Josh Bell | May 1st, 2022 |Intro to Artificial Intelligence

Written Assignment #2

1. **Prove the following entailment in three different ways.**
   a. Prove that (A => ⌐B) |= ⌐ (B Λ A) with truth tables.

A	B	A => ⌐B	⌐ (B Λ A)	(A => ⌐B) |= ⌐ (B Λ A)
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T

   b. Prove that (A => ⌐B) |= ⌐ (B Λ A) with logical equivalences.

(A => ⌐B) |= ⌐ (B Λ A)	Starting Statement
⌐A V ⌐B |= ⌐ (B Λ A)	Removing Implication
⌐A V ⌐B |= ⌐B Λ ⌐A	Moving Negation Inward
⌐A V ⌐B |= ⌐A Λ ⌐B	Cumulative Law

   c. Prove that (A => ⌐B) |= ⌐ (B Λ A) with resolution algorithm.
   Proving the Negation of the statement: (B Λ A)

Number	Statement	Obtained By …
S1	⌐ (B Λ A)	Initial
S2	⌐B V ⌐A	S1 with 'de Morgan' Law
S3	A	Goal Statement
S4	B	Goal Statement
S5	⌐B	Combine [S2 and S3]
S6	{}	Combine [S5 and S4]

---

2. **Decide whether each of the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or equivalence rules.**
   a. $((Study \wedge Rest) \Longrightarrow Pass) \wedge (Study \Longrightarrow \neg Rest) \wedge (Rest \Longrightarrow \neg Study) \wedge Study$ = **Neither**

S	R	P	(S Λ R) => P	S => ⌐R	(R => ⌐S) Λ S	((S Λ R) => P) Λ (S => ⌐R) Λ ((R => ⌐S) Λ S)
T	T	T	T	F	F	F
T	T	F	F	F	F	F
T	F	T	T	T	T	T
F	T	T	T	T	F	F
F	F	T	T	T	F	F
T	F	F	T	T	T	T
F	T	F	T	T	F	F
F	F	F	T	T	F	F

b. $((Pass \wedge \neg Study) \Longrightarrow Rest) \Longrightarrow (\neg Pass \vee Study \vee Rest)$ = **Valid**

S	R	P	(P ∧ ⌐S) => R	⌐P V S V R	((P ∧ ⌐S) => R) => (⌐P V S V R)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
F	T	T	T	T	T
F	F	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

---

3. Convert the following sentences to CNF

a. P => ⌐ (Q V R)

P => ⌐ (Q V R)	Starting Statement
⌐P V (⌐ (Q V R))	Removing Implication
⌐P V (⌐Q ∧ ⌐R)	De Morgan Law
(⌐P V ⌐Q) ∧ (⌐P V ⌐R)	Distributivity Law

b. (P ∧ Q) ⇔ S

(P ∧ Q) ⇔ S	Starting Statement
((P ∧ Q) => S) ∧ (S => (P ∧ Q))	Removing Biconditional
(⌐ (P ∧ Q) V S) ∧ (⌐S V (P ∧ Q))	Removing Implication
((⌐P V ⌐Q) V S) ∧ (⌐S V (P ∧ Q))	Moving Negation Inward
(⌐P V ⌐Q V S) ∧ ((⌐S V P) ∧ (⌐S V Q))	De Morgan Law
(⌐P V ⌐Q V S) ∧ (⌐S V P) ∧ (⌐S V Q)	Removing Parenthesis

---

4. Consider the following KB.

Number	Statement
S1	A V B
S2	⌐B V ⌐C
S3	⌐C V D
S4	B V ⌐E
S5	⌐D V E

a. Use the resolution algorithm to determine whether the following KB entails $\neg D$.

Number	Statement	Obtained By …
S6	D	Goal
S7	E	Combine [S6 and S5]
S8	B	Combine [S7 and S4]
S9	⌐C	Combine [S8 and S2]

From there the knowledge loops not solving the case proving the KB does not entail $\neg D$.

b. Use the resolution algorithm to determine whether the following KB entails $B \vee \neg D$.

Number	Statement	Obtained By …
S6	D	Goal
S7	⌐B	Goal
S8	E	Combine [S6 and S5]
S9	B	Combine [S8 and S4]
S10	{}	Combine [S9 and S7]

Finding the empty clause proves the KB entails $B \vee \neg D$.