D.A.R.T Collision Simulation Analysis Report

by

Josh Farkas
Kyle Williams
Nora Osmanova

Fundamentals Of Computer Simulation Theory And Application

Problem

Asteroids pose a serious threat to our planetary system. With this concern in mind, there are already planetary defense systems in the works. In 2022, NASA's Double Asteroid Redirection Test(DART) successfully collided with an asteroid and redirected its trajectory.

Our model attempts to visualize this process and analyze results. This simulation incorporated orbital mechanics, elastic collisions mechanics and conservation of momentum checks to verify the model. The simulation also attempted to do sensitivity analysis of the DART itself by varying its speed and mass. The results showed that it was a lot more difficult to simulate orbital mechanics and our model did not successfully predict the asteroid's trajectories.

Model Design

Simplifying Assumptions:

We assume that the solar system is a closed system not affected by any objects or phenomenon outside of it.

We initially restricted our model to 2D but switched to 3D after logical errors.

We assumed that the only forces acting on these objects are gravity and kinetic collisions, and that all objects are perfect spheres. The model was constrained by the current most accurate measurements of planets and asteroids.

We only used Newtonian physics, and ignored things like General Relativity.

Objects were indestructible, so collisions only transfer energy between them and not destroy them. All asteroids spawned with direction towards Earth.

Our model also assumed DART targets the asteroids perfectly. All these assumptions were made with the rationale that these assumptions will make the model run faster while being constrained to the law of physics.

How the Model Works:

Simulation data consists of Sun as the center and treated as a planet, 8 other planets, small, medium, large sized asteroids, DART spacecraft

Gravity: Every object obeys Newton's gravity law that every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of distances between their centers.

Time Steps: We utilized the runge-kutta 4 timestepping method. While much more accurate than Euler's method, it is still prone to error. Analysis was performed with a 60 delta and animations were produced with a 60,000 delta.

Collisions: Collisions were tracked for analysis but were not considered for updating trajectories of bodies. Asteroids were deployed in random locations around the sun, making it difficult to achieve asteroid to earth collisions.

Tunable and end Parameters:

dt, collision_elasticity, dart_mass, dart_speed, dart_distance, num_small(asteroids), num_medium(asteroids), num_large(asteroids), asteroid_distance_mean, asteroid_distance_SD, asteroid_speed_mean, asteroid_speed_SD, asteroid_radius_small, asteroid_mass_small, asteroid_radius_medium, asteroid_radius_large, asteroid_mass_large, small_detection(asteroids), medium_detection(asteroids), large_detection(asteroids), duration, seed, mass_multi, vel_multi

Metrics: Conservation of momentum, conservation of energy, dart mass vs interception rate of asteroids, position of planets (mercury).

Model Solution:
How it is built: Our software is built using four key modules, analysis, body, model, and animation. The model produces a list of lists, the elements of the outer list represent time steps and the elements of the inner lists are body objects. At each time step, the body module updates the body's new location using a runge-kutta time stepping system and a net acceleration formula. Analysis and animation accept this list of list and perform analytical studies and animations.
How it runs: You can either run the program by running the analysis file with the appropriate functions being called, or you can run the model file to simply produce a model and create an animation.
Equations:
Gravitational Force: $F = G m_1 \; m_2 \frac{1}{r^2}, G = 6.674 * (10 ** (-11)$

Kinetic Energy: $KE = \frac{1}{2} mv^2$
Potential Energy: $PE = M * g * H$

Net Acceleration: $\vec{F}_{12} = G \frac{m_1 m_2}{r^2} \hat{r}_{12}$
Force: *F* = *ma*

Runge-Kutta: $$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$
$$t_{n+1} = t_n + h$$

$$
\begin{aligned}
k_1 &= f(t_n, y_n), \\
k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right) \\
k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right) \\
k_4 &= f(t_n + h, y_n + hk_3)\,.
\end{aligned}
$$

Verification: We verified our results by implementing kinetic energy and potential energy equations and expected conservation of momentum and energy to be mostly unchanged.

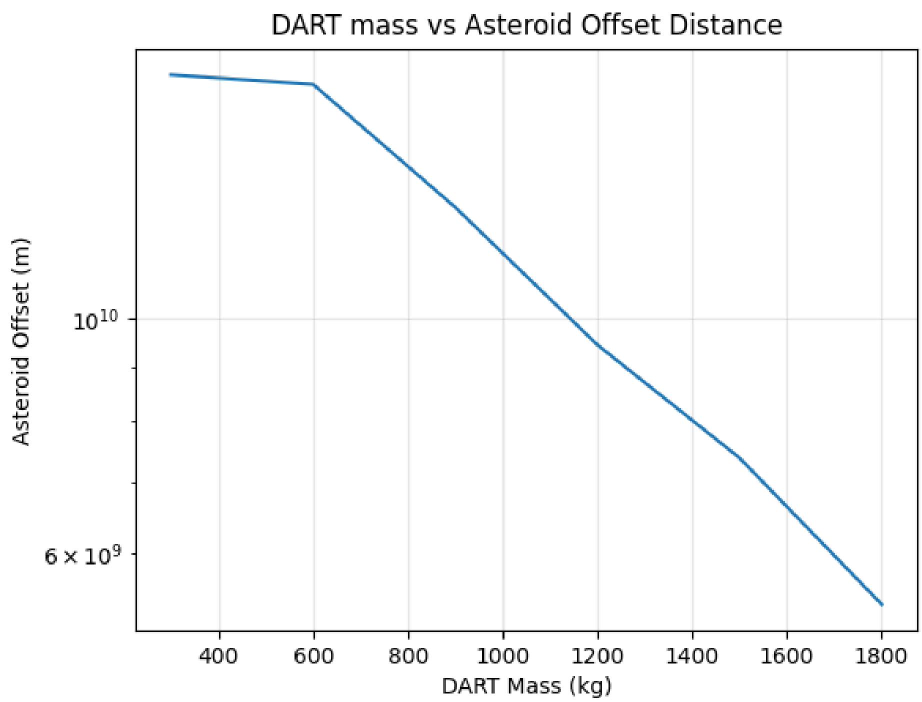
Results and Conclusion

Our results showed us that the simulation gravitational model is a lot more complex. The initial intent to do sensitivity analysis with DART did not give the results expected. Since DART always targets the asteroids, changing the spacecraft's speed or mass did not yield any results and interception rate ended up staying constant at 0. The model was also not able to predict an asteroid's path or have it collide with Earth, so our protection rate was always at 100%.

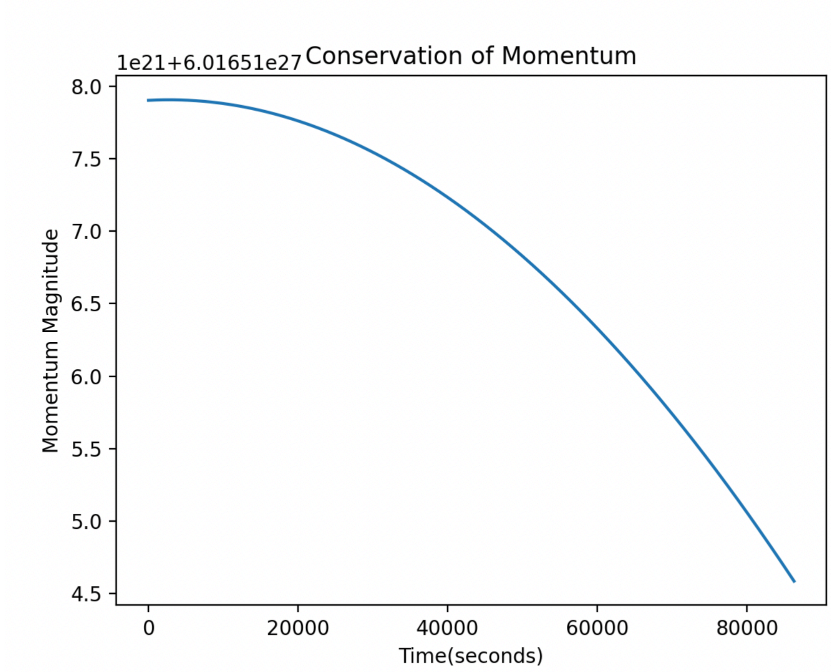
Energy Check: Total energy of the system was calculated at a timestep by summing up the KE and PE for all bodies at the moment of collision. This result was plotted over time and the result gave us the expected flat line with relative error of $\approx -\ 4.66e^{-15}$. This means the energy was conserved, further proving the fact that in a closed system with no outside forces, energy can't be created or destroyed.

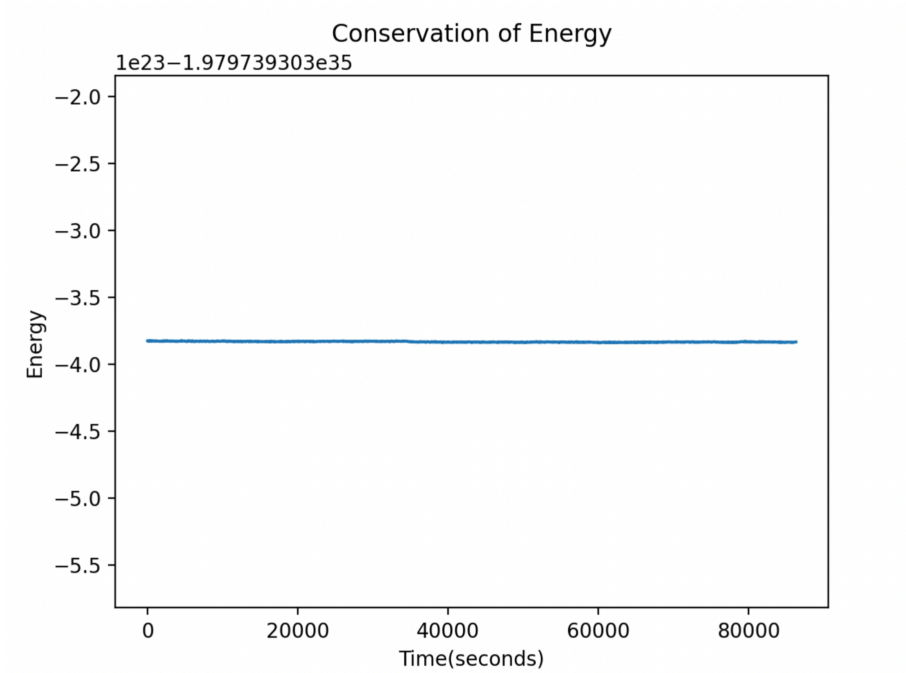
Momentum Check: Momentum was calculated by summing the momentum of all bodies involved in the collision. Momentum was calculated by using the directional x and y vector and plot magnitude of them over time. This also gave us the expected result of momentum conservation with relative error of $\approx\ 5.51e^{-07}$.

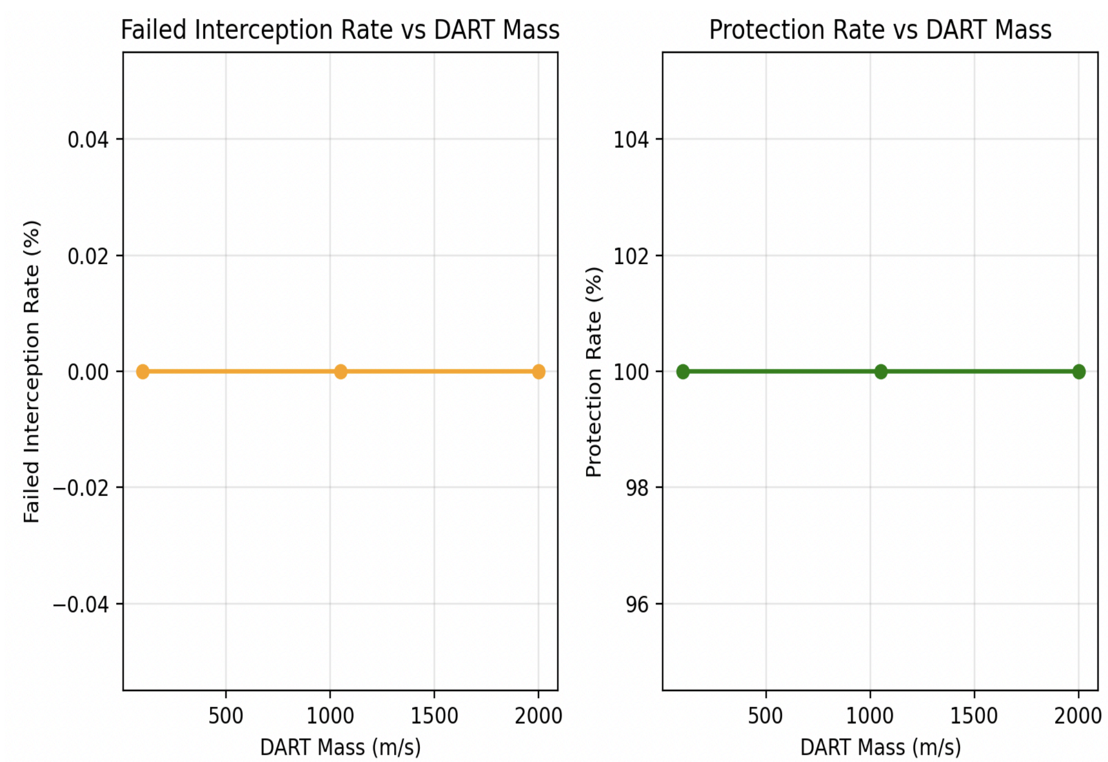
For both of these checks the graphs are provided below.

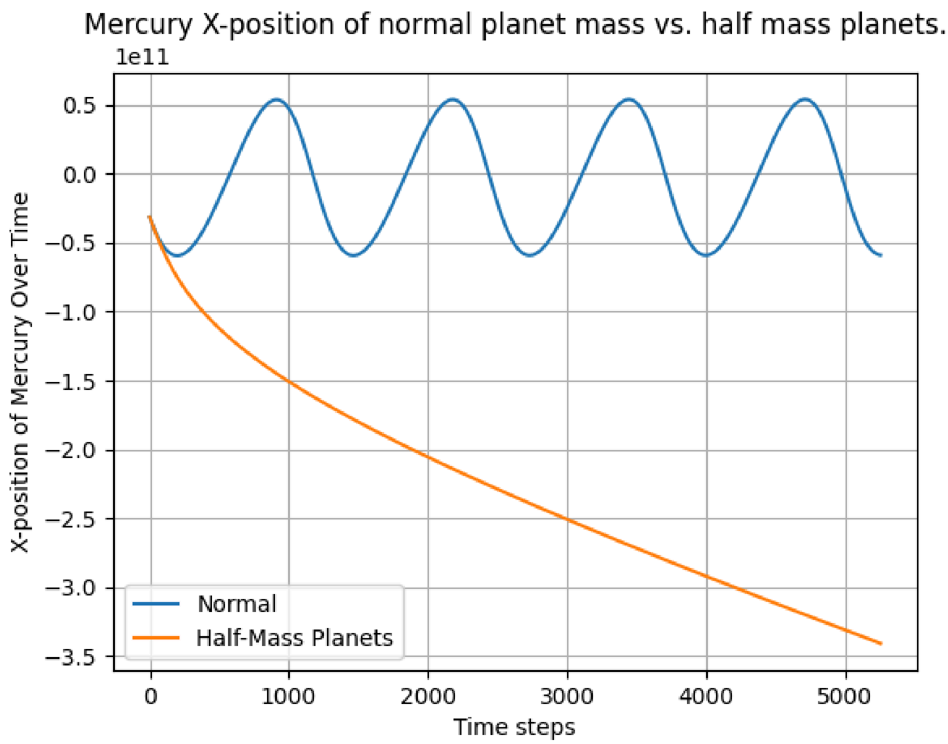

DART mass vs Asteroid Offset Distance
Asteroid Offset (m)
10^{10}
6×10^9
400
600
800
1000
1200
1400
1600
1800
DART Mass (kg)

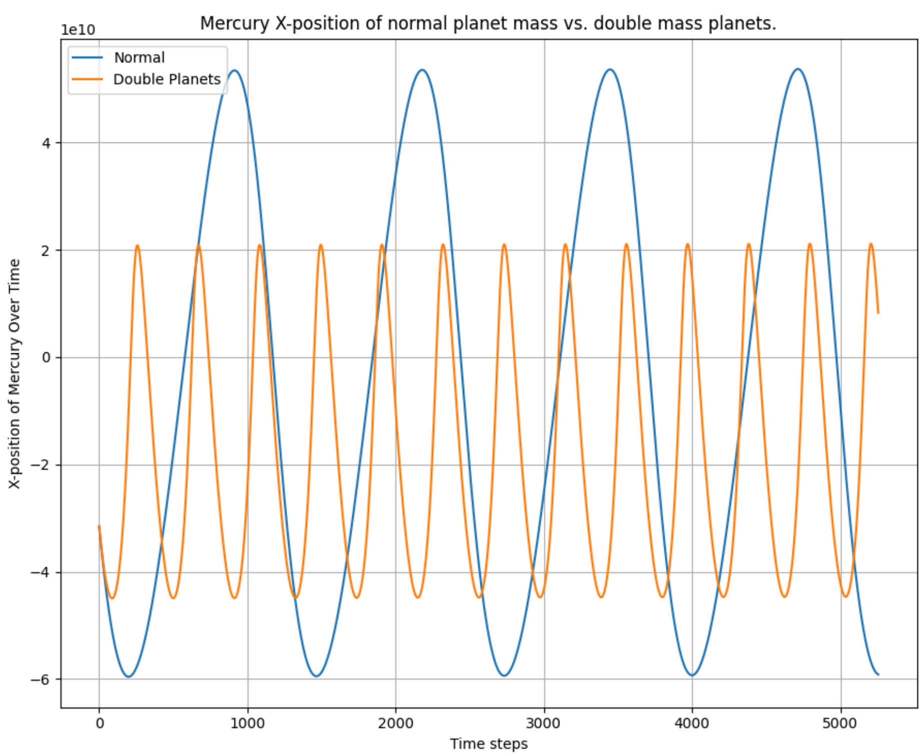
Graphs:

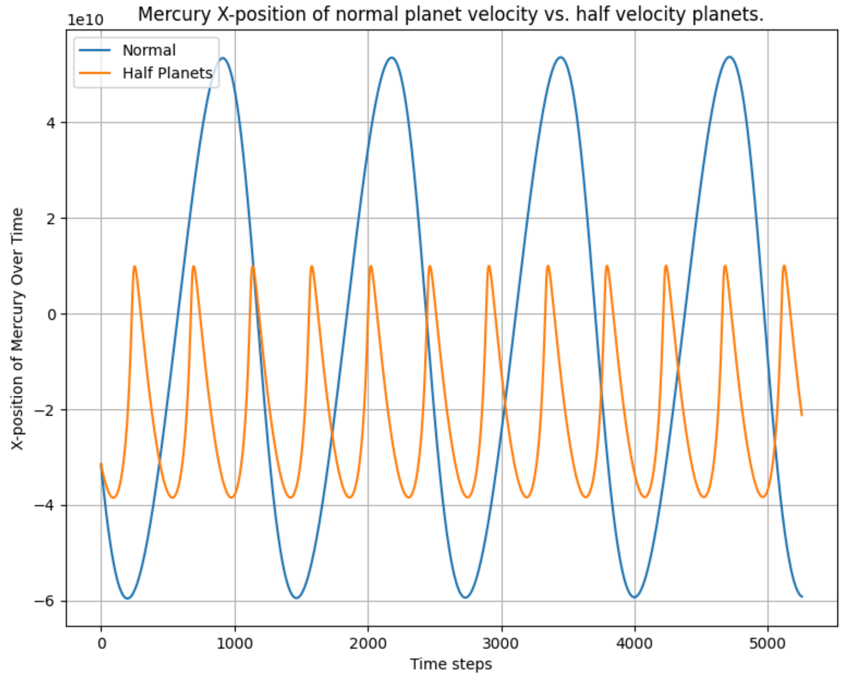

1e21+6.01651e27
Conservation of Momentum
Momentum Magnitude
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
0
20000
40000
60000
80000
Time(seconds)

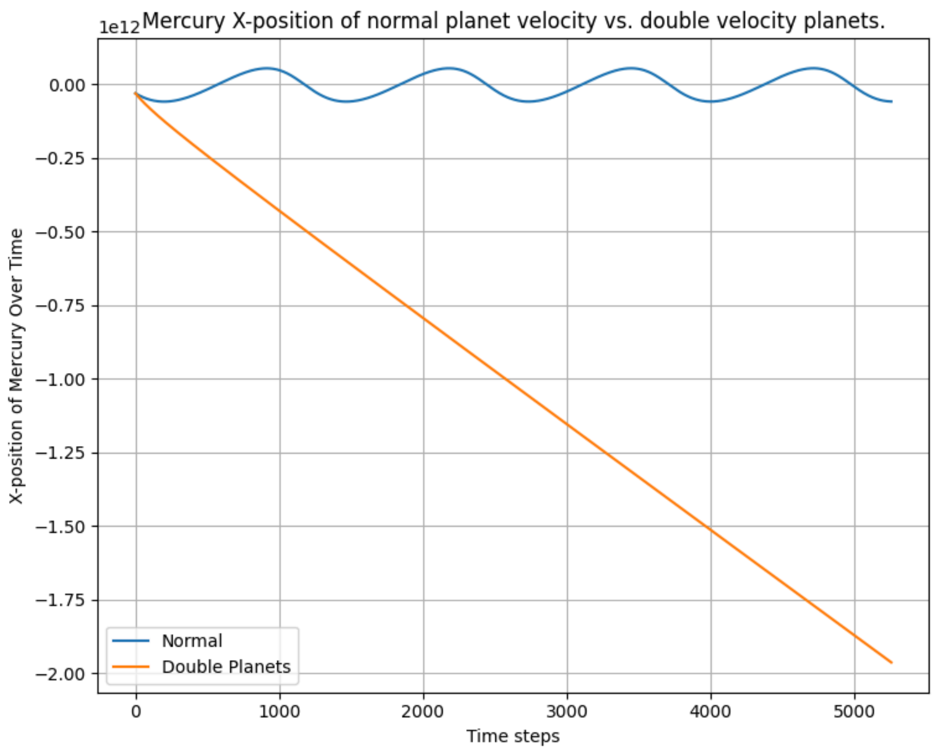

Conservation of Energy
1e23−1.979739303e35
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
Energy
0
20000
40000
60000
80000
Time(seconds)

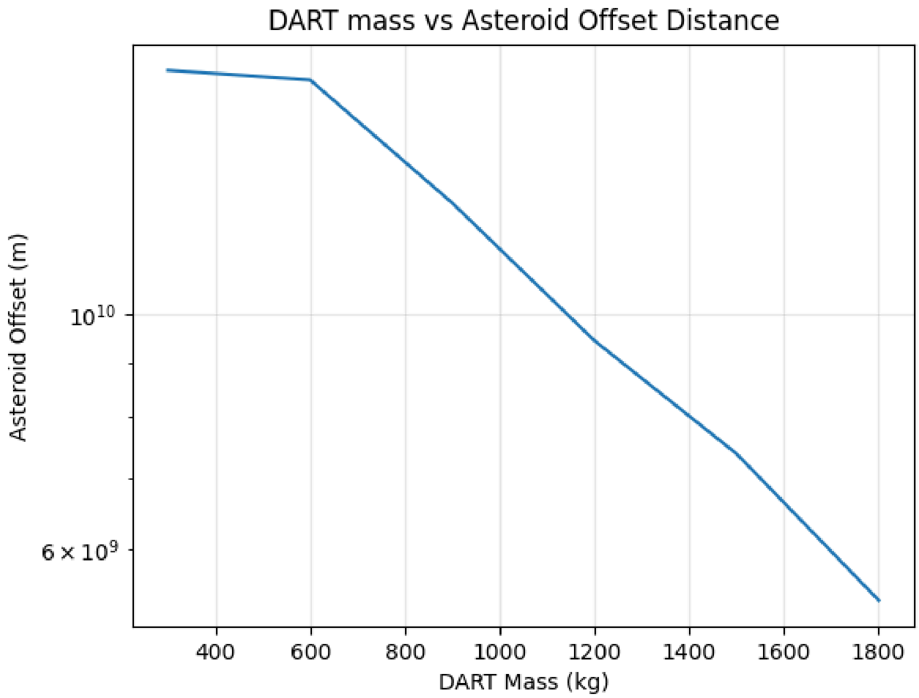

Failed Interception Rate vs DART Mass
0.04
0.02
0.00
−0.02
−0.04
Failed Interception Rate (%)
500
1000
1500
2000
DART Mass (m/s)
Protection Rate vs DART Mass
104
102
100
98
96
Protection Rate (%)
500
1000
1500
2000
DART Mass (m/s)

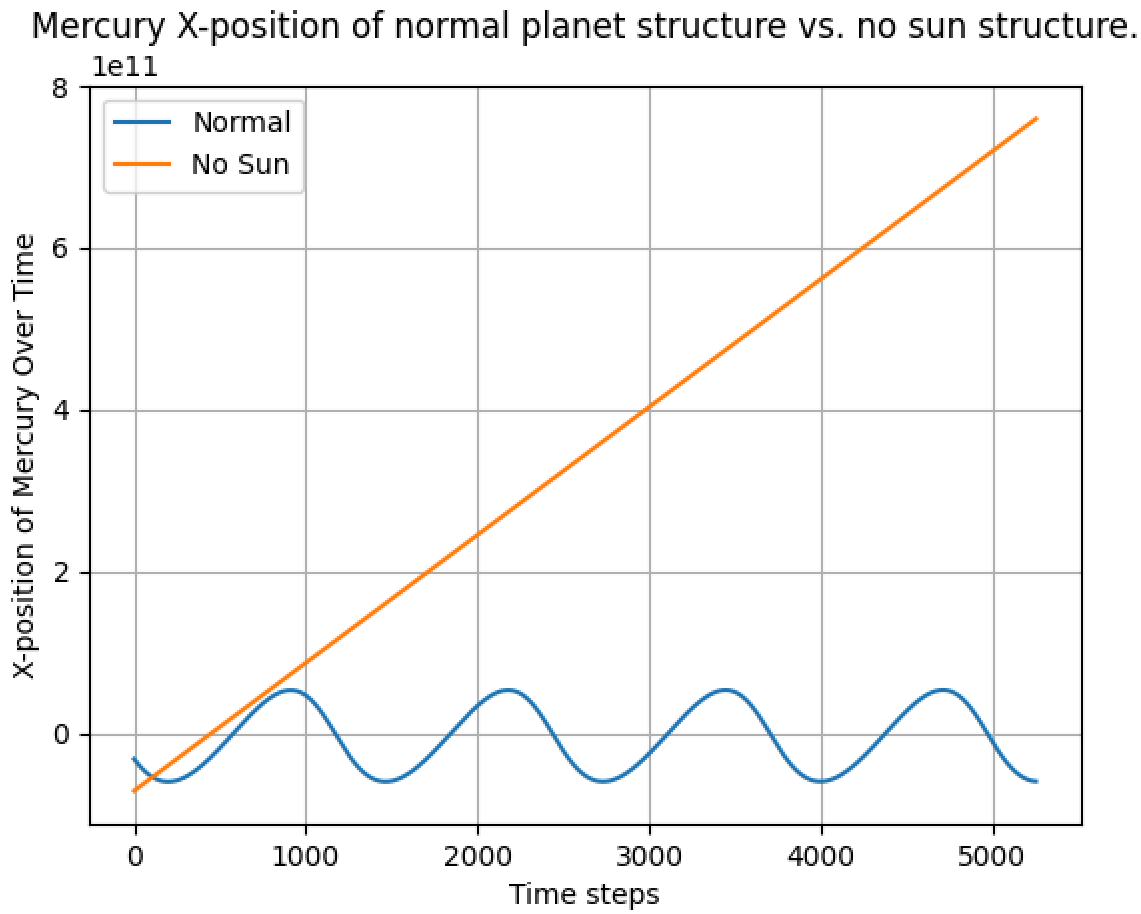

Mercury X-position of normal planet mass vs. half mass planets.
1e11
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
X-position of Mercury Over Time
Normal
Half-Mass Planets
0
1000
2000
3000
4000
5000
Time steps

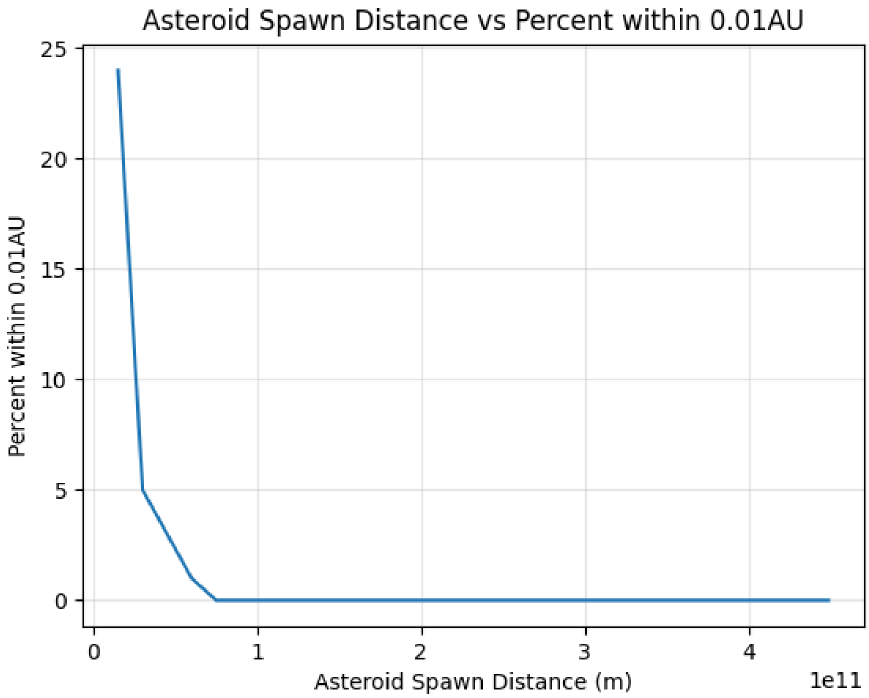

Mercury X-position of normal planet mass vs. double mass planets.
1e10
Normal
Double Planets
4
2
0
−2
−4
−6
X-position of Mercury Over Time
0
1000
2000
3000
4000
5000
Time steps


Mercury X-position of normal planet velocity vs. half velocity planets.
1e10
Normal
Half Planets
X-position of Mercury Over Time
Time steps
4
2
0
−2
−4
−6
0
1000
2000
3000
4000
5000


Mercury X-position of normal planet velocity vs. double velocity planets.
1e12
Normal
Double Planets
X-position of Mercury Over Time
Time steps
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
−2.00
0
1000
2000
3000
4000
5000


DART mass vs Asteroid Offset Distance
Asteroid Offset (m)
10^{10}
6×10^9
400
600
800
1000
1200
1400
1600
1800
DART Mass (kg)


Mercury X-position of normal planet structure vs. no sun structure.
1e11
8
6
4
2
0
Normal
No Sun
X-position of Mercury Over Time
0
1000
2000
3000
4000
5000
Time steps




```
Mercury's position after 24 hours in kilometers:
[-3.526395e+07, 3.443029e+07, 6.091106e+06]
Mercury's expected position after 24 hours in kilometers:
[-3.526394e+07, 3.443028e+07, 6.091105e+06]
Distance difference between calculated and real: 9.098470e+00
```

What results mean

Our system did not have a 0-change in conservation of momentum, meaning that there was an instability in momentum. We believe this to be caused by not calculating collision trajectory changes.

We had a 0-change in energy conservation. This means that energy did not change throughout the simulation, which is a realistic result.

We detected a non-correlation between DART mass and interception rate of DARTs to asteroids. This was due to mass not correlating with interception and interception being 100% to begin with.

Halving the mass off all planets including the sun caused mercury to fly away in a near straight line away from the sun. This implies the mass of the planets has a great effect on orbital stability (particularly the sun).

Doubling the mass of all planets including the sun caused mercury to orbit the sun in a tighter radius with decreased periodicity. We interpret this as there being a correlation between mass and orbital distance.

Halving the velocity of all planets including the sun caused mercury to again orbit the sun in a tighter radius with decreased periodicity. We interpret this as there being a correlation between initial velocity and orbital distance.

When doubling the planet's velocity including the sun, mercury immediately flew in a near straight line away from the sun. We interpret this as mercury reaching escape velocity.
As dart mass increased, collisions with an asteroid resulted in the asteroid having a closer resultant distance to its non-collision end distance.
Removing the sun from our system caused mercury to fly in a straight line out of the system. We interpret this as there not being enough net force in the system to affect mercury's velocity without a central sun structure.
The number of asteroids in our system that spawned further away from the sun exponentially decreases. This is a result of our asteroid spawning methodology.

Problems

We ran into problems implementing runge-kutta timestepping. After finely tuning it we were able to get it into an acceptable range, but there is still some level of error. The level of error is relatively minimal, we experienced a 9 kilometer positional difference of Mercury after a 24 earth hour cycle.
Having asteroids strike earth while maintaining random asteroid generation proved to be a problem. We wanted to maintain a natural field of asteroids, but couldn't figure out a way to influence them to strike earth while maintaining system integrity.
Analysis was slow due to an inefficient system involving making deep copies of objects many thousands of times. This made it difficult to troubleshoot and adjust analyses quickly.

Future Work

Including updated trajectories based on collisions. When an object collides with another, each object will have the force of the collision considered in their trajectory vector.
Updating our asteroid logic to have better control of starting position, velocity, and trajectory. This would allow us to test collisions with planets and DARTs easier.

References

Park, R. (n.d.). *Horizons System*. Ssd.jpl.nasa.gov. https://ssd.jpl.nasa.gov/horizons/app.html#/

OpenStax. (n.d.). 6.5: Newton's Universal Law of Gravitation. Physics LibreTexts.

https://phys.libretexts.org/Bookshelves/College_Physics/College_Physics_1e_(OpenStax)/06%3A_Uniform_Circular_Motion_and_Gravitation/6.05%3A_Newtons_Universal_Law_of_Gravitation

OpenStax. (n.d.). Conservation of Energy. Physics LibreTexts.

https://openstax.org/books/university-physics-volume-1/pages/8-3-conservation-of-energy

OpenStax. (n.d.). Conservation of Momentum. Physics LibreTexts.

https://openstax.org/books/physics/pages/8-2-conservation-of-momentum