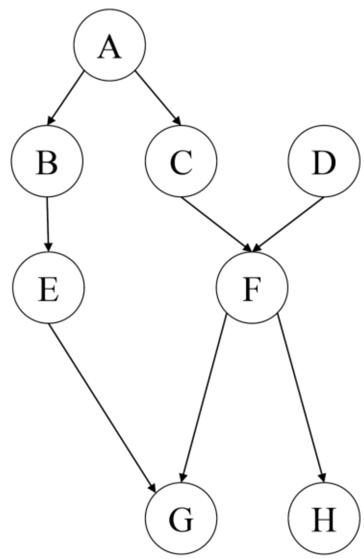
Josh Bell | May 21st 2022 | Intro to Artificial Intelligence

Written Assignment #4

1. Consider the Bayesian network below. Answer true or false for the following questions on d-separation. Show the blocked paths for partial credit.



a. I (D, G | { })

$D \rightarrow \mathrm{F} \rightarrow \mathrm{G}$ [Casual Chain] = *Active Chain*

$I\ (D, G\ |\{\}) = \boldsymbol{FALSE}$

b. I (B, C | { })

$B \leftarrow \mathrm{A} \rightarrow \mathrm{C}$ [Common Cause] = *Active Chain*

$I\ (B, C\ |\{\}) = \boldsymbol{FALSE}$

c. I (B, C | {A})

$B \leftarrow \mathrm{A} \rightarrow \mathrm{C}$ [Common Cause] = *Inactive Chain*

$B \rightarrow \mathrm{E} \rightarrow \mathrm{G} \rightarrow \mathrm{F} \rightarrow \mathrm{C} = (B \rightarrow \mathrm{E} \rightarrow \mathrm{G}) + (\mathrm{E} \rightarrow \mathrm{G} \rightarrow \mathrm{F}) + (\mathrm{G} \rightarrow \mathrm{F} \rightarrow \mathrm{C})$

$B \rightarrow \mathrm{E} \rightarrow \mathrm{G}$ [Casual Chain] = Active Chain

$E \rightarrow \mathrm{G} \leftarrow \mathrm{F}$ [Common Effect] = Inctive Chain

$G \leftarrow \mathrm{F} \leftarrow \mathrm{C}$ [Casual Chain] = Active Chain

$(B \rightarrow \mathrm{E} \rightarrow \mathrm{G}) + (\mathrm{E} \rightarrow \mathrm{G} \rightarrow \mathrm{F}) + (\mathrm{G} \rightarrow \mathrm{F} \rightarrow \mathrm{C}) =$ *Inactive Chain*

$I\ (B, C\ |\{A\}) = \boldsymbol{TRUE}$

d. I (B, C | {A, G})

$B \leftarrow \mathrm{A} \rightarrow \mathrm{C}$ [Common Cause] $=$ *Inactive Chain*

$B \rightarrow \mathrm{E} \rightarrow \mathrm{G} \leftarrow \mathrm{F} \leftarrow \mathrm{C} = (B \rightarrow \mathrm{E} \rightarrow \mathrm{G}) + (\mathrm{E} \rightarrow \mathrm{G} \leftarrow \mathrm{F}) + (\mathrm{G} \leftarrow \mathrm{F} \leftarrow \mathrm{C})$

$B \rightarrow \mathrm{E} \rightarrow \mathrm{G}$ [Casual Chain] $=$ Active Chain

$E \rightarrow \mathrm{G} \leftarrow \mathrm{F}$ [Common Effect] $=$ Active Chain

$G \leftarrow \mathrm{F} \leftarrow \mathrm{C}$ [Casual Chain] $=$ Active Chain

$(B \rightarrow \mathrm{E} \rightarrow \mathrm{G}) + (\mathrm{E} \rightarrow \mathrm{G} \leftarrow \mathrm{F}) + (\mathrm{G} \leftarrow \mathrm{F} \leftarrow \mathrm{C}) =$ *Active Chain*

$I\ (B, C\ |\{A\}) = \boldsymbol{FALSE}$

e. I (B, C | {A, C, G})

$B \leftarrow \mathrm{A} \rightarrow \mathrm{C}$ [Common Cause] $=$ *Inactive Chain*

$B \rightarrow \mathrm{E} \rightarrow \mathrm{G} \leftarrow \mathrm{F} \leftarrow \mathrm{C} = (B \rightarrow \mathrm{E} \rightarrow \mathrm{G}) + (\mathrm{E} \rightarrow \mathrm{G} \leftarrow \mathrm{F}) + (\mathrm{G} \leftarrow \mathrm{F} \leftarrow \mathrm{C})$

$B \rightarrow \mathrm{E} \rightarrow \mathrm{G}$ [Casual Chain] $=$ Active Chain

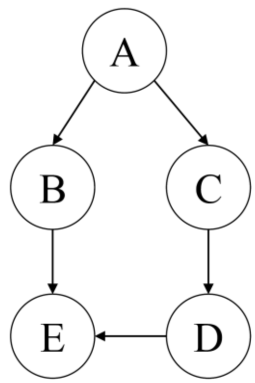
$E \rightarrow \mathrm{G} \leftarrow \mathrm{F}$ [Common Effect] $=$ Active Chain

$G \leftarrow \mathrm{F} \leftarrow \mathrm{C}$ [Casual Chain] $=$ Active Chain

$(B \rightarrow \mathrm{E} \rightarrow \mathrm{G}) + (\mathrm{E} \rightarrow \mathrm{G} \leftarrow \mathrm{F}) + (\mathrm{G} \leftarrow \mathrm{F} \leftarrow \mathrm{C}) =$ *Active Chain*

$I\ (B, C\ |\{A\}) = \boldsymbol{FALSE}$

2. Calculate the following probabilities using the Bayesian network below. The CPTs for each node are shown below the network. You may need to use the various probability formulas such as marginalization, the chain rule, conditional independence, Bayes rule, etc.



Conditional probability tables are given below:

A	**P(A)**
true	0.4
false	0.6

A	**B**	**P(B|A)**
true	true	0.9
true	false	0.1
false	true	0.25
false	false	0.75

C	**D**	**P(D|C)**
true	true	0.75
true	false	0.25
false	true	0.9
false	false	0.1

B	**D**	**E**	**P(E|B,D)**
true	true	true	0.1
true	true	false	0.9
true	false	true	0.2
true	false	false	0.8
false	true	true	0.3
false	true	false	0.7
false	false	true	0.4
false	false	false	0.6

A	**C**	**P(C|A)**
true	true	0.25
true	false	0.75
false	true	0.8
false	false	0.2

a. **P(A=true, B=false, C=true, D=false, E=false)**

$$P(A = True) * P(B = False|A = True) * P(C = True|A = True) * P(D = False|C = True) * P(E = False|B = True, D = True)$$

$$= 0.4 * 0.1 * 0.25 * 0.25 * 0.6 = \mathbf{0.0015}\ \boldsymbol{OR}\ .\mathbf{15}\%$$

b. P(B=false, C=false)

$$= \sum_a P(A = a, B = False, C = False)$$

$$= \sum_d \sum_a P(D = d, A = a, B = False, C = False)$$

$$= \sum_e \sum_d \sum_a P(E = e, D = d, A = a, B = False, C = False)$$

$$= \sum_e \sum_d \sum_a P(A = a) * P(B = False|A = a) * P(C = False|A = a)$$
$$* P(D = d|C = False) * P(E = e|B = False, D = d)$$

$$= \sum_a P(A = a) \sum_d P(D = d|C = False) * P(B = False|A = a)$$
$$* P(C = False|A = a) \sum_e P(E = e|B = False, D = d)$$

$$= \sum_a P(A = a) * P(B = False|A = a) * P(C = False|A = a)$$

$$= (P(A = True) * P(B = False|A = True) * P(C = False|A = True))$$
$$+ (P(A = False) * P(B = False|A = False)$$
$$* P(C = False|A = False))$$

$$= (0.4 * 0.1 * 0.75) + (0.6 * 0.75 * 0.1)$$

$$= 0.03 + 0.045$$

$$= \mathbf{0.075}\ \boldsymbol{OR}\ \mathbf{7.5\%}$$

c. P(A=true | B=false, C=false)

$$= \frac{P(A = True, B = False, C = False)}{P(B = False, C = False)}$$

$$= \frac{\sum_d P(A = True, B = False, C = False, D = d)}{\sum_a \sum_d P(A = a, B = False, C = False, D = d)}$$

$$= \frac{\sum_e \sum_d P(A = True, B = False, C = False, D = d, E = e)}{\sum_e \sum_a \sum_d \sum_d P(A = a, B = False, C = False, D = d, E = e)}$$

$$= \frac{\sum_e \sum_d P(A = True) * P(B = False|\, A = True) * P(C = False|A = True) * P(D = d|C = False) * P(E = e|B = False, D = d)}{\sum_e \sum_a \sum_d P(A = a) * P(B = False|\, A = a) * P(C = False|A = a) * P(D = d|C = False) * P(E = e|B = False, D = d)}$$

$$= \frac{P(A = True) * P(B = False|\, A = True) * P(C = False|A = True) * \sum_d P(D = d|C = False) * \sum_e P(E = e|B = False, D = d)}{\sum_a P(A = a) * P(B = False|\, A = True) * P(C = False|A = True) * \sum_d P(D = d|C = False) * \sum_e P(E = e|B = False, D = d)}$$

$$= \frac{P(A = True) * P(B = False|\, A = True) * P(C = False|A = True)}{\sum_a P(A = a) * P(B = False|\, A = True) * P(C = False|A = True)}$$

$$= \frac{P(A = True) * P(B = False|\, A = True) * P(C = False|A = True)}{\sum_a P(A = True) * P(B = False|\, A = True) * P(C = False|A = True)} + \frac{P(A = True) * P(B = False|\, A = True) * P(C = False|A = True)}{\sum_a P(A = False) * P(B = False|\, A = True) * P(C = False|A = True)}$$

$$= \frac{0.4 * 0.1 * 0.75}{(0.4 * 0.1 * 0.75) + (0.6 * 0.1 * 0.75)} = \frac{0.03}{0.03 + 0.045} = \frac{0.03}{0.075} = \mathbf{0.4}\ \boldsymbol{OR}\ \mathbf{40\%}$$