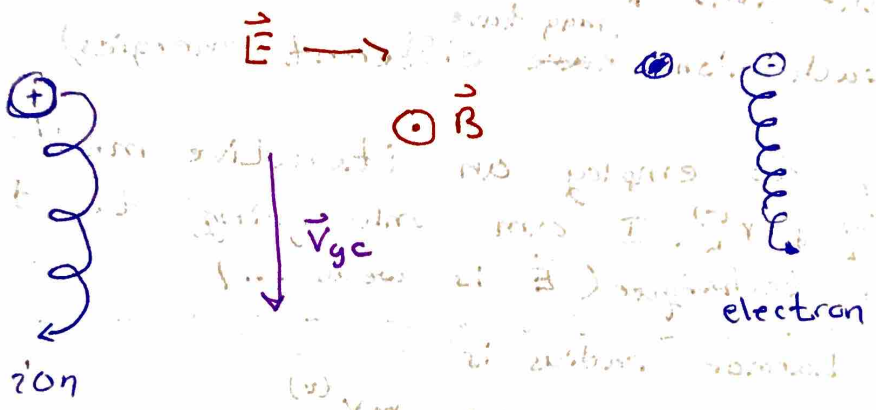
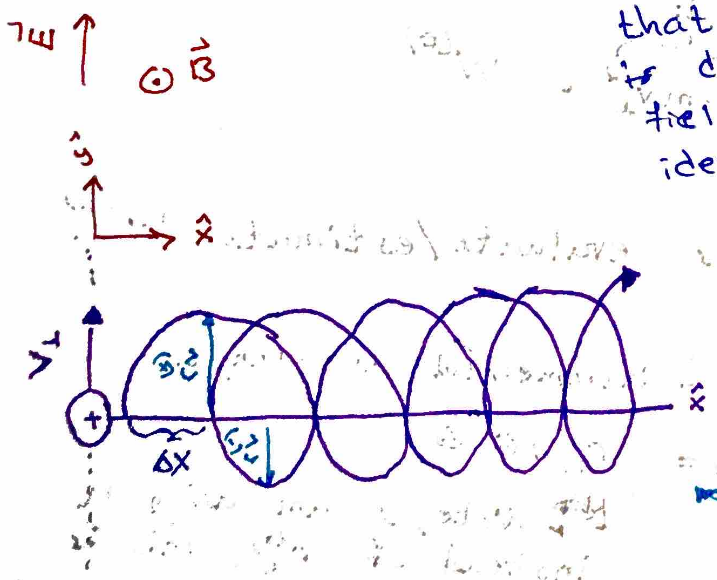
2.4] Show that v_E is the same for two ions of equal mass + charge but different energies, by using the following physical picture (see Fig 2-2).



(2-2)

Approximate the right half of the orbit by a semicircle corresponding to the ion energ after accelerating by the $\vec{E}$-field, and the left by a semicircle corresponding to the energy after ~~decl~~ deceleration. You may assume that $\vec{E}$ is weak, so that the fractional change in $v_\perp$ is small.



$r_L^{(+)}$ - radius of semicircle with accelerated $v_\perp$

$r_L^{(-)}$ - radius of semicircle with decelerated $v_\perp$.

• What I want to do is show that within one period the Δx ~~is~~ depends only ~~the~~ on the electric field + the magnetic field, for identical particles.

* Before moving on we are going to define the period as the time the ion takes to pass the x-axis with the same velocity. With the state, semicircle, model we can see that the change in position is the difference between the diameters of the semicircles.

$2r_L^{(+)} + 2r_L^{(-)}$

Some comments:

• The Coulomb force is a conservative force. This means that every time the ions reach the x-axis they have the same energy. (Each ion may have ~~have~~ different energies)

• We are going to employ an iterative method to estimate $r_L^{(+)}$ & $r_L^{(-)}$. I am only going to do one iteration. ~~this technique~~ ($\vec{E}$ is weak ...)

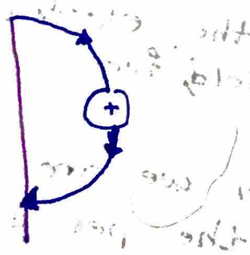
By definition the Larmor radius is

$$r_L = \frac{m v_\perp}{|q| B} \longrightarrow r_L^{(+)} = \frac{m v_\perp^{(+)}}{|q| B}$$

What we need is $v_\perp^{(+)} = v_\perp + v_\perp^{(E)}$

↳ increase/change in speed due to electric field.

Let us look at the ~~change in~~ energy at the point the ion reaches it's max y-position



$$\longrightarrow E^{(+)} = \overbrace{\frac{1}{2} m v_\perp^2}^{E^{(0)}} + W^{(E)}$$

What we need to do is evaluate/estimate $W^{(E)}$.

$$W^{(E)} = \int q\vec{E}\cdot d\vec{s}$$

; ds = incremental arc-length

$$d\vec{s} = r_L \, d\theta \, \hat{\theta}$$

$$= qE \int_0^{\pi/2} \hat{y}\cdot r_L \, d\theta \, \hat{\theta}$$

Note, I am using r_L instead of $r_L^{(+)}$. This is because I am using what is called a zeroth-order approximation to obtain a first-order approximation.

where $\hat{\theta} = -\sin(\theta)\hat{x} + \cos(\theta)\hat{y}$

$$\therefore \quad W^{(E)} = q E r_L \int_0^{\pi/2} \cos(\theta)\, d\theta = q E r_L$$

$$E^{(+)} = \frac{1}{2} m v_\perp^2 + q E r_L$$

$$= \frac{1}{2} m v_L^{(+)2}$$

$$v_L^{(+)} = v_\perp \sqrt{1 + \frac{2 q r_L E}{m v_\perp^2}}$$

~~We are going to assume~~ this ~~value is small~~

Note: $\frac{r_L}{v_L} = \frac{m}{|q| B}$

$$\therefore \quad \frac{q r_L}{m v_\perp} \left(E / v_\perp \right) = \frac{E/B}{v_\perp}$$

$$\therefore$$

$$v_L^{(+)} = v_\perp \sqrt{1 + 2 \frac{(E/B)}{v_\perp}}$$

We are going to assume this value is small (Taylor expansion)

$$v_L^{(+)} = v_\perp \left(1 + \overset{(2/2)}{1} \frac{(E/B)}{v_\perp} \right)$$

$$v_L^{(+)} = v_\perp + E/B$$

Now we can estimate $r_L^{(+)}$, recall: $r_L^{(+)} = \frac{m}{|q| B} v_L^{(+)}$

$$r_L^{(+)} = \frac{m}{|q| B} \left(v_\perp + E/B \right)$$

We are not done yet. We need $r_L^{(-)}$. To do this I am going to use the conservative force argument, that is at the start of the second semicircle the energy of the ion reverts back to $E^{(0)} = \frac{1}{2} m v_\perp^2$. This means we repeat the above approach to determine $r_L^{(-)}$; except directions change.

$$r_L^{(-)} = \frac{m}{|q|B} v_L^{(-)} ; \quad v_L^{(-)} = v_\perp - E/B$$

$$\therefore$$

$$r_L^{(-)} = \frac{m}{|q|B} (v_L - E/B)$$

Comments:

If you decide to do the ~~approach~~ show the work note that $\int_{\theta_1}^{\theta_2} \ldots d\theta$; $\theta_1 = \cancel{\pi}\ \pi_L + \theta_2 = \cancel{3\pi/2}\ 3\pi/2$

The displacement ~~in~~ of the ~~pa~~ ion is

$$\Delta x = r_L^{(+)} - r_L^{(-)} = \frac{m}{|q|B}\left[(v_L + E/B) - (v_L - E/B)\right]$$

$$\boxed{\Delta x = \frac{2mE}{|q| B^2}}$$

. This states that the displacement within one period is independent of the particles initial energy.

. The displacement represents the movement of the guiding center.

. Note we are not done.

We have the displacement, but what is the average speed of the guiding center. To determine this we need the period, τ.

$$\tau = \frac{\tau^{(+)}}{2} + \frac{\tau^{(-)}}{2}$$

→ The ion ~~speeds~~ is in the $r_L^{(+)}$ orbit for halve a period$^{(+)}$ and in the $r_L^{(-)}$ orbit for halve a period$^{(-)}$. (Based on model)

$$\tau^{(\pm)} = \frac{2\pi r_L^{(\pm)}}{v_L^{(\pm)}}$$

$$\tau^{(\pm)} = 2\pi \left(\frac{m}{|q|B} \right)$$

$\therefore$

$$\tau = \frac{2\pi m}{|q|B}$$

.... Now we can estimate ~~an average~~ the average speed of the guiding center...

$$\frac{\Delta x}{\tau} = \frac{\left[\frac{\cancel{2} \cancel{m} E}{\cancel{|q|} B^{\cancel{2}}} \right]}{\left[\frac{\cancel{2} \pi \cancel{m}}{\cancel{|q|} \cancel{B}} \right]} = \frac{E/B}{\pi}$$

$$V_{gc} = \frac{V_E}{\pi} \; ; \quad V_E = E/B$$

* Equation (2.16) from Book 2nd edition *

$$V_{gc} = \frac{V_E}{\sigma_1} \sim V_E \quad \square$$

- In plasma physics we consider order of magnitudes, hence $V_{gc} \sim V_E$.

- The guiding center of any ion is independent of its initial energy.

- Note there is no ~~ma~~ mass or charge dependence this means that all ~~particles +~~ charges have the same guiding center speed, i.e. the entire plasma moves.