PS1 - Chih Han Yeh

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Q.1

a) $V_2 = P_2 - P_1$
$= (5, 3, -7) - (1, 6, 5)$
$= \underline{(4, -3, -12)}$ //

b) $V_3 = P_3 - P_1$
$= (1, 6, 4) - (1, 6, 5)$
$= \underline{(0, 0, -1)}$ //

c) $\|V_2\| = \sqrt{4^2 + (-3)^2 + (-12)^2}$
$= \sqrt{16 + 9 + 144}$
$= \sqrt{169} = \underline{\underline{13}}$

$\|V_3\| = \sqrt{(-1)^2} = \underline{\underline{1}}$ //

d) $\hat{V}_2 = \frac{V_2}{\|V_2\|} = \underline{\left(\frac{4}{13}, \frac{-3}{13}, \frac{-12}{13}\right)}$ //

$\hat{V}_3 = \frac{V_3}{\|V_3\|} = \underline{(0, 0, -1)}$ //

Q.2

a) $V_2 \times V_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & -12 \\ 0 & 0 & -1 \end{vmatrix}$

$= \hat{i}(3) - \hat{j}(-4) + \hat{k}(0)$
$= \underline{(3, 4, 0)}$ //

b) $V_3 \times V_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -1 \\ 4 & -3 & -12 \end{vmatrix}$

$= \hat{i}(-3) - \hat{j}(4) + \hat{k}(0)$
$= \underline{(-3, -4, 0)}$ //

c) $V_3 \cdot V_2 = (0 \cdot 4) + (0 \cdot -3) + (-12 \cdot -1)$
$= \underline{12}$ //

Q.3) Orthogonal vectors would have a scalar product of 0.

Q.4)

(a) $\|\vec{v}\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2}$

$= \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

Not unit vector

(b) $\|\vec{v}\| = \sqrt{(-1)^2}$

$= \sqrt{1} = 1$

is a unit vector

c) $\|\vec{v}\| = \sqrt{\left(\frac{-2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} \rightarrow \sqrt{\frac{49}{49}} = 1$

is a unit vector

Q.5)

$\theta = \cos^{-1}\left[(u \cdot v) / \|u\| \|v\|\right]$

or

$\theta = \sin^{-1}\left[\|u \times v\| / \|u\| \|v\|\right]$

a) $\cos\theta = \frac{(u \cdot v)}{\|u\| \|v\|}$

b) $\sin\theta = \frac{\|u \times v\|}{\|u\| \|v\|}$

c) $\left.\begin{array}{l} \vec{a} \perp u \\ \text{and} \\ \vec{a} \perp v \end{array}\right\} (u \times v)$

Q.6 a) $(QRS)^{-1} = S^{-1}R^{-1}Q^{-1}$; $Q^{-1}R^{-1}S^{-1} \Rightarrow$ False

b) $QR \neq RQ$; False

c) $(QRS)^T = S^T R^T Q^T$; True

d) $(R+S)Q = RQ + SQ$; $SQ + RQ \rightarrow$ False

Q.7)

a) Dot product is 0

b) Inverse of A is A^{-1} $(A^T = A^{-1})$