

Homework 1

Due: October 20, 2021 in class

Note: No late homework will be accepted. You may discuss with your classmates but **you may not plagiarize.** You need to turn in **your analysis and also your code** (printout) written in Octave or Matlab.

In this homework we will use Lagrange polynomial interpolation and cubic spline interpolation to interpolate $(N + 1 = 11)$ data points (x_i, y_i) , where $i = 0, 1, 2, \dots, 10$. The independent variable x is in the range of $[-1, 1]$. There are two sets of $(N + 1 = 11)$ data points posted on CEIBA where you may download.

Part A. (25%)

Please refer to the file hw1AB.dat for the 11 data points. Here x_i , where $i = 0, 1, 2, \dots, 10$, are uniformly distributed in $[-1, 1]$.

A.1 Plot the Lagrange polynomial $L_j(x)$, where $j = 0, 1, 2, \dots, 10$. Note that $L_j(x_i) = 0$ when $i \neq j$ and $L_j(x_i) = 1$ when $i = j$.

A.2 Plot the interpolating polynomial that goes through the 11 data points

$$P(x) = \sum_{j=0}^{10} y_j L_j(x)$$

Part B. (25%)

Please refer to the file hw1AB.dat for the 11 data points. Here x_i , where $i = 0, 1, 2, \dots, 10$, are uniformly distributed in $[-1, 1]$.

B.1 Here we use cubic spline interpolation and we should assume the second derivative at the points, $g''(x_i)$ where $i = 0, 1, 2, \dots, 10$, as unknowns. Let's use free run-out condition for $g''(x_0) = g''(x_{10}) = 0$. What are the values of $g''(x_i)$ where $i = 1, 2, \dots, 9$?

B.2 Plot the cubic spline interpolation for the whole range of $x \in [-1, 1]$.

Part C. (25%)

Please refer to the file hw1CD.dat for the 11 data points. Here x_i , where $i = 0, 1, 2, \dots, 10$, are non-uniformly distributed in $[-1, 1]$.

C.1 Plot the Lagrange polynomial $L_j(x)$, where $j = 0, 1, 2, \dots, 10$. Note that $L_j(x_i) = 0$ when $i \neq j$ and $L_j(x_i) = 1$ when $i = j$.

C.2 Plot the interpolating polynomial that goes through the 11 data points

$$P(x) = \sum_{j=0}^{10} y_j L_j(x)$$

Part D. (25%)

Please refer to the file hw1CD.dat for the 11 data points. Here x_i , where $i = 0, 1, 2, \dots, 10$, are non-uniformly distributed in $[-1, 1]$.

D.1 Here we use cubic spline interpolation and we should assume the second derivative at the points, $g''(x_i)$ where $i = 0, 1, 2, \dots, 10$, as unknowns. Let's use free run-out condition for $g''(x_0) = g''(x_{10}) = 0$. What are the values of $g''(x_i)$ where $i = 1, 2, \dots, 9$?

D.2 Plot the cubic spline interpolation for the whole range of $x \in [-1, 1]$.