# Price optimisation and capacity allocation of Dartford Crossing, UK

1. **Problem Description**

The aim of this report is to investigate the performance of different pricing strategies and optimal capacity allocation in accordance to different objectives for Dartford Crossing. Dartford Crossing belongs to one of the busiest highways M25 in England. The current traffic toll strategy for Dartford Crossing is single pricing based on vehicle type (Table 2), and drivers are charged between 6am to 10pm, while at other time periods it is free. Considering that the heavy traffic remains a big challenge to the government, we would like to propose two pricing strategies to improve the efficiency of the Crossing. The two approaches are designed to leverage the difference in preferences and needs, for example the preference of peak hour crossing for private cars and the need of midnight crossing for night shift trucks, and optimised based on the government's potential objectives – maximise toll revenue, minimise emissions, or maximise throughput.

Two strategies proposed and assessed in this study: 1. Time-based differentiation by vehicle type through Peak-Period Pricing; 2. Capacity allocation of high-occupancy toll (HOT) lanes based on vehicle occupancy.

1. **Methodologies**

**i. Time-based differentiation through Peak-Period Pricing**

**Data simulation and assumptions**

To simulate the willingness-to-pay (WTP) data (Figure 1), we first use the M25 road traffic statistics collected from the Department for Transport to get the vehicle type mix and assume the same proportion for the Dartford Crossing. Then a data frame of 2000 observations (drivers) is simulated using this vehicle type mix. Each driver's WTP per hour is determined by their vehicle type and reference to the current pricing structure (see Table 2) with the assumption listed in Table 3. The simulated data has 2000 rows with 24 columns representing the WTP for each hour in a day, and 1 column containing the vehicle type information (see Table 4). The peak (7-9am, 4-6pm) hours are determined by the research done by the RAC foundation(2020). Figure 2 visualises revenue as a function of peak and non-peak price by vehicle type.

**Assumptions for constraints and features**

Four constraints are considered for the setting: capacity, operating cost, speed and emission limit.
**Capacity:** Given the daily flow of the Crossing is 216,108 vehicles, the hourly flow would be 9004. Assuming the capacity is higher than the actual hourly flow, it is set at 10000 vehicles per hour.
**Operating cost:** The operation cost is assumed to be £102m annually, as suggested by theconstructionindex (2021). The daily operating cost is used in a constraint where total revenue should be large enough to cover the operating cost.
**Speed:** The speed limit of the bridge is 48 km/h. The average speed for each car is calculated by the current throughput of the bridge. The formula is referenced from Ceryan(2022).
**Emission:** Please refer to flowchart 1 in Appendix for the derivation.

**Approach:**

1. Obtain the parametric demand function for each vehicle type using the simulated WTP data by linear regression. The regression results (R-squared and adjusted R-squared) can be found in Table 8.
2. Use non-linear programming to find the optimal peak and non-peak prices, based on the demand derived in step 1, the 4 constraints defined and the objective function. In this

analysis, we are proposing 3 scenarios for different objective functions that are applicable to the real-life situation: i. maximise total revenue; ii. minimise total emission; iii. maximise total throughput

### ii. High Occupancy toll lane (HOT) allocation

#### Background and methodologies

In order to improve crossing efficiency and minimise emissions, our second approach is to introduce one HOT lane to the existing 4-lane Dartford Crossing. HOT lane is a type of lane that is available only to vehicles that have more than 1 occupancy. In our proposed setting, all lanes will charge a low constant fee across all types of vehicles except for solo drivers using the HOT lane which will be charged at a premium rate. Hence, it is a single product multi-fare class with mixed arrival problems.

In this setting, we only consider the number of passengers on each vehicle regardless of their type and size. Therefore, there are two vehicle types: single-occupancy (SO) and multi-occupancy (MO) vehicles. Restricting SO vehicles to use HOT lanes is a means to encourage people to carpool and utilise public transportation. This can also ensure that the capacity of the HOT lane is not largely occupied by solo drivers. Hence, the number of SO vehicles may potentially reduce as people shift towards ride sharing or the use of transit which in turn maximises the throughput of the bridge and reduces emissions. Therefore, the objective is to determine the optimal allocation of SO vehicles at HOT lane in order to minimise emissions and maximise crossing throughput.

To model this setting, we used linear programming to obtain optimal allocation over a time horizon of 10 minutes. The constraints include: 1) allocation number has to be less than or equal to the expected demand of SO and MO vehicles in 10 minutes, 2) the revenue from the HOT lane has to be higher than the daily operational cost of the bridge, and 3). Total allocation number has to be less than the capacity of the bridge in 10 minutes. 4) The allocation number of MO vehicles has to be higher than that of SO vehicles.

#### Assumptions

Six assumptions were made to model setting, including capacity, time horizon, price, operating costs, probability of arrival and emissions.

**Capacity:** The capacity of the vehicles per 10 minutes was calculated using the previously mentioned assumption on 10000 vehicles per hour. Hence, 1667 vehicles will pass the bridge every 10 minutes.

**Time Horizon:** Using the capacity calculation, 2.5 vehicles will pass the bridge per second in which there is one car passing in every 0.4 seconds. Since we are modelling over a 10-minute period (600 seconds), we divided this time period by 0.4 seconds to obtain the total number of stages for our model (600/0.4 = 1500). Hence, there is one car in each stage.

**Price:** A constant price is charged across all vehicle types. The price for SO vehicles to use the HOT lane is assumed to be £3.50, the price for MO vehicles is assumed to be £2.50 which is the current standard toll pricing for a car between 6am and 10pm.

**Operating Costs:** Given the price, revenue per 10 minutes needs to be higher than the operating costs per 10 minutes. This operating cost is calculated by: annual operating cost (£102m as indicated in the previous approach) / 365(days) / 24 (hour) / 6 ( per 10 minutes interval in an hour) = £1944.44

**Probability of arrival:** On average, drivers travelling alone in the UK make 0.5 car ride a day as compared to 0.6 for drivers travelling with passengers (Nimblefins, 2021). Hence, the probability of no arrival in 10 minutes was assumed to be 0.1 and the probabilities of SO and MO vehicles to arrive at the HOT lane in 10 minutes were assumed to be 0.4 and 0.5.

**Emissions:** The total emissions per vehicle were calculated using the speed of the 4 types of vehicles using the emission equation found in Table 5, with the assumption that speed is greater than 25 km/h. Emissions of MO vehicles were assumed to be half of the emissions of SO vehicles. This is due to the assumption that each MO vehicle consists of 1 driver and 1 passenger, and the passenger could have driven their own car which will result in double the emission. Hence, the emissions were set to 325.10 $GCO_2e$ for MO vehicles and 650.21 $GCO_2e$ for SO vehicles.

**Approach**

1. Calculate the expected demand by multiplying the arrival probabilities by the time horizon.
2. Use linear programming to find the optimal allocation of SO vehicles per 10 minutes, based on the objective function, the 4 defined constraints and the expected demand.

## 3. Results and analysis

### a. Time-based differentiation through Peak-Period Pricing

| Scenario | 1. Maximise Revenue | | 2. Minimise emission | | 3. Maximise throughput | |
|---|---|---|---|---|---|---|
| | Price | Demand | Price | Demand | Price | Demand |
| Nonpeak motorbike | 1.00 | 20 | 0.97 | 38 | 0.05 | 25 |
| Peak motorbike | 1.00 | 22 | 1.70 | 2 | 0.88 | 25 |
| Non-peak cars | 2.00 | 5777 | 2.00 | 6716 | 0.50 | 4138 |
| Peak cars | 3.19 | 1176 | 3.68 | 0 | 0.50 | 4786 |
| Nonpeak vans | 3.00 | 2789 | 3.00 | 1491 | 0.90 | 3577 |
| Peak vans | 6.00 | 0 | 4.02 | 0 | 6.00 | 0 |
| Nonpeak trucks | 5.29 | 1414 | 5.25 | 356 | 5.12 | 2261 |
| Peak trucks | 10.00 | 0 | 5.32 | 233 | 10.00 | 0 |
| Optimal result | Revenue = £563,393<br>Throughput = 204,792 | | Emission/h = 5,527 $KGCO_2$<br>Throughput = 172,969<br>Revenue = £401,233 | | Throughput = 219,153<br>Revenue = £280,000 | |

Table 1. Result of peak-period pricing based on 3 objectives 1. Maximise revenue, 2. Minimise emission, 3. Maximise throughput

**Scenario 1: Maximise Revenue**

With the optimal pricing structure, peak demand for trucks and vans are 0. The WTPs of vans and trucks are quite flat across peak and non-peak hours, implying that drivers have no strong preference to cross the bridge during peak hours. Therefore a high price could be charged to encourage them to cross during non-peak hours, and allow more cars, which are more sensitive to time, to pass.

**Scenario 2: Minimise Emission**

Since emission is a function of speed, there is an optimal speed for the highest fuel economy. The number of vehicles on the bridge will affect the average vehicle speed, hence to minimise emissions, there is a limit imposed on the throughput. The demand for peak crossing is the highest for cars, therefore based on the throughput limitation, the peak pricing for cars is high, and the high price deters car drivers to cross during peak hours, hence the peak demand for cars in this scenario is 0. Both van and truck emissions are high, but because trucks bring in more revenue, we are allowing more trucks to pass through to meet the revenue constraint. Therefore peak demand for vans is 0.

**Scenario 3: Maximise throughput**

Due to the high peak demand from cars, the peak price for cars is set quite low to let the maximum number of cars to cross during peak hours. The peak demand for vans and trucks are 0 because they are relatively more elastic with time, i.e. they do not have a strong preference to drive during peak hours. Therefore high peak prices are imposed to encourage them to cross during non-peak hours.

The current daily flow of the Crossing is 216,108, and the revenue based on daily flow is £411,415. In scenario 1, the revenue is higher yet the total number of throughput is lower, meaning that the higher prices have deterred some demand. In scenario 2, both revenue and throughput are lower due to the high prices during peak hours to minimise emissions. In scenario 3, even though throughput is the highest, revenue is the lowest due to the low prices, especially for cars during peak hours.

### b. HOT Lanes Allocations

Our decision model showed that 18 SO and 750 MO vehicles should be allocated to the HOT lanes every 10 minutes in order to obtain the minimum emissions (Table 9). This brings to a total of 2592 and 108000 for SO and MO vehicles on the HOT lane daily, respectively.

From the allocation result, we recommend a limited number of HOT lane tickets to be sold for SO vehicles and solo drivers would need to purchase them online in advance. The optimal ticket allocations for multi-occupancy vehicles can serve as a reference for the company if they are to control the emission level of the HOT lane.

## 4. Recommendation/Conclusion

Depending on the government's objective, whether it is to maximise toll revenue, minimise emission, or maximise throughput, if a more targeted peak pricing strategy is imposed at the Dartford Crossing in place of the current pricing structure, different improvements will be achieved.

The proposed strategy for the HOT lane approach is to limit the number of HOT lane tickets for solo drivers to maximise the HOT lane throughput. Drivers who want to purchase HOT toll would need to purchase online in advance, but the rest of the ticket can be purchased online in advance or pay by mid-night the following day, and with no capacity limit. Similar to the current pricing strategy, drivers would need a ticket to cross the bridge between 6am and 10pm.

### 5. References


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## 6. Appendix

| Vehicle class | 06:00-22:00 | 22:00-06:00 |
|---|---|---|
| Motorbike | Free | Free |
| Cars, motorhomes, minibuses | £2.50 | Free |
| HGVs and vans with two axles | £3.00 | Free |
| Multi-axle goods vehicles | £6.00 | Free |

Table 2: Current pricing structure for Dartford Crossing by vehicle type.
Ref: UK Government (2022)

| Vehicle Type | Hours | Distribution |
|---|---|---|
| Motorbikes | 8-10am, 5-7pm | ~ N(1.2, 0.2) |
| | 12am-7am, 11am-4pm, 8pm-12am | ~ N(1, 0.2) |
| Private Cars | 8-10am, 5-7pm | ~ N(3, 0.2) |
| | 12am-7am, 11am-4pm, 8pm-12am | ~ N(2.5, 0.2) |
| Vans | 6am-10pm | ~ N(3.75, 0.2) |
| | 12am-5am, 11pm-12am | ~ N(3, 0.2) |
| Multi-axle goods vehicles | All day | ~ N(6, 0.2) |

Table 3: Simulated data assumptions and distributions for peak-period pricing.

| Index | hour1 | hour2 | hour3 | hour4 | … | hour24 | vehicle_ type |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | motorbike |
| 2 | | | | | | | private_cars |
| 3 | | | | | | | vans |
| 4 | | | | | | | multi_goods |
| … | | | | | | | … |
| 1999 | | | | | | | multi_goods |
| 2000 | | | | | | | multi_goods |

Table 4: WTP data schema

| Object | Formula |
|---|---|
| Average speed | $Average\ speed\ =\ 48\ -\ 0.0625\ *\ (\#\ of\ cars\ in\ thousands,\ ‘000)$ |

| | |
|---|---|
| Fuel economy | $speed <= 25km/h: fuel\ economy = 0.354006 * (1.48 * speed)$<br>$speed > 25km/h: fuel\ economy = 0.354006 * (45.33 - 0.33 * speed)$ |
| Emission (g) for two kilometres per car | $speed <= 25km/h:$<br>$emission = average\ emission\ per\ fuel * 2.872 * /(0.354 * (1.48 * speed))$<br>$speed > 25km/h:$<br>$emission = average\ emission\ per\ fuel * 2.872 * /(0.354 * (45.33 - 0.33 * speed$ |
| Emission allowance for motorbike | $\sum_{i=1}^{n=time\ period} Demand_i * 1.7 * 2.872/13.84 * 1000$ |
| Emission allowance for car | $\sum_{i=1}^{n=time\ period} Demand_i * 1.7 * 2 * 2.872/13.84 * 1000$ |
| Emission allowance for van | $\sum_{i=1}^{n=time\ period} Demand_i * 1.2 * 10 * 2.872/13.84 * 1000$ |
| Emission allowance for truck | $\sum_{i=1}^{n=time\ period} Demand_i * 1.7 * 2 * 5 * 2.872/13.84 * 1000$ |
| Emissions of motorbike | $speed <= 25km/h:$<br>$2200 * 2.872/((0.354 * (1.48 * speed))$<br>$speed > 25km/h:$<br>$2200 * 2.872/(0.354 * (45.33 - 0.33 * speed))$ |
| Emissions of cars | $speed <= 25km/h:$<br>$2392 * 2.872/(0.354 * (1.48 * speed))$<br>$speed > 25km/h:$<br>$2392 * 2.872/(0.354 * (45.33 - 0.33 * speed))$ |
| Emissions of two-axle goods vehicles (vans) | $speed <= 25km/h:$<br>$2500 * 2.872/((0.354 * (1.48 * speed))$<br>$speed > 25km/h:$<br>$2500 * 2.872/(0.354 * (45.33 - 0.33 * speed))$ |
| Emissions of multi-axle goods vehicles | $speed <= 25km/h:$<br>$2600 * 2.872/((0.354 * (1.48 * speed))$<br>$speed > 25km/h:$<br>$2600 * 2.872/(0.354 * (45.33 - 0.33 * speed))$ |

Table 5. The formulas for average speed, fuel economy, emission for two kilometers per car, emission allowance and emissions for the four types of vehicles.

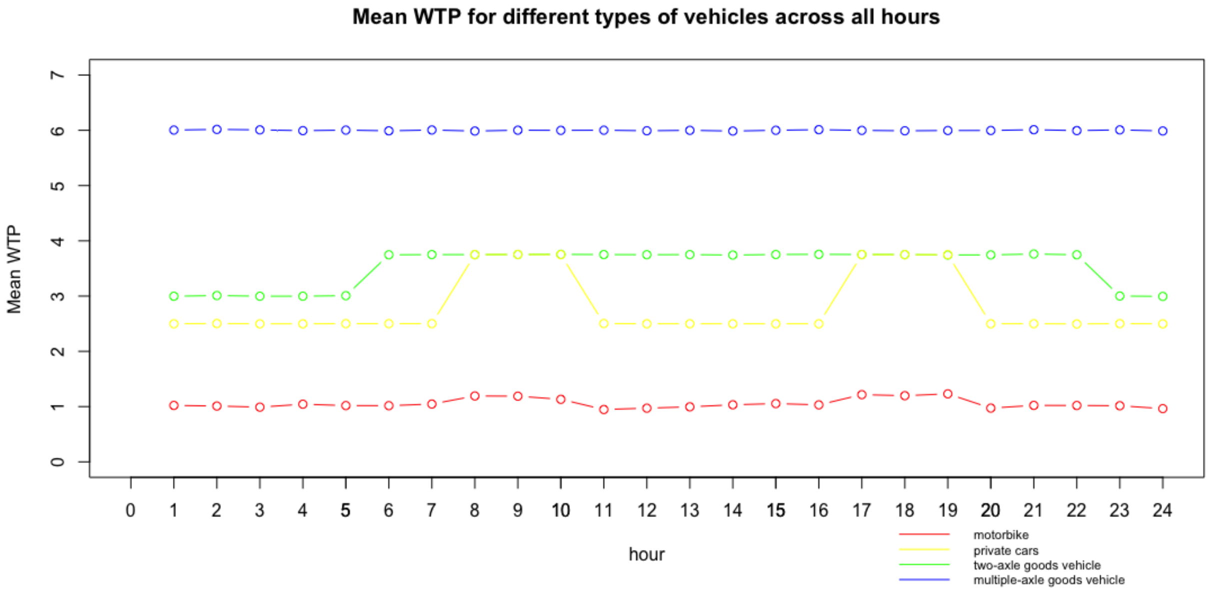


Figure 1. The mean WTP for the four types of vehicles (motorbikes, cars, two-axle, and multi-axle) across all hours.

| | | |
|---|---|---|
| Real Peak Hours | 7-8 am | 4-5 pm |
| Assumption | 7-9 am | 4-6 pm |

Table 6. Assumption for time-based differentiation through Peak-Period Pricing
Ref: RAC Foundation(2022)

| Motorbike | Private cars | Two-axle goods vehicles | multiple-axle goods vehicles | All motor vehicles |
|---|---|---|---|---|
| 1094 | 165759 | 30935 | 18320 | 216108 |

Table 7. Vehicle mix - average daily flow
Ref: Department for Transport (2022)

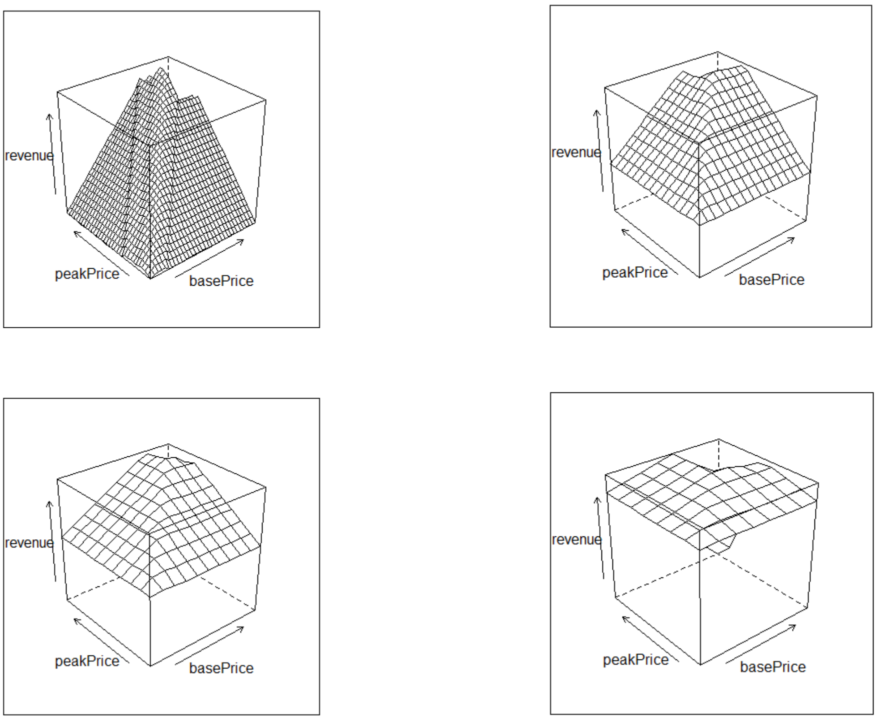


Figure 2. The revenue function of peak price and non peak price.

| Vehicle types | Motorbike | | Cars | | Vans | | Trucks | |
|---|---|---|---|---|---|---|---|---|
| Time period | Non Peak | Peak | Non Peak | Peak | Non Peak | Peak | Non Peak | Peak |
| R2 | 0.801 | 0.820 | 0.750 | 0.752 | 0.770 | 0.767 | 0.794 | 0.785 |
| Adjusted R2 | 0.800 | 0.820 | 0.748 | 0.750 | 0.766 | 0.763 | 0.787 | 0.778 |

Table 8. R2 and adjusted R2 for different vehicle types regression results

| | |
|---|---|
| Allocations of SO vehicles | 18 |
| Allocations of MO vehicles | 750 |
| Emission ($GCO_2e$) | 253372.4 |

Table 9. shows the number of single- and multi-occupancy vehicles allocated to the HOT lanes every 10 minutes at minimum emission.

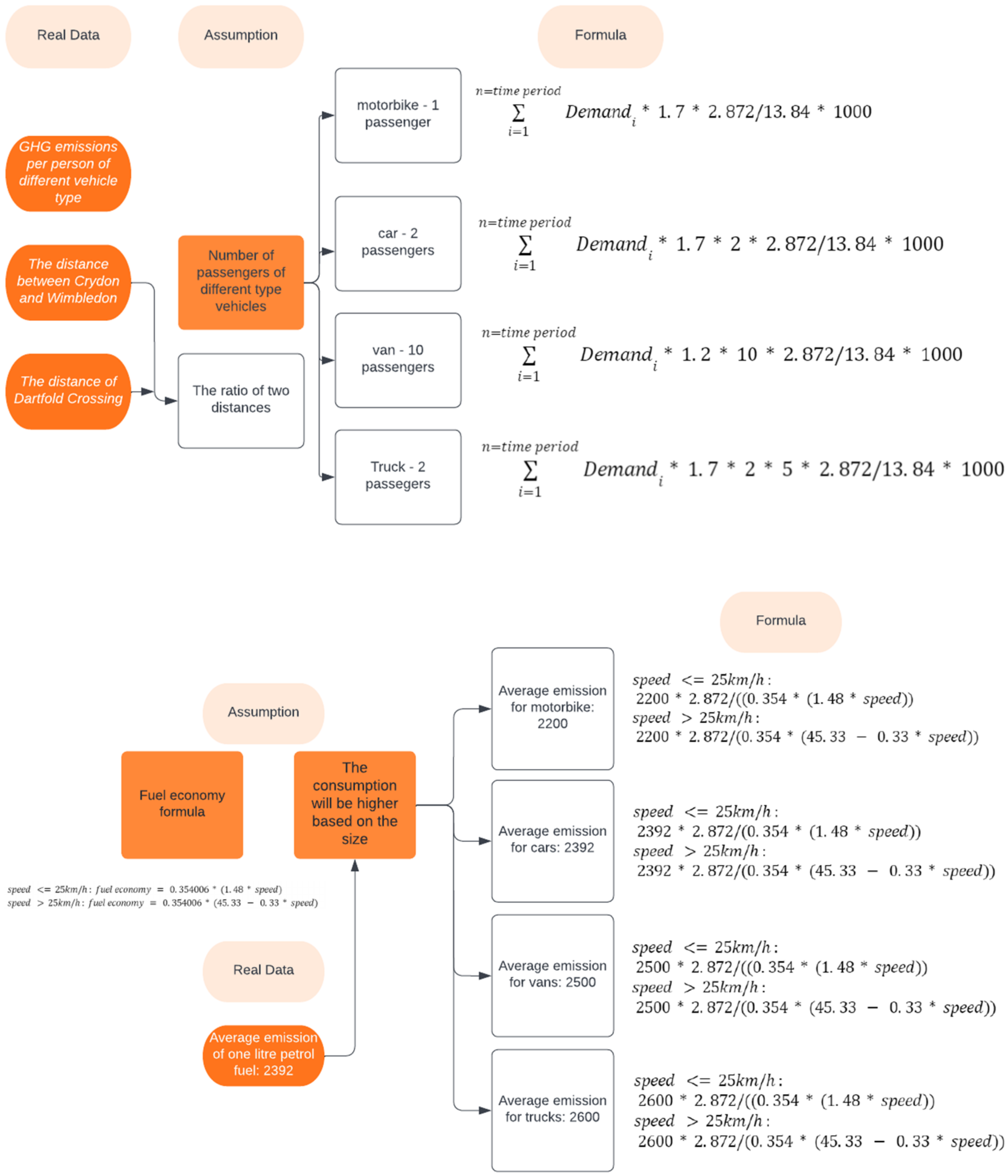


Fig.3. Flowchart of emission assumptions