1.5 In a strictly steady state situation, both ions & electrons will follow the Boltzmann relation

$$n_j = n_0 \exp(-q_j \phi / KT_j).$$

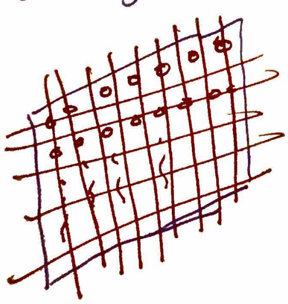
For the case of an infinite, transparent grid charged to a potential ϕ_0, show that the shielding distance is then given approximately by

$$\lambda_D^{-2} = \frac{ne^2}{\epsilon_0}\left(\frac{1}{kT_e} + \frac{1}{kT_i}\right)$$ * note $n = n_\infty$ from the text.

Show that λ_D is determined by the temperature of the colder species.

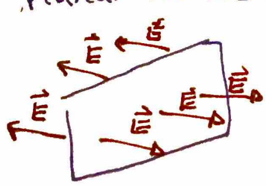
We are going to follow the procedure in the ~~to~~ book. ~~Before~~, However, I am going to "justify" the 1-D situation.

The infinite transparent grid charged to a potential has a particular symmetry.



→ the same symmetry as a parallel plate charged to a potential ϕ_0

→ & We know that parallel plates emit electric fields ~~perpendicular to the plane~~ parallel to the planar surface. Hence, 1-D situation.



We start with the Poisson equation:

$$-\nabla^2\phi = \rho/\epsilon_0$$ charge density ; $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$; $\vec{E} = -\vec{\nabla}\phi$

$-\nabla^2\phi = \rho/\epsilon_0$

What is the charge density?

$$\rho = \rho_i + \rho_e = \cancel{en_i - e}\; en_i - en_e$$

$$= e(n_i - n_e)$$

$\therefore$

$$-\epsilon_0\nabla^2\phi = e(n_i - n_e)$$

This is where we diverge from the ~~plasma~~ book; we are ~~not~~ going to assume that the ions ~~have~~ ~~do not~~ have ~~enough~~ enough time, ~~that is~~ to react to the potential, steady-state.

- $n_i \neq n_\infty = n_0$ but follow the Boltzmann relation

$$n_i = n_0 \exp(-q_i\phi(x)/KT_i)\,;\quad q_i = e \qquad Z = 1$$ * A hydrogen plasma.

$$n_e = n_0 \exp(-q_e\phi(x)/KT_e)\,;\quad q_e = -e$$

$\therefore$

$$-\epsilon_0 \frac{\partial^2}{\partial x^2}(\phi(x)) = +en_0\left[\exp(-e\phi(x)/KT_i) - \exp(e\phi(x)/KT_e)\right]$$

We assume the thermal energies are much greater than the grid-charge pontential energies; $\frac{|q_j\phi(x)|}{KT_j} \ll 1 \quad j\in\{e,i\}$

$\cdot\ e^{(x)} \simeq 1 + x + \dots \quad + \cdot\ e^{(-x)} \simeq 1 - x + \dots$

$\therefore$

$$-\epsilon_0 \frac{\partial^2}{\partial x}\left(\phi(x)\right) = e n_0 \left[\left(1 - \frac{e}{kT_i}\phi(x)\right) - \left(1 + \frac{e}{kT_e}\phi(x)\right)\right]$$

$$\therefore \quad = -e^2 n_0 \left(\frac{1}{kT_i} + \frac{1}{kT_e}\right)\phi(x)$$

$$\phi''(x) = +\frac{e^2 n_0}{\epsilon_0}\left(\frac{1}{kT_i} + \frac{1}{kT_e}\right)\phi(x)$$

$$\hookrightarrow \quad \phi(x) = \phi_0 \exp\left(-|x|/\lambda_D\right)$$

$$\boxed{\lambda_D^{-2} \equiv \frac{n_0 e^2}{\epsilon_0}\left(\frac{1}{kT_i} + \frac{1}{kT_e}\right)} \quad (I)$$

Now by inspecting (I) we can see that the colder species, ions or electrons, strongly influences the Debye length: (ions = protons)

$$T_1 < T_2 \Rightarrow \frac{1}{T_2} < \frac{1}{T_1} \quad \square$$