CS 480/CS680 PSet 3: Curves & Surfaces

Due: November 13, 2025 at 11:59 pm

<u>Submission guidelines:</u>
Please prepare your answers neatly written or typed. Submit on Gradescope. If your answers are hand-written please either upload a scan or photos of your answer sheets.

<u>Total points:</u> 480: 100 pts + 10 pts max extra credit / 680: 120 pts

1. (40 pts)
   We are given the implicit function for a one-sheeted hyperboloid, where c_x, r are given constants:
   $$f(x,\, y,\, z) = (x - c_x)^2 - y^2 + z^2 - r^2.$$
   a) (10 pts) Give the 4 × 4 matrix $\boldsymbol{Q}$ for this cylindrical surface, such that $\boldsymbol{p}^T\boldsymbol{Q}\boldsymbol{p} = 0$ for any homogeneous point $p = (x, y, z, 1)$ that is on the cylindrical surface.
   b) (10 pts) Using the implicit function, derive a function that gives a unit normal vector at any point on the surface (x, y, z).
   c) (10 pts) Give an equivalent parametric equation for the surface, in terms of θ and u. Assume that $-\pi \le \theta \le \pi$ and $u_{min} \le u \le u_{max}$, for some given bounds u_{min} u_{max}.
   d) (10 pts) Using the parametric equation, derive the function that gives the unit normal vector at any point on the sheet with parameter $(\theta,\, u)$.

2. (30 pts)
   Consider a 3D surface of revolution where the curve is rotated around the x-axis and given by the equation $y = \frac{1}{x}$, with $x \ge 1$.
   a) (10 pts) Derive a simple parameterization for the surface of revolution $p(\theta,\, u)$.
   b) (10 pts) Derive the equation for the surface normal $N(\theta,\, u)$.
   c) (10 pts) Sketch the resulting surface. It need not be perfect, but should communicate the basic idea. It might be helpful to use a graphing calculator or software to see the curve before it is rotated.

3. (30 pts)
   We are given a 2D cubic Bezier curve segment, which has the following control points:
   $\boldsymbol{p_0}$ = (-1, 1)
   $\boldsymbol{p_1}$ = (0, 3)
   $\boldsymbol{p_2}$ = (3, 1)
   $\boldsymbol{p_3}$ = (1, 2)

   a) (7.5 pts) Draw the convex hull for this 2D Bezier curve segment.
   b) (7.5 pts) Compute the value of $p'(1)$ for this 2D Bezier curve segment. What does this value represent?
   c) (7.5 pts) We are now given a second 2D Bezier curve segment, which has the control points:

$\boldsymbol{p_0}$ = (1, 2)
$\boldsymbol{p_1}$ = (3, -1)
$\boldsymbol{p_2}$ = (0, -2)
$\boldsymbol{p_3}$ = (-1, 0)

Does this segment join the previous segment with C1 continuity? Give a mathematical justification for your answer.

d) (7.5 pts) Which control point above (for the second curve in (c)) may we change to achieve C1 continuity? Write down the new position of the point that will achieve this.

4. (680: required, 20 pts; 480: extra credit, 10 pts max)
As background for this problem, we recommend that students read Section 14-7 of the Hearn, Baker, Carithers text (see Piazza Resources).
We are given the following boundary conditions for a cubic spline section:

$$\boldsymbol{P}(0) = \boldsymbol{p}_k$$
$$\boldsymbol{P}(1) = \boldsymbol{p}_{k+1}$$
$$\boldsymbol{P}'(0) = \tfrac{1}{2}[(1+b)(\boldsymbol{p}_k - \boldsymbol{p}_{k-1}) + (1-b)(\boldsymbol{p}_{k+1} - \boldsymbol{p}_k)]$$
$$\boldsymbol{P}'(1) = \tfrac{1}{2}[(1+b)(\boldsymbol{p}_{k+1} - \boldsymbol{p}_k) + (1-b)(\boldsymbol{p}_{k+2} - \boldsymbol{p}_{k+1})]$$

In the textbook, we see this is a Cardinal Spline (Kochanek-Bartels spline with t=0 and c=0). In this case $M_{geom} = [p_{k-1}\ \ p_k\ \ p_{k+1}\ \ p_{k+2}]^T$ and the boundary conditions can be written:

$$\begin{bmatrix} \boldsymbol{P}(0) \\ \boldsymbol{P}(1) \\ \boldsymbol{P}'(0) \\ \boldsymbol{P}'(1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{2}(1+b) & b & \frac{1}{2}(1-b) & 0 \\ 0 & -\frac{1}{2}(1+b) & b & \frac{1}{2}(1-b) \end{bmatrix} \begin{bmatrix} \boldsymbol{p}_{k-1} \\ \boldsymbol{p}_k \\ \boldsymbol{p}_{k+1} \\ \boldsymbol{p}_{k+2} \end{bmatrix}$$

a) (5 pts) Show how to compute the 4×4 coefficient matrix M_C given the boundary conditions written above. You do not need to compute a matrix inverse to find M_C (the relevant one is given in the textbook anyway). Just give the specific equations for M_C.

b) (5 pts) Given M_C write out the blending functions for this curve.

c) (5 pts) Do adjacent segments satisfy C1 continuity? Give a mathematical justification.

d) (5 pts) Does adjusting b change the tangent direction at the endpoints or only magnitude?