Josh Bell | May 23rd 2022 | Intro to Artificial Intelligence

Written Assignment #5

1. **Consider the game represented in normal form below.**

	B: S1	B: S2	B: S3
A: S1	A = 5, B = 9	A = 3, B = 6	A = 1, B = -4
A: S2	A = 4, B = -4	A = 2, B = 2	A = -1, B = 6
A: S3	A = 0, B = 0	A = 2, B = -2	A = 0, B = 0

a. Indicate all the pure strategy Nash equilibria in the matrix.

If A chooses …

S1: B chooses S1
S2: B chooses S3
S3: B chooses Either S1 or S3

If B chooses …

S1: A chooses S1
S2: A chooses S1
S3: A chooses S1

This shows that the only pure strategy in this game is choosing (S1,S1)

b. Does Player A have a strictly dominant strategy? If yes, state what it is. If no, explain why not.

Player A does have a strictly dominant strategy, no matter what Player B chooses Player A will always go with option S1

c. Does Player B have a strictly dominant strategy? If yes, state what it is. If no, explain why not.

Player B does not have a strictly dominant strategy, their choices change depending on what Player A chooses.

d. What is the Pareto optimal outcome in this game?

The Pareto optimal outcome for this game is S1 due to the values given to Player A and B given are the highest value each player is given on the matrix.

e. Is this a zero-sum game?

This is not a zero-sum game, while sum outcomes can be seen as zero sum not all options are zero sum like S1 for A and B, A gains 5 while B gains 9 making the net profit not 0 but rather +14 which is good for the players but resulting in a non-zero-sum game.

2. Consider the game represented in normal form below.

	B: S1	B: S2
A: S1	A = 2, B = -2	A = -5, B = 5
A: S2	A = -3, B = 3	A = 4, B = -4

a. Calculate the mixed strategy Nash equilibrium for this game. Clearly indicate the probability associated with each strategy for each player.
Suppose that Player B chooses S1 with probability p and S2 with probability (1-p) and vice versa …

Player A Pure Strategy	Player B Pure Strategy
$\boldsymbol{S_1} = (2p) + \big(-3(1-p)\big)$ $= 2p + 3p - 3$ $= 5p - 3$ $\boldsymbol{S_2} = (-5p) + \big(4(1-p)\big)$ $= -5p - 4p + 4$ $= -9p + 4$ $\boldsymbol{S_1 = S_2}$ $5p - 3 = -9p + 4$ $14p = 7$ $\boldsymbol{p = \frac{1}{2}}$	$\boldsymbol{S_1} = (-2p) + \big(5(1-p)\big)$ $= -2p - 5p + 5$ $= -7p + 5$ $\boldsymbol{S_2} = (3p) + \big(-4(1-p)\big)$ $= 3p + 4p - 4$ $= 7p - 4$ $\boldsymbol{S_1 = S_2}$ $-7p + 5 = 7p - 4$ $-14p = -9$ $14p = 9$ $\boldsymbol{p = \frac{9}{14}}$

b. Calculate the expected payoffs for each player at the mixed strategy Nash equilibrium you calculated

Player A's Expected Payoff	Player B's Expected Payoff
$5p - 3$ $p = \frac{1}{2}$ $5(\frac{1}{2}) - 3$ $\frac{5}{2} - 3$ $\frac{5}{2} - \frac{6}{2}$ $\boldsymbol{-\frac{1}{2}}$	$7p - 4$ $p = \frac{9}{14}$ $7(\frac{9}{14}) - 4$ $\frac{63}{14} - 4$ $\frac{63}{14} - \frac{56}{14}$ $\boldsymbol{\frac{1}{2}}$