The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is $\$ 5$ per unit on special risk insurance and $\$ 2$ per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work-Hours per Unit		Work-Hours Available
	Special Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

1. Formulate a linear programming model for this problem:
   a. Decision Variables:
      i. Let x_1 be units of special risk insurance products
      ii. Let x_2 be units of mortage products
   b. Objective: Maximize Profit P such that $P = 5x_1 + 2x_2$
   c. Constraits: Department Hour Availability
      i. $3x_1 + 2x_2 \leq 2400$
      ii. $0x_1 + 1x_2 \leq 800$
      iii. $2x_1 + 0x_2 \leq 1200$
      iv. $x_1 \geq 0$ and $x_2 \geq 0$
2. Use the graphical method to solve this model

3. Verify the exact value of your optimal solution from part by solving algebraically for the simultaneous solution of the relevant two equations
   a. Since we obtained the maximum possible profit $P = 3600$ through graphing method, we will determine the values of the decision variables by solving the systems of inequalities in the given constraints:

$$\begin{cases} 3x_1 + 2x_2 \leq 2400 \\ 0x_1 + 1x_2 \leq 800 \Rightarrow x_2 \leq 800 \\ 2x_1 + 0x_2 \leq 1200 \Rightarrow x_1 \leq 600 \end{cases}$$

Since $P = 5x_1 + 2x_2$, thus each x_1 units mostly influence the maximum profit P thus direct substitute $x_1 = 600$ to equation 1:

$$\begin{cases} 3x_1 + 2x_2 \leq 2400 \Rightarrow 2x_2 \leq 2400 - 3(600) \Rightarrow 2x_2 \leq 600 \Rightarrow x_2 \leq 300 \\ x_2 \leq 800 \\ x_1 \leq 600 \end{cases}$$

This means, that considering $x_1 = 600$, then x_2 has least upper bounded by 300

$$\begin{cases} x_2 \leq 300 \\ x_2 \leq 800 \\ x_1 \leq 600 \end{cases}$$

Thus, the decision therefore $x_1 = 600$ and $x_2 = 300$ to achieve maximum profit $P = 3600$

Consider the model

$$\text{Minimize } Z = 40x_1 + 50x_2$$

that is subject to

$$2x_1 + 3x_2 \geq 30$$

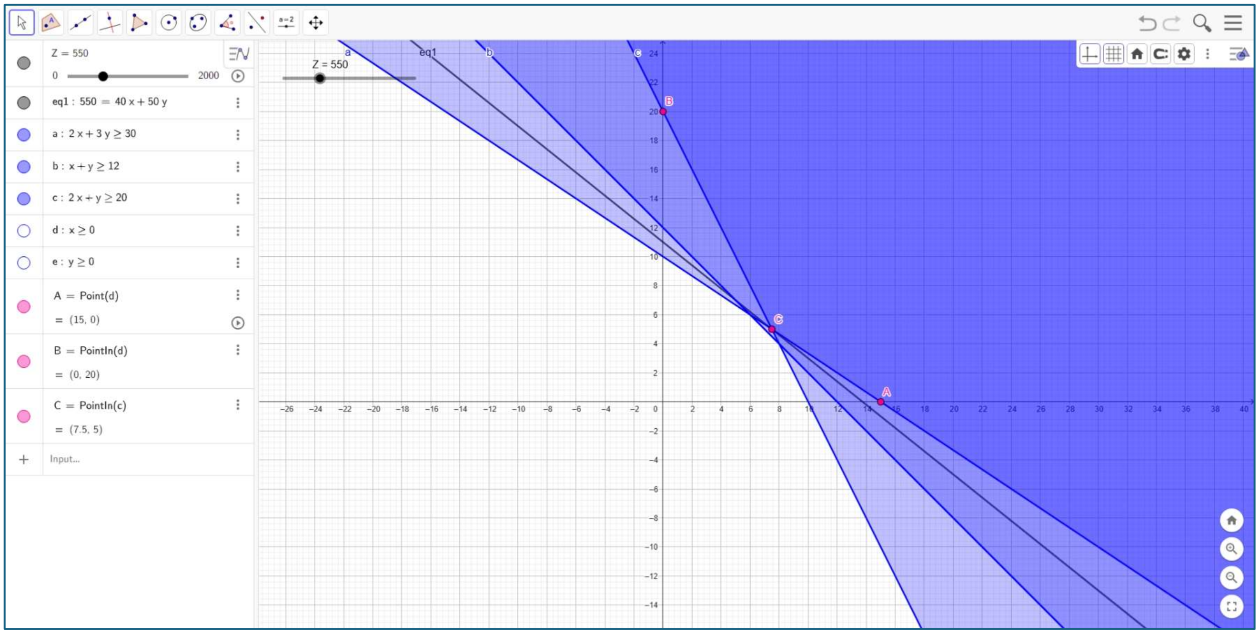
$$x_1 + x_2 \geq 12$$

$$2x_1 + x_2 \geq 20$$

and

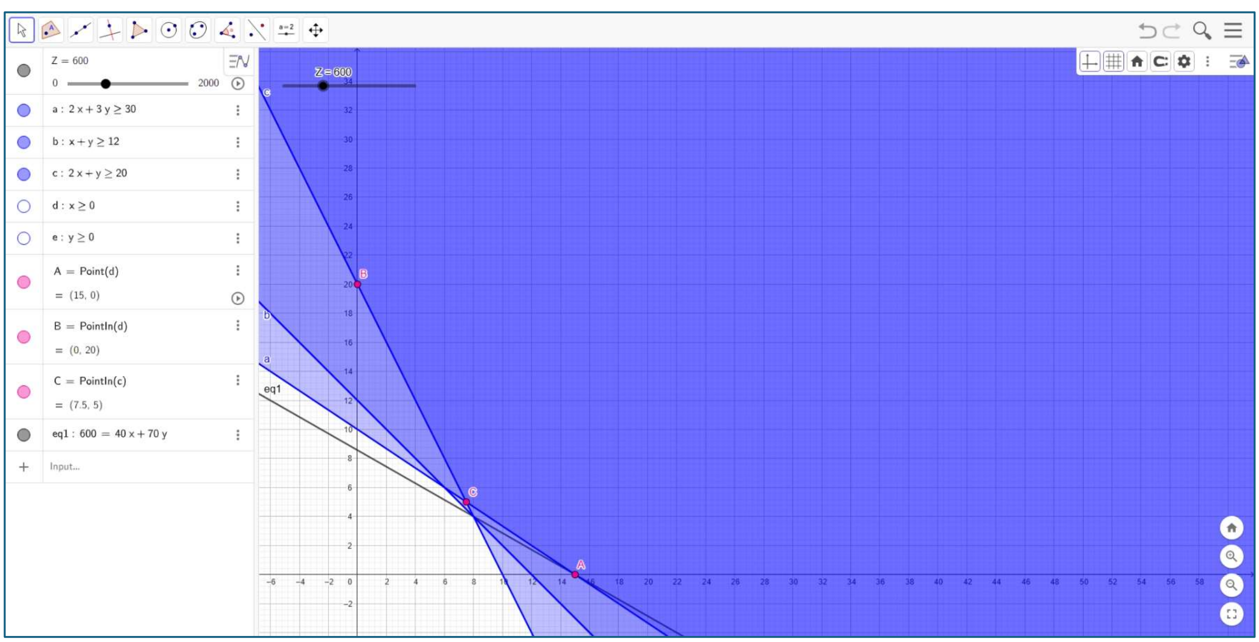
$$x_1 \geq 0, x_2 \geq 0$$

1. Use graphical model to solve the model



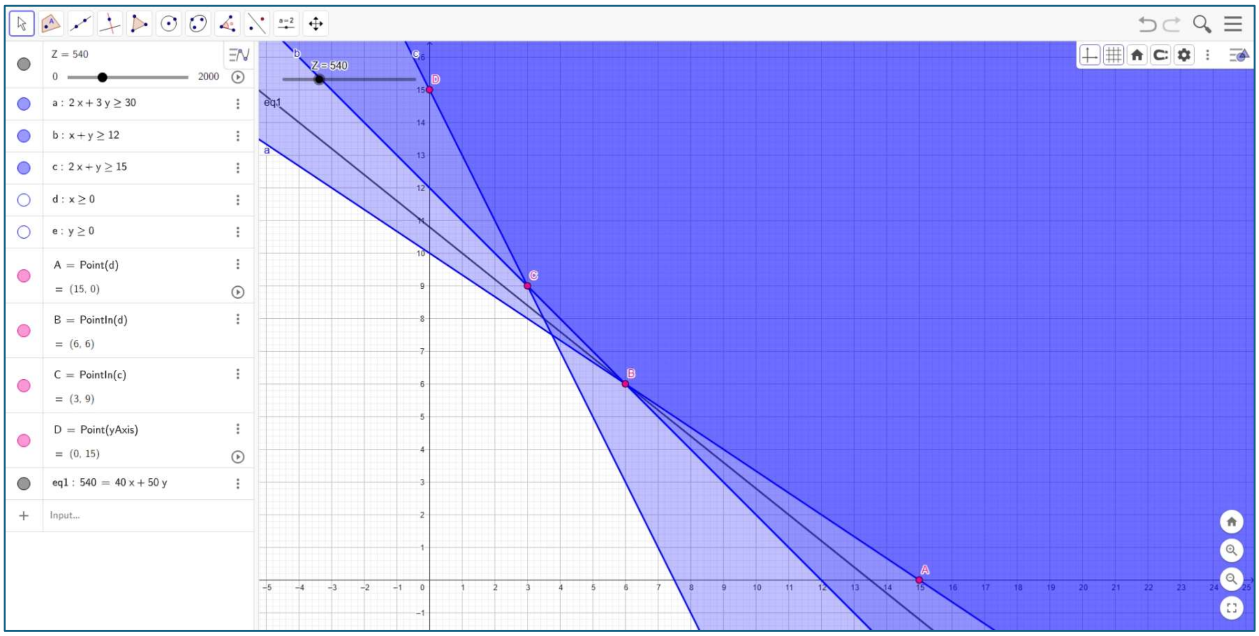
The decision shall be made to minimize Z are when $x_1 = 7.5$ and $x_2 = 5$; graphically, this is where point C intersects within the feasible region that would give $Z = 550$, this is relatively lowest amount as compared to point A or B.

2. How does the optimal solution change if the objective function is changed to $Z = 40x_1 + 70x_2$



As each unit of x_2 increases, it became more valuable in Z from $40x_1 + 50x_2$ to $40x_1 + 70x_2$ thus the line gets way steeper with respect to x_2, that is why we need at least $Z = 600 = 40(15) + 70(0)$ to achieve minimum amount of Z given $x_1 = 15$ and $x_2 = 0$ units respectively.

3. How does the optimal solution change if the third functional constraints are changed to $2x_1 + x_2 \geq 15$



As we go back to the original $Z = 40x_1 + 50x_2$ but the third functional constraint were changed, the entire feasible region or solution has also been altered, such that it gives us 4 possible decisions due to the new premise that the third constraints had to offer. The minimum amount of Z that satisfies all constraints shall be $Z = 540$ where the decisions are $x_1 = 6$ and $x_2 = 6$.