MMCS Assessment2

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1 Exercise 1 *Written*

1.1 Analysis

- For seven products, set a *index* which contains 7 elements to identify different *products*, their sale *prices* (pound) and *production time* (hours).
- For different kind of employees, set a *index* which contains 2 elements ('1' presents full time employee and '2' presents part time employees), to identify the different *working hours* during a day and the different *salaries*.
- As all employees have to work for constitutive 5 days and rest for 2 days, and there are 7days in a week, therefore, I divided all company's employees into 7 *groups*, group 1, group 2 to group 7. The group number i means this group's workers starting working on the ith day in a week. (e.g. group1: start working on Monday)
- To arrange work and rest for different groups, set a 2-dimension matrix *arrange*. Columns represent different group number, rows represent different days in a week. 1 represent work and 0 represent rest.

$$\begin{matrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{matrix}$$

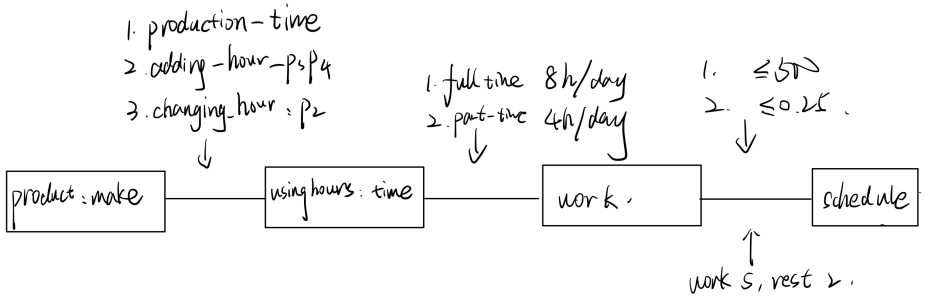
- There are several constrains for this problem. The first is employees need to rest 2 days after working for 5 days. The second is part-time employees working hour should less or equal to the total working hour. The third is worker need to be less and equal to 500 everyday. The forth is the amount of products and workers should be integer. which is not clearly mentioned in the problem. And there are 3 other constrains which need to be solved by big-M method.
- The rest 3 constrains are non-linear, which need to be produced into linear.

- For product7, the constrain is about if we produce the product or not. If we produce product7, we add 2000 in cost. Therefore I set a variable *adding_cost_p7* to record the adding cost on each days, and I also set a auxiliary variable *a_p7* to judge if product7 is produced. And the M used to judge is $M\_p7 = 421$. (The largest working hour is employees are all full time, and I assume that the company use all hours to produce p7, $M = 8 * 400 \div 9.5 \approx 421$.)
- For product2, the constrain is about if the product2 is more than 100 units and change the producing time when the amount is greater to 100. I divide the production of product2 into *p2_1* and *p2_2*, which represent the amount of production below 100 and greater than 100. And I set *a_p2_1* and *a_p2_2* to judge f production is less and equal to 100 and if there exit $p2\_2 > 0$ to make production greater to 100. The using M in this part is $M\_p2\_2 = (8 * 4000 - (2 * 100)) \div 3 \approx 1226$.
- For product3 and 4, the constrain is about if we produce both products or not. I set a variable *adding_hour_p3p4* to record the adding hour on each days. To judge if 2 products are produced at the same day, I set 2 auxiliary variable *a_p3* and *a_p4* to judge if product3 and product4 is produced separately. The M used to judge are $M\_p3 = 8 * 4000 \div 350 \approx 1481$ and $M\_p3 = 8 * 400 \div 490 \approx 1667$. And I also set a auxiliary variable *a_p3p4* to judge if $a\_p3 > 0$ and $a\_p4 > 0$, which means they are produced at same time.

- For this problem, we need to find out the *maximum profit*. The profit is all about incomes and costs. The income is all about products selling, and the costs is all about the salary which company pays to employees. And the producing time is the key to link cost(salaries) and income(producing). So we can view the arrangement of which group work in which day as a sub-problem.

- I will set 4 variables: 1.*make*(the amount of different products produced on different days); 2.*time*(the time need to be used in producing different time on different days); 3.*work*(the number of different types' workers work in different days); 4.*group*(arrange workers into different groups); 5.*schedule*(the number of different groups and types of workers work on different days).

- The image below shows my basic logic in solving this problem.



- In the end, I output a overview results in Xpress window, and create 3 csv files to illustrate the detail figures of producing product, dividing employees and arrange employees' work.

1.2 Model

- Index sets

  P: different products, $p \in P = \{1, 2, 3, 4, 5, 6, 7\}$

  D: different days in a week, $d \in D = \{1, 2, 3, 4, 5, 6, 7\}$

  W: different work types (1 is full time, 2 is part time), $w \in W = \{1, 2\}$

  N: different groups, $n \in N = \{1, 2, 3, 4, 5, 6, 7\}$

- Variables

  $make_{p,d}$: product p produced on day d

  $time_{p,d}$: presents used hours in producing product p on day d

  $work_{w,d}$: the amount of w type employees work on d day

  $group_{w,n}$: the amount of w type employees in group n

  $schedule_{w,d,n}$: the amount of type w employees in group n work on d day.

- Auxiliary Variables 1(For M method)

  $a\_p7_d, a\_p2\_1_d, a\_p2\_2_d, a\_p3_d, a\_p4_d, a\_p3p4_d$

- Auxiliary Variables 2(For constrains)

  $adding\_hour\_p3p4_d$: the adding hour when product 3 and 4 are produced at same time on day d.

  $adding\_cost\_p7_d$: the adding cost if product7 is produced on day d

  $p2\_1_d$: the amount of producing product2 in day d when the production is less and equal to 100 units.

  $p2\_2_d$: the amount of producing product2 in day d when the production is greater than 100 units.

- Parameters

  $price_p$: the price of each products p

  $production\_time_p$: the producing hours of each products p

  $salaries_w$: the salary of different work types w

  $worktime_w$: the working hours of different work types w

  $arrange_{d,n}$: the arrangement of each group n work or rest on each day d, which I have illustrated on page 1, if the value equal to 1, then work, if it equal to 0, then rest.

  $M\_p7 = 421$ $M\_p2\_2 = 1226$

  $M\_p3 = 1481$ $M\_p4 = 1667$

- Objective function

$$\max \sum_{p \in P, d \in D} make_{p,d} * price_p - \sum_{d \in D} adding\_cost\_p7_m - \sum_{w \in W, d \in D} salaries_w * worktime_w * work_{w,d} \tag{1}$$

- Constraints

  s.t.

  $\sum_{w=2,d\in D} work_{w,d} \leq \sum_{p\in P,d\in D} make_{p,d} \times 0.25$

  $\sum_{w\in W} work_{w,d} \leq 500, d \in D$

  $make_{p=7,d} \leq a\_p7_d \times M\_p7, d \in D$

  $adding\_cost\_p7_d = a\_p7_d \times 2000, d \in D$

  $make_{p=3,d} \leq a\_p3_d \times M\_p3, d \in D$

  $make_{p=4,d} \leq a\_p4_d \times M\_p4, d \in D$

  $a\_p3_d + a\_p4_d \leq a\_p3p4_d + 1, d \in D$

  $adding\_hour\_p3p4_d = a\_p3p4_d \times 75, d \in D$

  $p2\_1_d - 100 \leq a\_p2\_1_d, d \in D$

  $p2\_2_d \leq a\_p2\_2_d \times M\_p2\_2, d \in D$

  $a\_p2\_2_d \leq a\_p2\_1_d, d \in D$

  $make_{p=2,d} = p2\_1_d + p2\_2_d, d \in D$

  $schedule_{w,d,n} = group_{w,n} \times arrange_{d,n}, w \in W, d \in D, n \in N$

  $time_{p,d} = make_{p,d} \times production\_time_p, p \in P \setminus \{2\}, d \in D$

  $time_{p=2,d} = p2\_1_d \times 2 + p2\_2_d \times 3, d \in D$

  $time_{p=3,d} = time_{p=3,d} + adding\_hour\_p3p4_d, d \in D$

  $\sum_{p\in P} time_{p,d} = work_{w=1,d} \times 8 + work_{w=2,d} \times 4, d \in D$

  $\sum_{n\in N} schedule_{w,d} = \sum_{w\in W} work_{w,d}, d \in D$

  $work_{w,d}, group_{w,n}, schedule_{w,d,n}, group_{w,n} \in \mathbb{Z}, w \in W, p \in P, n \in N$

  $work_{w,d}, group_{w,n}, schedule_{w,d,n}, group_{w,n} \geq 0, w \in W, p \in P, n \in N$

  $a\_p7_d, a\_p2\_1_d, a\_p2\_2_d, a\_p3_d, a\_p4_d, a\_p3p4_d \in \{1, 0\}, d \in D$

2 Exercise 2 *Computer*

Files and code are attached in the assessment_2.zip package.

The result is: produce 1585 units of product4 and 98 units of product2 everyday. Only use full time employees, the total amount of employees is 700.

There are 500 employees working everyday and 200 employees are resting everyday. Every group has 100 employees. On Monday, group1,4,5,6,7 work, group 2,3 rest. On Tuesday, group1,2,5,6,7 work, group 3,4 rest. On Wednesday, group1,2,3,6,7 work, group 4,5 rest. On Thursday, group1,2,3,4,7 work, group5,6 rest. On Friday, group1,2,3,4,5 work group 6,7 rest. On Saturday, group2,3,4,5,6 work, group1,7 rest. On Sunday, group3,4,5,6,7 work, group2,3 rest.

The maximum profit is 2644670 a week.