1 Rules for the Connectives

Exercise 2

Apply both implication rules to prove $P \implies R$ from the givens $P \implies Q$, $P \implies (Q \implies R)$.

Exercise 4

Assume that $n, m \in \mathbb{N}$.
Show: (m is odd $\wedge$ n is odd) $\implies$ $m + n$ is even.

Answer of exercise 4

- Prove that (m is odd $\wedge$ n is odd) $\implies$ $m + n$ is even when:
- Prove that $m + n$ is necessarily even if m and n are both odd.

− $n \in \mathbb{N}$

− $m \in \mathbb{N}$

⊩ { assume m and n are odd, and then derive that $m + m$ is even to demonstrate implication }

- Prove that $m + n$ is even when:

(1) m is odd

(2) n is odd

[3] { definition of odd-ness }
$\exists x \in \mathbb{Z} \mid m = 2x + 1$

[4] { definition of odd-ness }
$\exists y \in \mathbb{Z} \mid n = 2y + 1$

Exercise 5

Show:

1. From $P \iff Q$ it follows that $(P \implies R) \iff (Q \implies R)$,
2. From $P \iff Q$ it follows that $(R \implies P) \iff (R \implies Q)$.

Exercise 7

Produce proofs for:

1. Given: $P \implies Q$. To show: $\neg Q \implies \neg P$,
2. Given: $P \iff Q$. To show: $\neg P \iff \neg Q$.

Harder

Exercise 9

Show that from $(P \implies Q) \implies P$ it follows that P.

Hint: Apply Proof by Contradiction. (The implication rules do not suffice for this admittedly exotic example.)

Exercise 11

Assume that A, B, C, and D are statements.

1. From the given $A \implies B \quad \vee \quad C$ and $B \implies \neg A$, derive that $A \implies C$.

2. From the given $A \quad \vee \quad B \implies C \quad \vee \quad D$, $C \implies A$, and $B \implies D$.

Exercise 15

Show that for any $n \in \mathbb{N}$, division of n^2 by 4 gives a remainder 0 or 1.

2 Rules for the Quantifiers

Exercise 18

Show, using $\forall$-introduction and Deduction Rule: if from Γ, $P(c)$ it follows that $Q(c)$ (where c satisfies P, but is otherwise "arbitrary"), then from Γ it follows that $\forall x\colon P(x) \implies : Q(x)$.