EXERCISES 18

PROBABILISTIC PROGRAMMING

18.1 Relational Probability Models

Exercise 18.SOCC

Three soccer teams A, B, and C, play each other once. Each match is between two teams, and can be won, drawn, or lost. Each team has a fixed, unknown degree of quality—an integer ranging from 0 to 3—and the outcome of a match depends probabilistically on the difference in quality between the two teams.

a. Construct a relational probability model to describe this domain, and suggest numerical values for all the necessary probability distributions.

b. Construct the equivalent Bayesian network for the three matches.

c. Suppose that in the first two matches A beats B and draws with C. Using an exact inference algorithm of your choice, compute the posterior distribution for the outcome of the third match.

d. Suppose there are n teams in the league and we have the results for all but the last match. How does the complexity of predicting the last game vary with n?

e. Investigate the application of MCMC to this problem. How quickly does it converge in practice and how well does it scale?

a. The classes are $Team$, with instances A, B, and C, and $Match$, with instances AB, BC, and CA. Each team has a quality Q and each match has a $Team_1$ and $Team_2$ and an $Outcome$. The team names for each match are of course fixed in advance. The prior over quality could be uniform and the probability of a win for team 1 should increase with $Q(Team_1) - Q(Team_2)$.

b. The random variables are $A.Q$, $B.Q$, $C.Q$, $AB.Outcome$, $BC.Outcome$, and $CA.Outcome$. The network is shown in Figure S18.1.

c. The exact result will depend on the probabilities used in the model. With any prior on quality that is the same across all teams, we expect that the posterior over $BC.Outcome$ will show that C is more likely to win than B.

d. The inference cost in such a model will be $O(2^n)$ because all the team qualities become coupled.

e. MCMC appears to do well on this problem, provided the probabilities are not too skewed. Our results show scaling behavior that is roughly linear in the number of teams, although we did not investigate very large n.

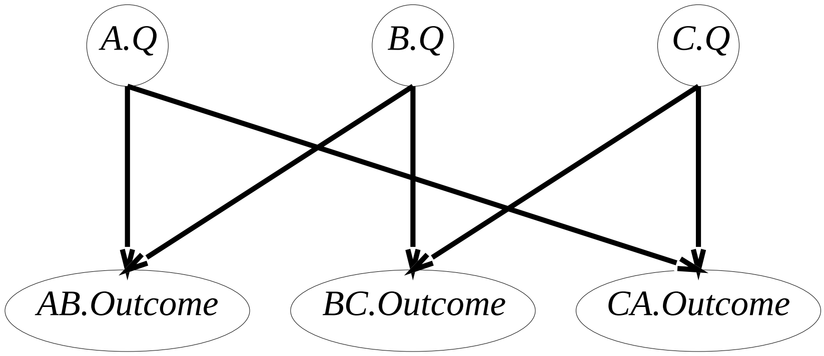

Figure S18.1 Bayes net showing the dependency structure for the team quality and game outcome variables in the soccer model.

[[need exercises]]

18.2 Open-Universe Probability Models

[[need exercises]]

18.3 Keeping Track of a Complex World

[[need exercises]]

18.4 Programs as Probability Models

[[need exercises]]