Homework 5

Due: January 5, 2022 in class

Note: No late homework will be accepted. You may discuss with your classmates but you may not plagiarize. You need to turn in your analysis and also your code (printout) written in Octave or Matlab.

Part A. (50%)

A.1 Plot the stability diagram for the second-order Runge-Kutta method. Where does the neutral curve intersect with $\mathrm{Re}(\lambda h)$?

A.2 Plot the stability diagram for the fourth-order Runge-Kutta method. Where does the neutral curve intersect with $\mathrm{Re}(\lambda h)$ and with $\mathrm{Im}(\lambda h)$?

Part B. (50%)

Consider the equation

$$\frac{dy}{dx} + (2 + 0.01x^2)y = 0,$$

subject to $y(0) = 4$ in the range $0 \leq x \leq 15$.

B.1 Solve this equation using the following numerical schemes: i) Euler, ii) backward Euler, iii) trapezoidal, iv) second-order Runge-Kutta and v) fourth-order Runge-Kutta. Use $\Delta x = 0.1, 0.5, 1.0$ and compare to the exact solution.

B.2 For each scheme, estimate the maximum Δx for stable solution over the given domain and discuss your estimate in terms of results of B.1.