1. a)

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Input:
v[0..N-1]                        // N polygon vertices (N >= 3)
P_in                             // A point inside the polygon

Output:
string "CCW" or "CW"       // "CCW" if counter-clockwise, "CW" if clockwise

// Step 1. Define the first edge vector
Vector e = v[1] - v[0];

// Step 2. Define the vector from the first vertex to P_in
Vector p = P_in - v[0];

// Step 3. Calculate the 2D cross product
float z_cross_product = (e.x * p.y) - (e.y * p.x);

// Step 4. Check whether z is + or - to determine the winding order
if (z_cross_prouct > 0.0):
     return "CCW";
else:
     return "CCW";
```

b)
- Iterate through all of the vertices of the polygon, form a triangle, identify the center point of this triangle as P-in.
- First we choose the vertex with the smallest x-coordinate, or y-coordinate if the x-coordinate is shared. (Vert)
- we add the 2 neighboring vertices, then average it out to find the center

2.

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \xrightarrow{M} \begin{pmatrix} x - \sqrt{3}\,z\,a \\ -y + b + 2 \\ \frac{\sqrt{3}}{2}x + \frac{1}{2}z + c \\ 1 \end{pmatrix} \Rightarrow M = \begin{bmatrix} 1 & 0 & \sqrt{-3} & a \\ 0 & -1 & 0 & (b+2) \\ \sqrt{3}/2 & 0 & 1/2 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} (1)x + (0)y + (-\sqrt{3})z + (a)1 \\ (0)x + (-1)y + (0)z + (b+2)1 \\ (\sqrt{3}/2)x + (0)y + (1/2)z + (c)1 \\ 0x + 0y + 0z + (1)1 \end{pmatrix}$$

b)

$$M = \begin{bmatrix} 1 & 0 & -\sqrt{3} & a \\ 0 & -1 & 0 & b+2 \\ \sqrt{3}/2 & 0 & 1/2 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} = TSRP_{in} \quad ; \quad T = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b+2 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation

Rotate than scale

R, S

R: rotate by y-axis ($\theta = -\pi/3$)

$$\begin{bmatrix} 1/2 & 0 & -\sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & 1/2 \end{bmatrix}$$

$$S = R_1 s \cdot R^{-1}$$

$$= \begin{bmatrix} 1 & 0 & -\sqrt{3} \\ 0 & -1 & 0 \\ \sqrt{3}/2 & 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \xrightarrow{M} \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

$$M = \begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{aligned} x' &= ax + by + c \\ y' &= dx + ey + f \end{aligned}$$

Using points from G:

$P_1 = (3, 5)$ $\quad$ $P_1' = (3,4) \rightarrow 3 = a(3) + b(5) + C, \quad 4 = d(3) + e(5) + f$

$P_2 = (1,1)$ $\quad \Rightarrow$ $P_2' = (1,1) \rightarrow 1 = a(1) + b(1) + C, \quad 1 = d(1) + e(1) + f$

$P_3 = (3,3)$ $\quad$ $P_3' = (3,2) \rightarrow 3 = a(3) + b(3) + C, \quad 2 = d(3) + e(3) + f$

Solve for $a \rightarrow f$: $b = C = 0$; $e = 1$

$a = 1$ $\quad$ $2d + 1 = 0$, $d = -1/2$, $f = 1/2$

$$M = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 1/2 \\ 0 & 0 & 1 \end{pmatrix}$$

4. $q_1 = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$ $q_2 = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$ let $q = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}$, $q = \begin{pmatrix} \cos(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2})\vec{u} \end{pmatrix} \begin{matrix} w \\ \vec{v} \end{matrix}$

a) $\cos \theta_1/2 = -\frac{1}{\sqrt{2}}$

$\underline{\theta_1 = 2 \times \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{2}}$

$\vec{u} \underbrace{\sin\frac{\theta_1}{2}}_{\frac{1}{\sqrt{2}}} = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) \Rightarrow u_1 = \sqrt{2}\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$= \underline{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$

b) $q_2 = (0, 0, -1, 0)$

$\cos\frac{\theta_2}{2} = w = 0$, $\underline{\theta_2 = \pi}$

$\vec{u} \underbrace{\sin\frac{\theta_2}{2}}_{1} = (0, -1, 0) \Rightarrow \underline{u_2 = (0, -1, 0)}$

c) $q_1 \cdot q_2 = (w_1 w_2 - \vec{v}_1 \cdot \vec{v}_2, w_1\vec{v}_2 + w_2\vec{v}_1 + (\vec{v}_1 \times \vec{v}_2))$

$q_2 \cdot q_1 = (w_2 w_1 - \vec{v}_1 \cdot \vec{v}_2, w_2\vec{v}_1 + w_1\vec{v}_2 + (\vec{v}_2 \times \vec{v}_1))$

$\Rightarrow \left(-\frac{1}{\sqrt{2}} \cdot 0\right) - \left(-\frac{1}{\sqrt{6}}\right) = \frac{1}{\sqrt{6}}$, $w_1\vec{v}_2 = (0, \frac{1}{\sqrt{2}}, 0)$ $\quad \vec{v}_1 \times \vec{v}_2 = (\frac{1}{\sqrt{6}}, 0, -\frac{1}{\sqrt{6}})$

$w_2\vec{v}_1 = (0, 0, 0)$

$q_1 q_2 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{6}}\right)$

$\Rightarrow w_2 w_1 - \vec{v}_1 \cdot \vec{v}_2 = \frac{1}{\sqrt{6}}$, $w_2\vec{v}_1 = 0$ $\quad \vec{v}_2 \times \vec{v}_1 = \left(-\frac{1}{\sqrt{6}}, 0, \frac{1}{\sqrt{6}}\right)$

$w_1\vec{v}_2 = (0, \frac{1}{\sqrt{2}}, 0)$

$q_2 q_1 = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}\right)$

$\underline{q_1 q_2 \neq q_2 q_1, \text{ Not commutative}}$

5.

$M = T_{out} R_{out} S R_{in} T_{in}$, let $\vec{u} = (u_x, u_y, u_z)$, $\vec{w} = \perp \vec{u}$, e.g.) $\frac{\vec{u} \times (1,0,0)}{\| \vec{u} \times (1,0,0) \|}$

$\vec{v} = \vec{w} \times \vec{u}$

$$T_{in} = \begin{pmatrix} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & 1 & -C_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad R_{in} = \begin{pmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad S = \begin{pmatrix} S_u & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_{out} = R_{in}^{-1} = R_{in}^T = \begin{pmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad T_{out} = \begin{pmatrix} 1 & 0 & 0 & C_x \\ 0 & 1 & 0 & C_y \\ 0 & 0 & 1 & C_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

let $s' = S_u - 1$

$t_x = (-s')(u_x C_x + u_y C_y + u_z C_z) u_x + C_x$

$t_y = (-s')(u_x C_x + u_y C_y + u_z C_z) u_y + C_y$

$t_z = (-s')(u_x C_x + u_y C_y + u_z C_z) u_z + C_z$

$$M = \begin{pmatrix} 1+s' u_x^2 & s' u_x u_y & s' u_x u_z & t_x \\ s' u_y u_x & 1+s' u_y^2 & s' u_y u_z & t_y \\ s' u_z u_x & s' u_z u_y & 1+s' u_z^2 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

6. $P' = T_{out} R_{out} \text{Mirror } R_{in} T_{in} \cdot P$, $P_0 = \frac{d}{\sqrt{a^2+b^2+c^2}} \cdot \vec{w}$

$ax + by + cz = d$, let $\vec{n} = (a, b, c)$

$\vec{w} = \frac{\vec{n}}{\sqrt{a^2+b^2+c^2}}$

$\vec{u}, \vec{v} \perp \vec{w}$

perpendicular formed through cross produces

$$T_{in} = \begin{pmatrix} 1 & 0 & 0 & -P_{0x} \\ 0 & 1 & 0 & -P_{0y} \\ 0 & 0 & 1 & -P_{0z} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad R_{in} = \begin{pmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Mirror (Reflect): $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$Rout = Rin^{-1} = Rin^{T}$$

$$= \begin{pmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Tout = \begin{pmatrix} 1 & 0 & 0 & P_{ox} \\ 0 & 1 & 0 & P_{oy} \\ 0 & 0 & 1 & P_{oz} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M = \frac{1}{a^2+b^2+c^2} \begin{pmatrix} -a^2+b^2+c^2 & -2ab & -2ac & 2ad \\ -2ab & a^2-b^2+c^2 & -2bc & 2bd \\ -2ac & -2bc & a^2+b^2-c^2 & 2cd \\ 0 & 0 & 0 & a^2+b^2+c^2 \end{pmatrix}$$