Computer Science Foundations
Puzzle-Solving Workshop and Seminar
Optional Bonus Material

Episode 1—September 30
Fall 2013

Paul Pham and Neal Nelson

September 30, 2013

This version of the handout is for people who already have some discrete math and programming experience and know how to do the required problems. You still have to do the required problems and turn them in, but these optional bonus problems are provided to stretch your thinking even further.

Solutions to them will reflect positively on your evaluation.

However, in this program, we are cultivating a community of learners and teachers. Learning from and teaching your peers and fellow students can be way more effective than listening to instructors. Therefore, this handout will also contain tips for you to teach one of your classmates how to do the problem.

You will be an assistant teacher in this situation, and your job will be to help your fellow student have an *aha* moment of realization or discovery. Remember the first time you figured out something difficult, something you didn't know how to do before and even thought was impossible? Overcoming that situation brings an awesome feeling that everyone should have.

The general philosophy to keep in mind is that a teacher must meet a student where he or she is at. Try figuring out where your classmate is stuck, and then providing *just enough* of a hint to push them over their difficulties. In the weight-lifting analogy, you are a spotter, or an assistant trainer. The weight may be too heavy for your classmate to lift on his or her own, but by providing a slight pull, you can help them do it. This costs you very little physical / intellectual effort; at that point, you are mostly being a cheerleader and providing encouragement.

Bring plenty of paper and something to write with!

0.1 Instructions

1. Work on these optional bonus problems. If you solve them, great! Turn them in. If you can't solve them after 15 minutes or so, also great! In

any case, stop and move to the next step.

2. Read the section Problem 1 Hints below. Then look up and see which of your classmates has their hands raised for help. Pick one, go over with pencil and pen, then commence tutoring.

1 Optional Problem 1a

If Problem 1 is too easy for you, Gauss's grouchy schoolmaster has another one that might cause you to sweat a little more. (Teachers have a great way of making up new problems on the spot). But don't worry, this is optional, you don't have to do this to get credit for the workshop.

What is the sum of the first n squares? A square isn't just someone who isn't cool, a square is also an integer multiplied by itself. Let's use a different Greek symbol now, like ζ:

$$\zeta = 1^2 + 2^2 + 3^2 + \ldots + n^2 \tag{1}$$

You will need your powers of suggestive picture drawing, and your original solution to Gauss's problem.

1.1 Optional Problem 1b

And if that weren't enough, the teacher gives Gauss a final secret extra challenge problem, which nevertheless is a natural generalization to the problems above.

What is the sum of the first n kth powers? That is, a kth power is an integer multipled by itself k times. Let's use the symbol ω

$$\omega = 1^k + 2^k + 3^k + \ldots + n^k \tag{2}$$

2 Problem 1 Hints

Find out whether the person likes to think with symbols or with pictures. Follow the appropriate subsection below.

2.1 Thinking in Symbols

If they like to think with symbols, write out the numbers $1, 2, 3, \ldots$ on the left side of a page, and $\ldots, 98, 99, 100$ on the right side of the page, with a blank in between. Ask them if they can see a pattern. If they can't, add the numbers 4 and 97 to the left and right of the page, respectively. If they still can't see a pattern, ask them just to add the first and last number, then add the second and next-to-last number.

Once they understand that each pair has the same sum, pause here and see if they can take the next step on their own.

If they can't, ask them how many such pairs there are, if you were to finish writing out all the numbers from 1 to 100.

At this point, you should stop giving them hints and let them ponder the rest of the problem on their own.

2.2 Thinking in Pictures

Sitting *opposite* (across a table) from your classmate: Draw out one square, then two squares underneath that one, then three squares underneath that one. Label the first row with a 1, the second row with a 2, and so on. You should be forming a (jagged) right-triangle that lines up on the left.

Ask your classmate if they can think of a simple way to count all the squares, simply by using the height and width of the triangle.

Pause here for a few minutes to let them think about it.

If they still don't see the pattern: ask your classmate to draw the same figure as you, upside down and off to one side from your drawing, on the same piece of paper. Then ask them the same question.

At this point, you should stop giving them hints and let them ponder the rest of the problem on their own.