

1. MAKING SYMBOLIC FORM EXPLICIT

Problem 2.22. Can you think of an argument showing that

Problem 2.23. Give structure trees of the following formulas:

- (1) $\forall x (A(x) \implies (B(x) \implies C(x)))$
- (2) $\exists x (A(x) \wedge B(x))$
- (3) $\exists x A(x) \wedge \exists x B(x)$

Problem 2.26. The following were rewritten to have restricted quantifiers.

- (1) $\exists x, y \in \mathbb{Q} \mid x < y$
- (2) $\forall x \in \mathbb{R} (\exists y \in \mathbb{R} \mid x < y)$
- (3) $\forall x \in \mathbb{Z} (\exists m, n \in \mathbb{N} \mid x = m - n)$

Problem 2.27. Write as formulas without restricted quantifiers:

Problem 2.31. Translate into formulas, taking care to express the intended meaning:

- (1) The equation $x^2 + 1 = 0$ has a solution.
 $\exists x \in \mathbb{C} \mid x = \pm\sqrt{-1}$
- (2) A largest natural number does not exist.
- (3) The number 13 is prime.
- (4) The number n is prime.
- (5) There are infinitely many primes.