SA2-Part 1

Espiritu, Joseph Raphael; Clores, Harneyyer Leosara

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1.Data Gathering and Info

- **Kaggle Data** (new link with same OHLC) (outdated previous link) (https://www.kaggle.com/datasets/mczielinski/bitcoin-historical-data)
  - Columns are **Timestamp**, **Open**, **High**, **Low**, **Close**, **Volume**
  - Timestamp (**Start time of time window (60s window), in Unix time**)
  - Open (**Open price at start time window**)
  - High (**High price at start time window**)
  - Low (**Low price at start time window**)
  - Close (**Close price at start time window**)
  - Volume (**Volume of BTC transacted in this window**)

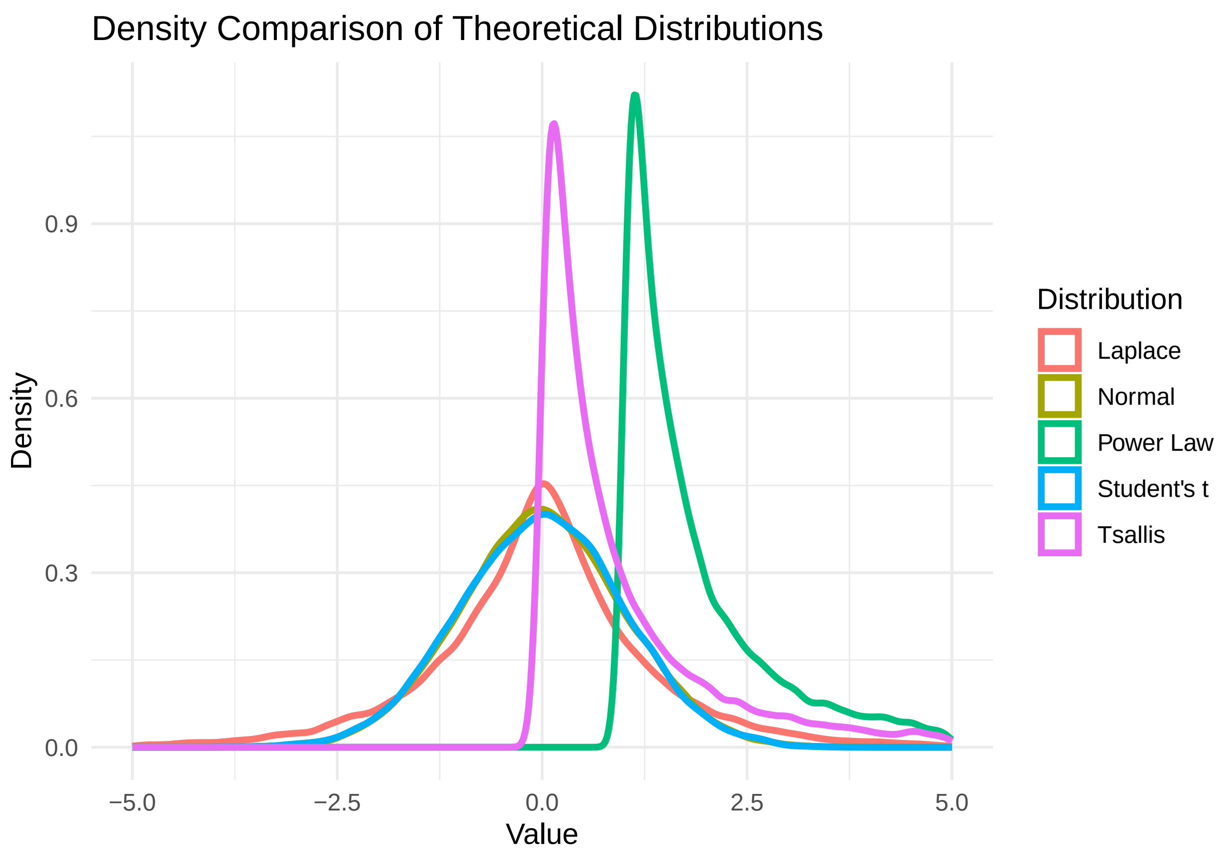
```r
library(lubridate)   # Unix Data easier
library(tidyverse)   # Data analysis package
library(tsallisqexp) # Tsallis
library(poweRlaw)    # power Law
library(VGAM)        # Lapalce
library(patchwork)   # Combining Graphs

#1. Load and inspect the data
bitstamp_data <- read.csv("btcusd_1-min_data.csv")
summary(bitstamp_data)
```

2. Processing Data and Graphing Distribution Tests

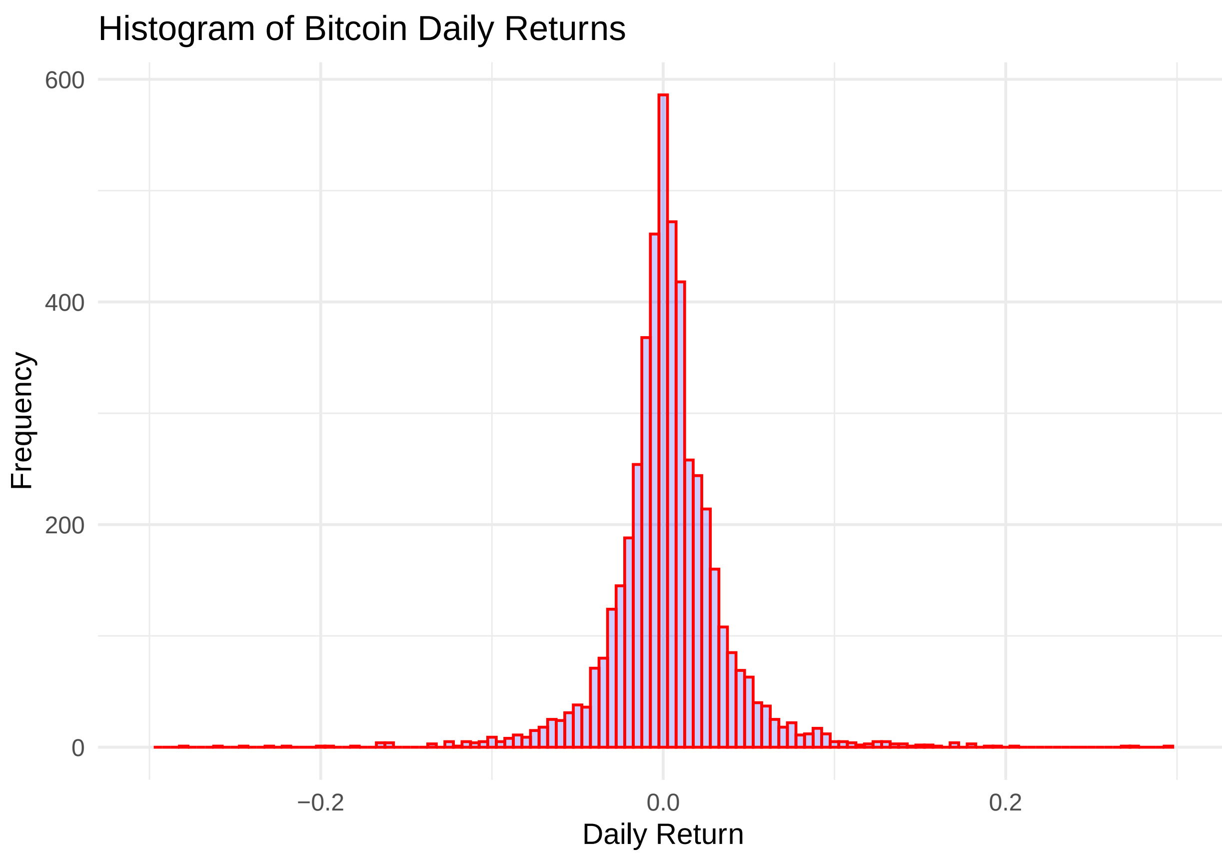
```
##    Timestamp              Open              High               Low
##  Min.   :1.325e+09   Min.   :     3.8   Min.   :     3.8   Min.   :     3.8
##  1st Qu.:1.431e+09   1st Qu.:   425.5   1st Qu.:   425.7   1st Qu.:   425.3
##  Median :1.536e+09   Median :  6630.8   Median :  6635.0   Median :  6627.0
##  Mean   :1.536e+09   Mean   : 17660.3   Mean   : 17667.3   Mean   : 17653.1
##  3rd Qu.:1.642e+09   3rd Qu.: 27550.0   3rd Qu.: 27555.0   3rd Qu.: 27544.0
##  Max.   :1.747e+09   Max.   :109111.0   Max.   :109356.0   Max.   :108794.0
##      Close              Volume
##  Min.   :     3.8   Min.   :   0.000
##  1st Qu.:   425.5   1st Qu.:   0.018
##  Median :  6630.7   Median :   0.465
##  Mean   : 17660.3   Mean   :   5.288
##  3rd Qu.: 27550.0   3rd Qu.:   3.022
##  Max.   :109036.0   Max.   :5853.852
```

```r
#2. Clean and preprocess the data
bitstamp_data <- bitstamp_data %>%
  drop_na(Close) %>%                                  # Remove rows with missing Close values
```

```r
  mutate(Date = as.Date(as.POSIXct(Timestamp, origin = "1970-01-01", tz = "UTC"))) %>%
  replace_na(list(Open = 0, High = 0, Low = 0, Close = 0, Volume = 0))  # Replace NAs in numeric column

# 3. Aggregate to daily level and calculate daily returns
daily_data <- bitstamp_data %>%
  group_by(Date) %>%
  summarise(
    Low = min(Low, na.rm = TRUE),
    High = max(High, na.rm = TRUE),
    .groups = "drop"
  ) %>%
  mutate(
    DailyMid = (Low + High) / 2,
    Return = (DailyMid / lag(DailyMid)) - 1
  ) %>%
  drop_na(Return)

# Store returns separately
returns <- daily_data$Return

# Sample size
sampleSize <- 20000
# Generate synthetic data from each distribution
set.seed(12345)  # for reproducibility

normal_sample    <- rnorm(sampleSize, mean = 0, sd = 1)
student_sample   <- rt(sampleSize, df = sampleSize - 2)
laplace_sample   <- rlaplace(sampleSize, location = 0, scale = 1)  # Laplace from VGAM
tsallis_sample   <- rtsal(sampleSize, shape = 1.5, scale = 1)      # Adjust shape/scale as needed
powerlaw_sample  <- rplcon(sampleSize, xmin = 1, alpha = 2.5)      # Must be > xmin

# Combine into one dataframe for ggplot
plot_data <- bind_rows(
  tibble(value = normal_sample, Distribution = "Normal"),
  tibble(value = student_sample, Distribution = "Student's t"),
  tibble(value = laplace_sample, Distribution = "Laplace"),
  tibble(value = tsallis_sample, Distribution = "Tsallis"),
  tibble(value = powerlaw_sample, Distribution = "Power Law")
)

# Plot all density curves on one plot
dist_plot <- ggplot(plot_data, aes(x = value, color = Distribution)) +
  geom_density(linewidth = 1.2) +
  labs(
    title = "Density Comparison of Theoretical Distributions",
    x = "Value",
    y = "Density"
  ) +
  theme_minimal() +
  xlim(-5, 5)

dist_plot
```




```r
# Plot 2: Bitcoin Returns Histogram
hist_plot <- ggplot(daily_data, aes(x = Return)) +
  geom_histogram(
    binwidth = 0.005,
    fill = "blue",
    color = "red",
    alpha = 0.2
  ) +
  labs(
    title = "Histogram of Bitcoin Daily Returns",
    x = "Daily Return",
    y = "Frequency"
  ) +
  xlim(-0.3, 0.3) +
  theme_minimal()

hist_plot
```



3. **Kolmogorov–Smirnov (K–S) Test: Explanation and Its Application to Bitcoin Returns**

- **Purpose of the K–S Test**
  - The Kolmogorov–Smirnov test is a **non-parametric test** that compares the **empirical distribution function (EDF)** of observed data with a **theoretical cumulative distribution function (CDF)**.

  - It calculates the **maximum distance (D-statistic)** between these two distributions.

  - A **smaller D value** suggests that the theoretical distribution is a good fit for the data.
- **Why Use the K–S Test on Bitcoin Returns?**
  - Bitcoin returns are known to exhibit **fat tails**, **volatility clustering**, and **extreme values**.

  - The K–S test is ideal for identifying whether known theoretical distributions (e.g., Normal, Student's t) can adequately model these returns.

Distributions Used in the K–S Test

1. **Normal Distribution**
   - Symmetrical bell-shaped curve; defined by its **mean** and **standard deviation**.

   - Assumes returns are independent and identically distributed (i.i.d.).

   - **Limitation**: Does **not model extreme events well**.
2. **Student's t-Distribution**
   - Similar to the normal distribution, but with **heavier tails**, depending on the **degrees of**

freedom n - 1.

- More suitable for data with **outliers** or **small sample sizes**.

3. **Laplace Distribution**
   - Also called **double exponential distribution**; has a **sharper peak** and **heavier tails** than the normal.
   - Well-suited for modeling **sudden jumps and frequent moderate shocks**.
4. **Tsallis Distribution**
   - Arises from **non-extensive statistical mechanics**; introduces a **q-parameter** that governs the tail thickness.
   - Designed for **systems with long-range dependencies**, like financial markets.
5. **Power Law Distribution**
   - Heavy-tailed distribution following a decay.
   - Captures **extreme rare events**—commonly used in finance and risk modeling.

```r
# Sample size
n <- length(returns)

# --- Normal distribution ---
normal_sample <- rnorm(n, mean = mean(returns), sd = sd(returns))
ks_normal <- ks.test(returns, normal_sample)
```

4. KS Test and D-Values from Bitcoin Data

```
## Warning in ks.test.default(returns, normal_sample): p-value will be approximate
## in the presence of ties
```

```r
# --- Student's t distribution ---
# Degrees of freedom = n - 2
student_sample <- rt(n, df = n - 2)
# Scale and shift to match mean and sd of returns
student_sample <- student_sample * sd(returns) + mean(returns)
ks_student <- ks.test(returns, student_sample)
```

```
## Warning in ks.test.default(returns, student_sample): p-value will be
## approximate in the presence of ties
```

```r
# --- Laplace distribution (VGAM) ---
# VGAM::rlaplace uses location (mean) and scale (b)
# Scale b = sd / sqrt(2)
laplace_scale <- sd(returns) / sqrt(2)
laplace_sample <- rlaplace(n, location = mean(returns), scale = laplace_scale)
ks_laplace <- ks.test(returns, laplace_sample)
```

```
## Warning in ks.test.default(returns, laplace_sample): p-value will be
## approximate in the presence of ties
```

```r
# --- Tsallis distribution ---
# Generate random sample from Tsallis with parameters mean and sd from data
tsallis_sample <- rtsal(n, mean(returns), sd(returns))
ks_tsallis <- ks.test(returns, tsallis_sample)
```

```
## Warning in ks.test.default(returns, tsallis_sample): p-value will be
```

```
## approximate in the presence of ties
```

```r
# --- Power Law distribution ---
# Generate random sample from Power Law with parameters (xmin = -0.3, alpha = sd)
powerlaw_sample <- rplcon(n, xmin = -0.3, alpha = sd(returns))
ks_powerlaw <- ks.test(returns, powerlaw_sample)
```

```
## Warning in ks.test.default(returns, powerlaw_sample): p-value will be
## approximate in the presence of ties
```

```r
# Print KS test results
ks_normal
```

```
##
##  Asymptotic two-sample Kolmogorov-Smirnov test
##
## data:  returns and normal_sample
## D = 0.10031, p-value < 2.2e-16
## alternative hypothesis: two-sided
```

```r
ks_student
```

```
##
##  Asymptotic two-sample Kolmogorov-Smirnov test
##
## data:  returns and student_sample
## D = 0.10051, p-value < 2.2e-16
## alternative hypothesis: two-sided
```

```r
ks_laplace
```

```
##
##  Asymptotic two-sample Kolmogorov-Smirnov test
##
## data:  returns and laplace_sample
## D = 0.045445, p-value = 8.308e-05
## alternative hypothesis: two-sided
```

```r
ks_tsallis
```

```
##
##  Asymptotic two-sample Kolmogorov-Smirnov test
##
## data:  returns and tsallis_sample
## D = 0.99447, p-value < 2.2e-16
## alternative hypothesis: two-sided
```

```r
ks_powerlaw
```

```
##
##  Asymptotic two-sample Kolmogorov-Smirnov test
##
## data:  returns and powerlaw_sample
## D = 0.7955, p-value < 2.2e-16
## alternative hypothesis: two-sided
```

5. Interpretation and Results

- **Normal Distribution**

The KS test shows a moderate statistic (D = 0.10031) but a highly significant p-value (< 2.2e-16), strongly rejecting the hypothesis that Bitcoin returns follow a normal distribution. This aligns with financial data characteristics, which usually display heavy tails and volatility clustering not captured by normality.

- **Student's t Distribution**
  With a KS statistic similar to the normal (D = 0.10051) and a very low p-value (< 2.2e-16), the Student's t distribution with default parameters still fails to model Bitcoin returns well, despite its heavier tails.
- **Laplace Distribution**
  The Laplace distribution shows the *smallest KS statistic* (D = 0.045445) among all tested distributions and although the p-value (8.308e-05) rejects perfect fit, it provides the *closest approximation* to the empirical returns distribution.
  This suggests that the Laplace's heavier tails effectively capture the sharp jumps and large fluctuations typical in Bitcoin returns.
- **Tsallis Distribution**
  The very large KS statistic (D = 0.99447) and highly significant p-value indicate that the Tsallis distribution does not fit the Bitcoin returns well with the current parameters.
- **Power Law Distribution**
  Similarly, the high KS statistic (D = 0.7955) and low p-value demonstrate a poor fit. Power law may capture tail behavior but not the full distribution.

Summary None of the tested **theoretical distributions perfectly fit Bitcoin daily returns**, but the Laplace distribution provides the *best practical approximation*, capturing heavy tails and jump behavior better than others.

```r
# Fit Laplace parameters
mu <- mean(returns, na.rm = TRUE)
b <- sqrt(var(returns, na.rm = TRUE) / 2)  # scale parameter for Laplace

# Create x values for Laplace density
x_vals <- seq(min(returns, na.rm = TRUE), max(returns, na.rm = TRUE), length.out = 1000)
laplace_density <- dlaplace(x_vals, location = mu, scale = b)

# Binwidth for histogram
binwidth <- 0.005

# Extend breaks to cover all data
min_break <- floor(min(returns, na.rm = TRUE) / binwidth) * binwidth
max_break <- ceiling(max(returns, na.rm = TRUE) / binwidth) * binwidth

# Calculate counts for scaling
counts <- hist(returns, breaks = seq(min_break, max_break, by = binwidth), plot = FALSE)$counts
scale_factor <- length(returns) * binwidth

# Prepare Laplace line data frame explicitly
laplace_df <- data.frame(
  x = x_vals,
  y = laplace_density * scale_factor
)

ggplot(data = data.frame(returns), aes(x = returns)) +
```

```r
geom_histogram(binwidth = binwidth, fill = "blue", color = "red", alpha = 0.2) +
geom_line(data = laplace_df, aes(x = x, y = y), color = "darkgreen", linewidth = 1.2) +  # specify da
labs(
  title = "Histogram of Bitcoin Daily Returns with Laplace Distribution Overlay",
  x = "Daily Return",
  y = "Frequency"
) +
xlim(-0.3, 0.3) +
theme_minimal()
```

6. Showcasing Overlay of Histogram and Laplace to see the Practical Approximation

```
## Warning: Removed 1 row containing non-finite outside the scale range
## (`stat_bin()`).

## Warning: Removed 2 rows containing missing values or values outside the scale range
## (`geom_bar()`).

## Warning: Removed 83 rows containing missing values or values outside the scale range
## (`geom_line()`).
```