EC516 HW**07** Solutions

Problem 7.1

a) $\Delta\omega = 0.25\pi$, $A = -20\log_{10}(0.15) \approx 16.478$. Therefore, $\beta = 0$, and

$$N = \frac{16.478 - 8}{2.285 \cdot 0.25\pi} \approx 4.724$$

Picking the smallest integer larger than the above, the length will be 5.

b) $\Delta\omega = 0.15\pi$, $A = -20\log_{10}(0.15) \approx 16.478$. Therefore, $\beta = 0$, and

$$N = \frac{16.478 - 8}{2.285 \cdot 0.15\pi} \approx 7.873$$

The length is 8.

c) $\Delta\omega = 0.25\pi$, $A = -20\log_{10}(0.09) \approx 20.915$. Therefore, $\beta = 0$, and

$$N = \frac{20.915 - 8}{2.285 \cdot 0.25\pi} \approx 7.196$$

The length is 8.

d) $\Delta\omega = 0.15\pi$, $A = -20\log_{10}(0.09) \approx 20.915$. Therefore, $\beta = 0$, and

$$N = \frac{20.915 - 8}{2.285 \cdot 0.25\pi} \approx 11.994$$

The length is 12.

Problem 7.2

a) Window Design will produce equiripple frequency response (same error in passband and stopband). Optimal FIR Filter is more flexible when you want to allow varying error in the passband and stopband.

b)

$$\frac{-10\log_{10}(0.10 \cdot 0.05) - 13}{14.6(0.75\pi - 0.5\pi)/(2\pi)} \approx 5.485$$

Smallest odd integer that is greater than the above value is 7 (or $L = 3$).

c) $\Delta\omega = 0.25\pi$, $A = -20\log_{10}(0.05) \approx 26.021$. Therefore,

$$N = \frac{26.021 - 8}{2.285 \cdot 0.25\pi} \approx 12.017$$

Smallest integer greater than the above value is 13. Kaiser window requires filter with longer length to achieve error within $\delta_s = 0.05$ in the stopband.

d) Note that by trig identity we get

$$\cos^k(\omega) = \sum_{m=0}^{k} a_m \cos(m\omega)$$

In order to have k-th power of $\cos(\omega)$, the frequency response must include $\cos(k\omega)$. That means the frequency response found by optimal filter design $H(e^{j\omega})$ can be written in the following form.

$$H(e^{j\omega}) = e^{j\omega n_0} \left(\alpha \cdot \cos(k\omega) + R(\omega)\right)$$

n_0, α are some arbitrary numbers and $R(\omega)$ is a real-valued function. Inverse DTFT of $\cos(k\omega)$ is

$$\begin{aligned}
\mathrm{DTFT}^{-1}\left\{e^{j\omega n_0}\cos(k\omega)\right\} &= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega n_0}\frac{1}{2}\left(e^{jk\omega} + e^{-jk\omega}\right)e^{-j\omega n} d\omega \\
&= \frac{1}{2}\delta[n - n_0 - k] + \frac{1}{2}\delta[n - n_0 + k]
\end{aligned}$$

This means that filter length must be at least $2k+1$ to have k-th power of $\cos(\omega)$. From the number we found from part (b), the highest order of $\cos(\omega)$ is 3.

Problem 7.3

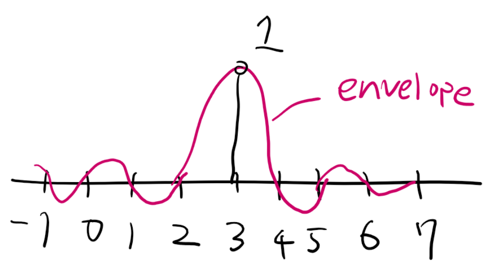
(a) Taking the DTFT of $\delta[n-3]$ will give us $X(e^{j\omega}) = e^{-j\omega}$. To find the envelope, we find the inverse CTFT of

$$E_x(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \pi \\ 0 & \text{else} \end{cases}$$

Taking inverse CTFT gives us

$$\frac{\sin(\pi(t-3))}{\pi(t-3)}$$

Note that the envelope is zero at any integer values except for $t = 3$.



(b) We take the CTFT of $\frac{\sin(\pi(t-0.5))}{\pi(t-0.5)}$, which is

$$E_x(j\omega) = \begin{cases} 1 & |\omega| < \pi \\ 0 & \text{else} \end{cases}$$

Taking the inverse DTFT gives us

$$x[n] = \frac{\sin(\pi(n-0.5))}{\pi(n-0.5)}$$

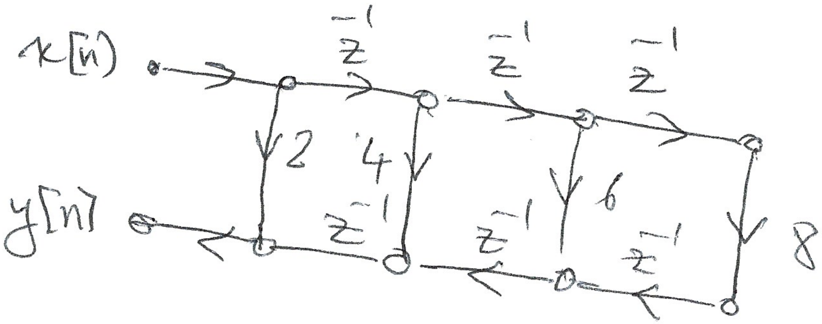
Problem 7.4

$$h[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 8\delta[n-3] + 6\delta[n-4] + 4\delta[n-5] + 2\delta[n-6]$$

This should have been 2, not 8.

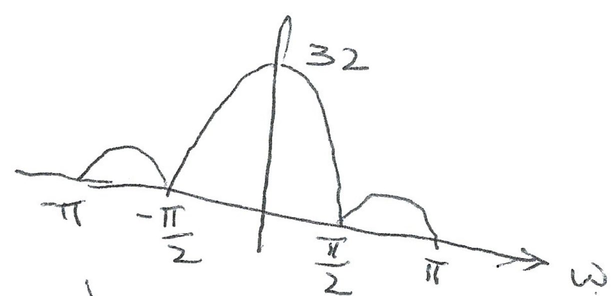
$= 7$ pt. triangle $=$ 4 pt. box $*$ 4 pt. box

(a)



(b) $$H(e^{j\omega}) = e^{-j3\omega}\left(\frac{\sin 2\omega}{\sin \omega/2}\right)^2$$

$$\therefore \; |H(e^{j\omega})| = \left(\frac{\sin 2\omega}{\sin \omega/2}\right)^2 =$$



(c) Yes, Type I.

(d) $\measuredangle H(e^{j\omega}) = -3\omega$