same as Route 4 by requiring to get an Uber to Jefferson Station Bus Terminal. From here, it involves taking a bus to New York City, where another bus will drive all the way to the Visitor's Center at West Point.

Rental Car

Traveling by rental car requires two main stops in the trip from Philadelphia back to West Point–one stop to pick up the rental car and one stop to drop off the rental car. As mentioned previously, the two most popular rental car companies in the area are Enterprise and Hertz. Both companies maintain several rental locations throughout the New York and Pennsylvania area; therefore, there are a few different feasible locations to pick up and drop off the rental car. After researching these different rental car locations, four different routes from Philadelphia back to West Point appeared to be feasible without considering any cost constraints. These four routes are graphically displayed as a directed network in Figure 3.



(a) Potential Routes (b) Legend

Figure 3: A visual representation of every feasible rental car route without constraints

Route 1 involves walking to the nearest Enterprise location in Philadelphia, then driving the rental car to the drop off site in New Windsor, and finally returning to West Point by Uber.

Route 2 begins by walking to the nearest Enterprise location in Philadelphia as well, yet involves driving the rental car to the drop off site in Newburgh instead, and again finishing the trip with an Uber back to West Point.

Route 3 consists of taking an Uber to the nearest Hertz location in Philadelphia, then driving the rental car to the drop off site in Newburgh, and finally returning to West Point by Uber.

Finally, Route 4 begins by taking an Uber to the nearest Hertz location in Philadelphia as well, yet involves driving the rental car to the drop off site in Poughkeepsie instead, and again finishing the trip with an Uber back to West Point.

Results

Public Transportation

Traveling efficiently via public transportation relies on understanding the schedules of every transshipment location and building a timeline that is feasible based on these schedules. The model in Figure 2 provided a shell for each feasible route, but lacked the details necessary for building a timeline to help decide the best time to leave Philadelphia in order to make it back to West Point and be on time for recall

formation. Each transshipment location does not make its schedules by coordinating with other locations, but rather by when the next train or bus is available for use. Figure 4 provides a more complete picture of the schedules at each transshipment location and the time it takes to get from one location to the next. It also includes the cost for two adults to travel at each scheduled time.

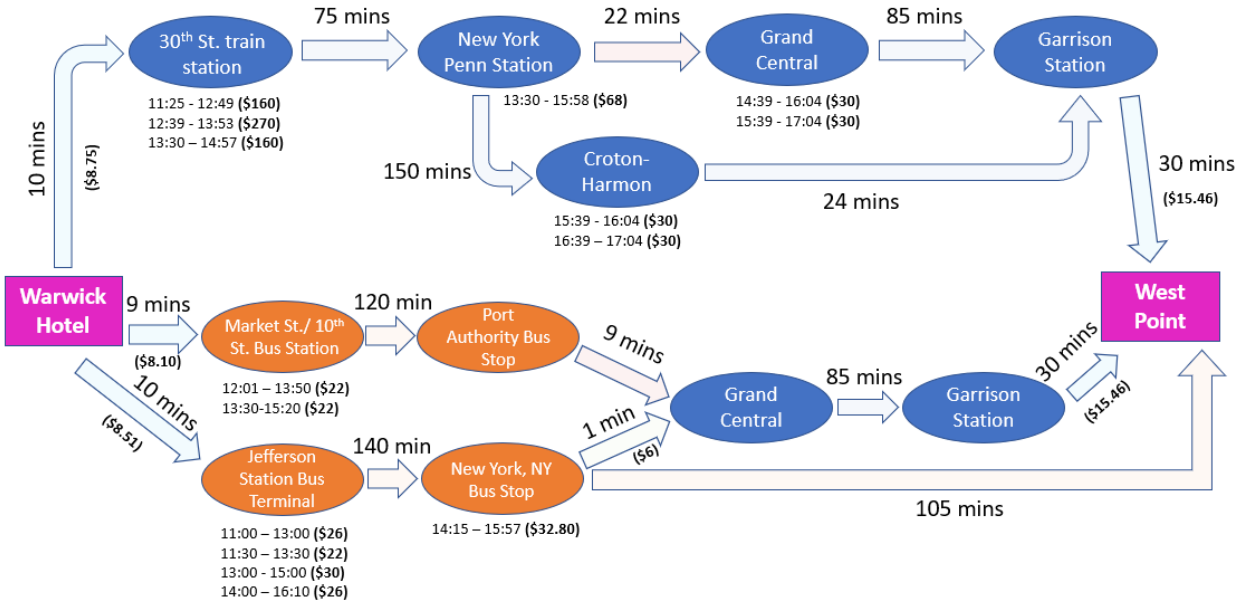


Figure 4: A visual display of each public transportation route including travel times, costs and schedules

Some of the schedules for the transshipment locations have overlapping times and some have large gaps between arrivals and the next available departure. As the analysis for the "best" route is conducted, it becomes essential to find a directed path that contains non-overlapping schedules at the transshipment locations and minimizes time spent in between stops while still allowing enough transition time. The ideal amount of wait time from arrival to next available departure is between 30-45 minutes. This ensures a stress free trip and also allows for some buffer given a delayed train or traffic.

Rental Car

Traveling efficiently using a rental car relies on comparing price estimates from different rental car companies and mapping out estimated travel times from each pickup location to each drop off site. The model in Figure 3 provides a shell for each feasible route, but lacks the details necessary to determine the best possible route from Philadelphia back to West Point in terms of time and cost. Figure 5 shown on the next page includes the approximate time and cost required for two adults to get from one location to the next. Each time is an estimate provided by Google Maps, and the probable time required to wait for an Uber driver or pickup/return the rental car is factored into each estimate as well. Each cost included in Figure 5 is the cheapest option available to either Uber to a location or rent a car from Enterprise or Hertz.

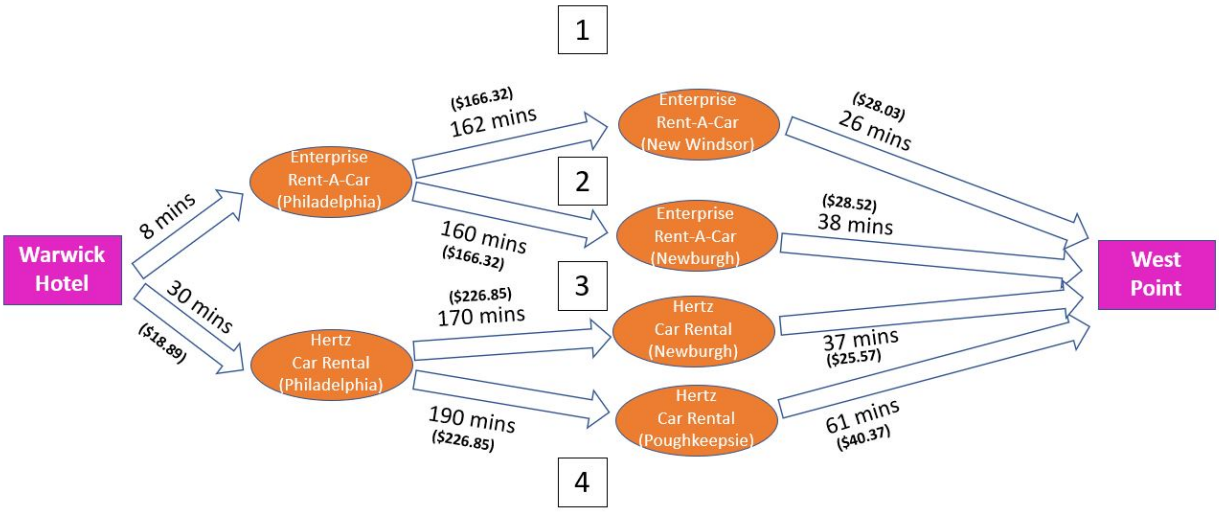


Figure 5: A visual display of each rental car route with its associated time and cost

Evaluation of Results

Public Transportation

At this point in the model, it is time to start analyzing each of the routes and determine if each one is feasible given the constraints. If so, the next step becomes determining what the latest start time would be to maximize the time spent in Philadelphia without being late to recall formation. After careful analysis of the results, Route 4 was determined to be the "best" route using public transportation. The route is graphically displayed in the model in Figure 6. Each of the routes will be analyzed in detail in the following subsections in order to verify and confirm this decision.

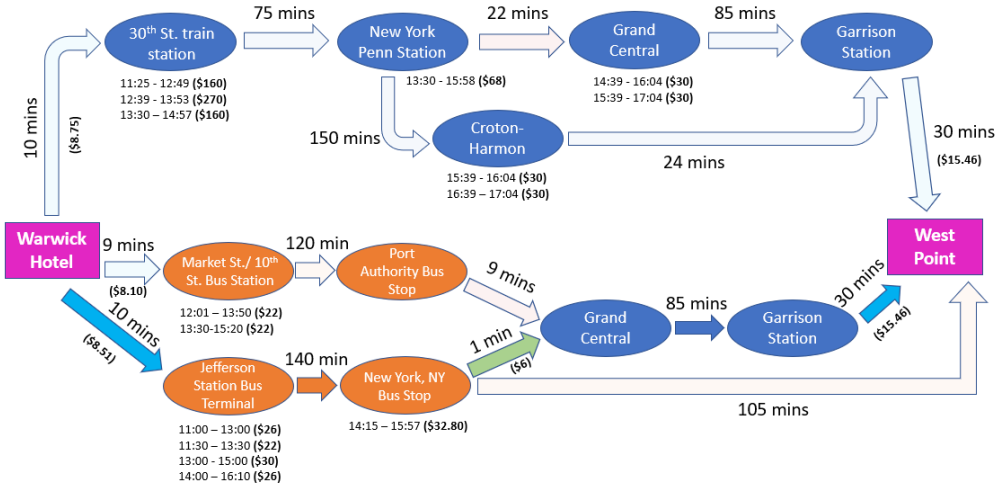


Figure 6: A visual display of the "best" public transportation route

Route 1

Route 1, the directed route taking trains only, meets the time constraint. By ordering an Uber on time from the hotel to catch the 1330 train at 30^{th} Street Train Station, this portion of the route is feasible. With a 1457 arrival time at New York Penn Station, there is approximately 40 minutes to walk a few blocks to Grand Central Station. Grand Central's next departing time is a train at 1539 with an expected arrival time of 1704 at Garrison. This gives approximately 40 minutes to order an Uber and travel back to West Point.

While the time constraint is met for Route 1, the cost constraint is not. The trip from 30^{th} Street Train Station to New York Penn Station alone is equal to or greater than $160. Given that only $90 can be spent, this route is deemed infeasible because it exceeds the amount allotted to spend on the trip.

Route 2

Much like Route 1, Route 2 also meets the time constraint. Route 2 follows a very similar path as Route 1 by requiring an Uber to be on time from the hotel to catch the 1125 train at 30^{th} Street Train Station. Upon arrival at 1249 to New York Penn Station, there is approximately 40 minutes to buy a ticket and wait for the next train toward Croton-Harmon Train Station at 1330. Once arriving at this station, there is another 40 minute wait to take the next train to Garrison, which is scheduled to arrive in Garrison at 1704. Much like Route 1, this gives approximately 40 minutes to order an Uber and travel back to West Point.

The time constraint is met, but since this route must take the same train as Route 1 from 30^{th} St. Train Station to New York Penn Station, this route may also be deemed infeasible due to the cost constraint. This leaves 3 possible routes remaining. All routes using only trains are too expensive and thus infeasible.

Route 3

Route 3 meets the time constraint as well. It requires taking an Uber to the bus station on Market Street and 10^{th} Street to catch the 1201 bus. This bus drives directly to the Port Authority bus stop and has an estimated arrival of 1350. From here there is roughly a 10 minute walk to Grand Central Station. The next train from here to Garrison leaves around 40 minutes upon arrival to Port Authority at 1439. There is a bus at this station that leaves at 1330 and has an estimated arrival at 1520 to Port Authority, but this only gives approximately 20 minutes to make a 10 minute walk from the Port Authority bus stop and buy a ticket to Garrison train station, which falls outside the 30-45 minute window to allow a stress free trip with some buffer for traffic or delays. The 1439 train is expected to arrive at 1604, and an Uber will then be taken back to West Point with plenty of time left to change and get back for recall formation at 1800.

Unlike Routes 1 and 2, Route 3 is feasible in terms of cost. In total, to get from the hotel in Philadelphia to West Point costs approximately $75.

Route 4

Route 4 is not much different from Route 3 in terms of the path being taken to get to West Point, but the scheduling produces more favorable conditions to maximize time spent in Philadelphia. The Uber from the hotel for this route goes to the Jefferson Station Bus Terminal and needs to arrive with enough time to make the bus scheduled to depart at 1:00. This bus goes from Jefferson Station to the bus stop in New York, NY, and has an estimated arrival time of 1500. This gives approximately 40 minutes to walk to the subway that runs every five minutes to Grand Central Station. Upon arrival, there will be a wait to take the next train to Garrison which is scheduled to leave at 1539 and arrive in Garrison at 1704. Much like Routes 1 and 2, this gives approximately 40 minutes to order an Uber and travel back to West Point.

This route has a few additional costs compared to Route 4, including the subway tickets and a slightly more expensive bus ticket. The total cost for this route is $89.97

Route 5

The final route, Route 5, is the most restrictive of all the routes, but still provides a feasible option. This route requires an Uber to be taken to the Jefferson Station bus terminal and arrive on time for an 1130 bus headed toward the New York, NY, bus stop. At this bus stop there is another bus that can be driven all the way to West Point's visitor center just outside the gate. However, there are only two buses on Sundays that make this trip, so scheduling freedoms are limited. The only bus that makes the trip feasible leaves at 1415 and arrives around 1600. From here, the rest of the trip is simply a walk back on campus or finding a fellow cadet to drive the rest of the distance. This route requires the purchase of two bus tickets for each member and ends up being the cheapest of the five possibilities at a total cost of $54.80. Although this route is certainly feasible, it does not maximize time spent in Philadelphia and is thus not the "best" route.

Table 1 provides a general summary of the times and costs associated with each public transportation route.

"Best" Route Analysis				
Route	Time Departure	Time Arrival	Cost	Feasible?
1	12:50	17:40	$214.21	No
2	10:50	17:40	$282.21	No
3	11:15	16:40	$75.56	Yes
4	12:15	17:40	$89.97	Yes
5	10:55	16:30	$54.80	Yes

Table 1: Public transportation route data

Rental Car

After careful analysis of the results involving travel via rental car, Route 1 was determined to be the "best" route using a rental car in terms of time. This route is highlighted in the model included as Figure 7 on the next page. Despite requiring the least amount of travel time, this route along with the other three routes do not meet the cost restraints introduced at the beginning of the report; therefore, none of the rental car routes are feasible. Each of the routes will be analyzed in greater detail in the following subsections to help verify and confirm this result.

Route 1

Route 1, which involves walking to the Enterprise location in Philadelphia, driving to the Enterprise in New Windsor, then taking an Uber back to West Point, is the most efficient route in terms of both time and cost. The total estimated time for the trip is 196 minutes and the cost is $194.35. Although this route would allow us to leave the city of brotherly love relatively late, we are not able to afford the cost associated with the route, as it is $104.35 over our budget. Therefore, although this is the best rental car route, it is not feasible given our cost constraints.

Route 2

Route 2, which is very similar to Route 1 except that it requires driving to the Enterprise in Newburgh rather than New Windsor, is the second most efficient route in terms of both time and cost. The total

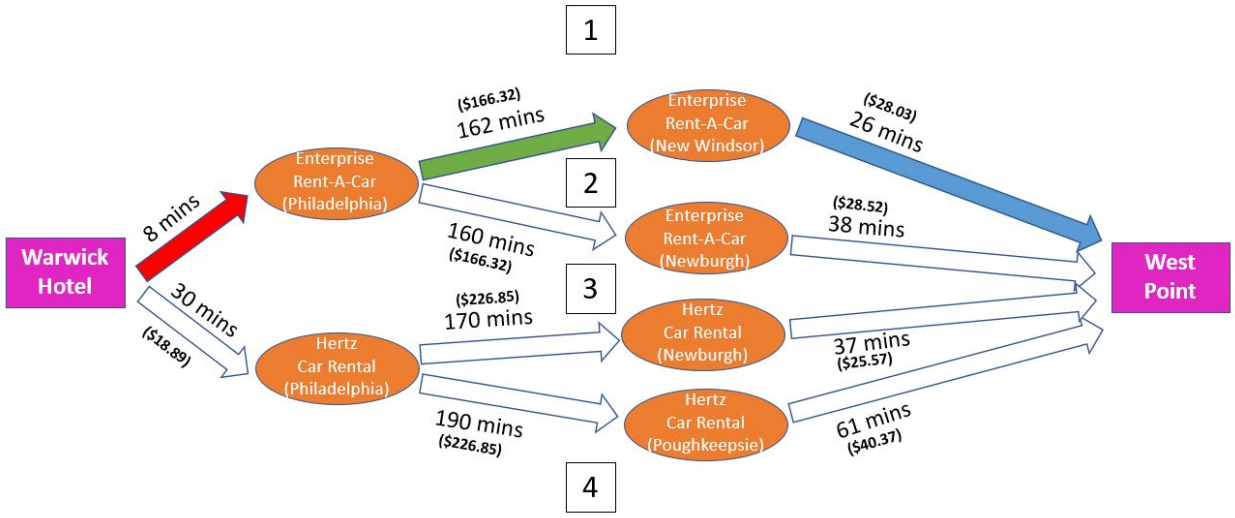


Figure 7: A visual display of the "best" rental car route in terms of time

estimated time for the trip is 206 minutes and the cost is $194.84. Although this route would also allow us to leave the city of brotherly love relatively late, we are not able to afford the cost associated with the route, as it is $104.84 over our budget. Therefore, this route is also not feasible given our cost constraints.

Route 3

Route 3 is quite different from Route 1 and Route 2. This route involves taking an Uber to the Hertz location in Philadelphia, driving to the Hertz in Newburgh, then taking another Uber to West Point. This route is not very efficient in terms of time or cost compared to Route 1 and Route 2. The total estimated time for the trip is 237 minutes and the cost is $271.61. Not only does this route require more time than the first two routes, it is also much more expensive due to more Uber trips and the rental car price offered by Hertz. Clearly, this route is not feasible given our cost constraints.

Route 4

Route 4, which is very similar to Route 3 except that it requires driving to the Hertz in Poughkeepsie rather than Newburgh, is the least efficient route in terms of both time and cost. The total estimated time for the trip is 281 minutes and the cost is $286.11. Clearly, this route is neither optimal nor feasible.

Table 2 provides a general summary of the times and costs associated with each rental car route.

"Best" Route Analysis			
Route	Travel Time (min)	Cost	Feasible?
1	196	$194.35	No
2	206	$194.84	No
3	237	$271.61	No
4	281	$286.11	No

Table 2: Rental car route data

"Best" Route Conclusion

None of the routes involving a rental car produced a feasible solution because they all fell outside the cost constraint. Three feasible routes were found using only public transportation to get from the Warwick Hotel back to West Point; therefore, the first decision was obviously that the two cadets must travel via public transportation to make it back to West Point. From here and after a detailed analysis of all routes using public transportation, Route 4 for public transportation was determined to be the "best" route. This route fell within the cost constraint at a cost of $89.97 and within the time constraint by leaving at 12:15 and arriving back to West Point at approximately 17:40.

Appendix A

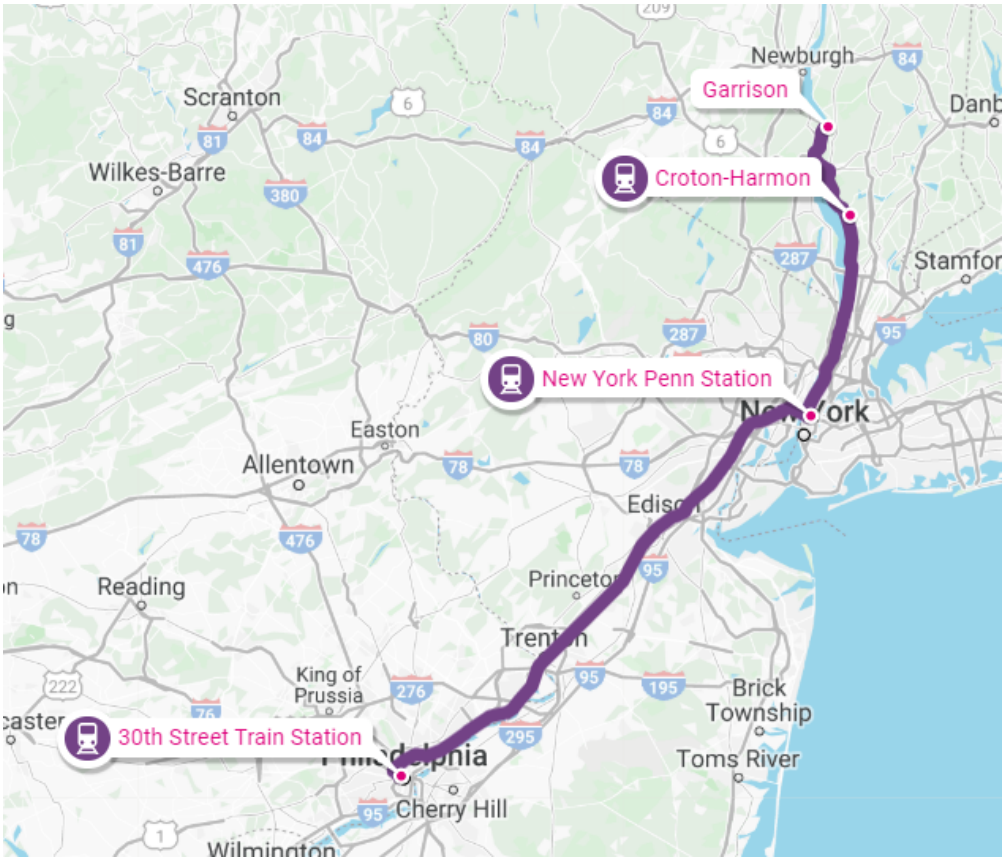


Figure 8: A visual display of Routes 1 and 2, which consists of only taking trains to get from Philadelphia to West Point

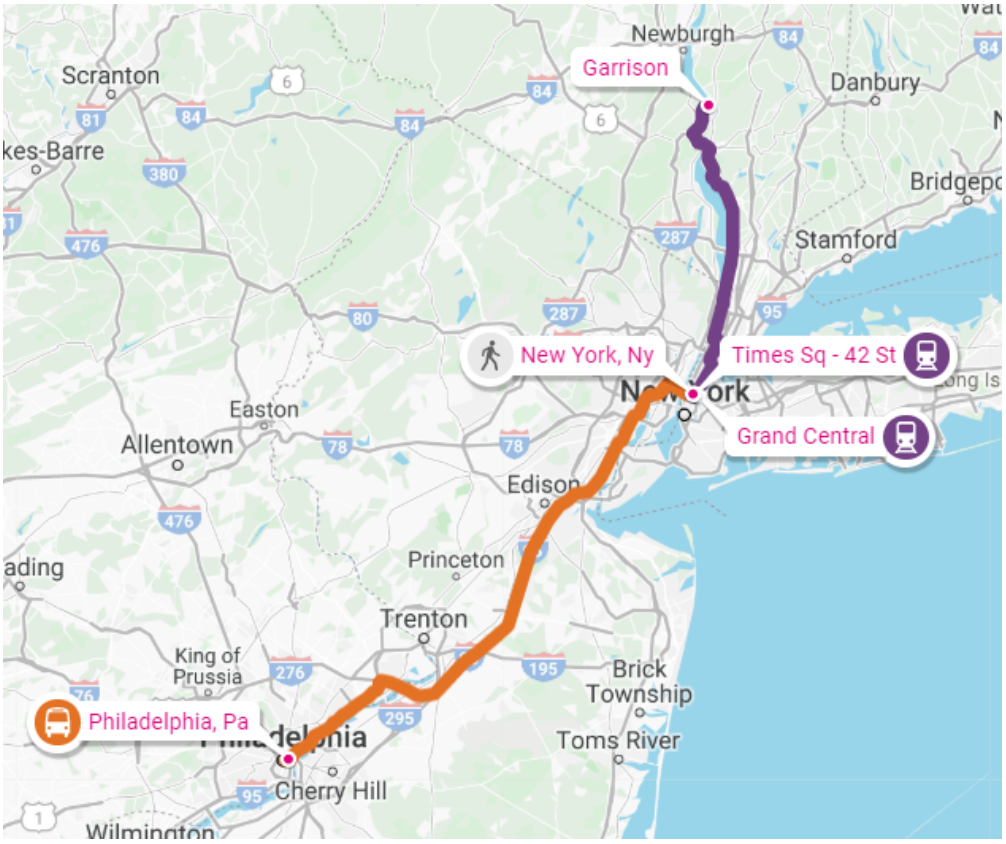


Figure 9: A visual display of Routes 3 and 4, which consists of taking both trains and buses to get from Philadelphia to West Point

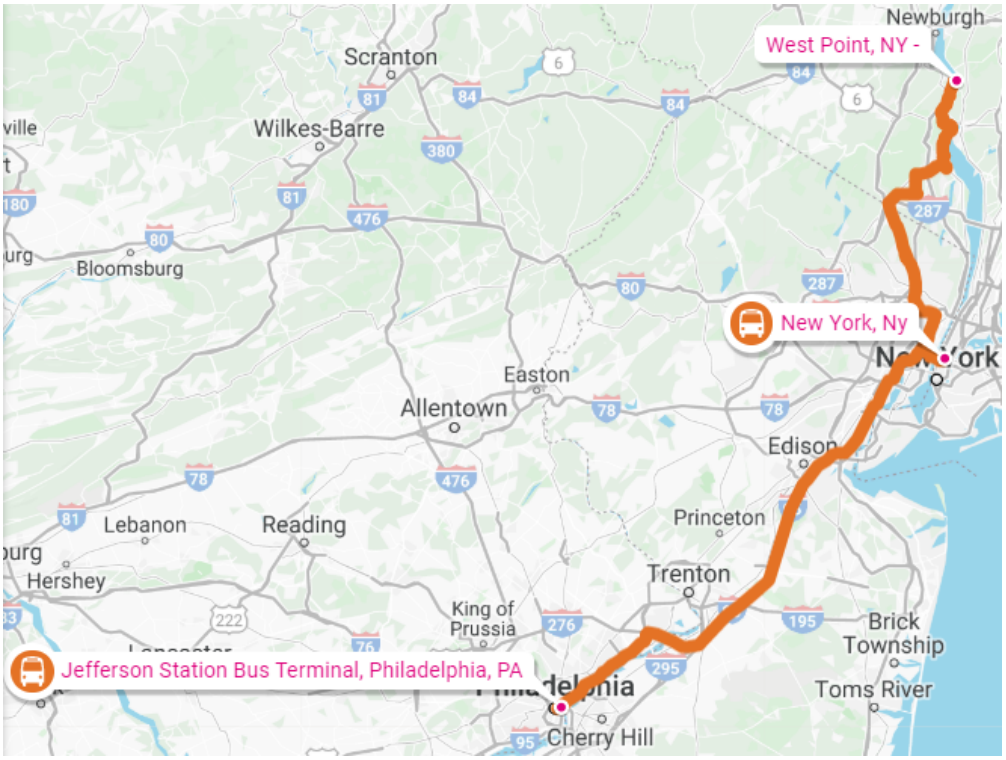


Figure 10: A visual display of Route 5, which consists of only taking buses to get from Philadelphia to West Point

Works Cited

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