FALL 25 EC516 Homework 02

Due: Sunday September 21 (Before 11:59pm)

You must submit your homework attempt on Blackboard. For this purpose, you must convert your homework attempt to a single pdf file and upload it at the corresponding homework assignment on Blackboard.

Problem 2.1 (DTFT Basics)

A) Consider the complex exponential signal $x[n] = e^{j0.375\pi n}$ to answer each of the following questions:
   a) Is the signal $x[n]$ periodic? If yes, what is its fundamental period?
   b) Plot the absolute value of $x[n]$ as a function of *n*.
   c) Specify a numerical value of n_0 such that $x[n_0] = -1$.

B) Determine the DTFT $X(e^{j\omega})$ of $x[n] = \delta[n-3]$ by using $X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ where $\delta[n]$ is the discrete-time unit impulse.

C) Calculate the DTFT of each of the following signals using $X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ and plot $|X(e^{j\omega})|$ as a function of ω.
   a) $x[n] = u[n+2] - u[n-3]$, where $u[n]$ is the discrete-time unit step.
   b) $x[n] = u[n] - u[n-5]$, where $u[n]$ is the discrete-time unit step.
   c) $x[n] = u[n] - u[n-4]$, where $u[n]$ is the discrete-time unit step.

Problem 2.2 (Analog Sampling)

Suppose $x(t)$ is a *real-valued* speech-signal whose CTFT is $X(j\omega)$ and it is known that $|X(j\omega)| = 0$ for $|\omega| \geq 10{,}000\pi$. Let $x[n] = x(nT)$ be the output of an analog sampler where T represents the sampling interval. Answer the following questions about $X(e^{j\omega})$, the DTFT of $x[n]$, for the specified values of *T*.

(a) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.0001$ secs. *Justify your answer.*

(b) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.00005$ secs. *Justify your answer.*

(c) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.00001$ secs. *Justify your answer.*

Problem 2.3 (Inverse DTFT and DTFT Properties)

A) For each DTFT given below, compute the corresponding signal using $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\, d\omega$ and plot $|x[n]|$ as a function of *n*.

a) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - 0.5\pi - 2\pi k)$
b) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \{\pi\delta(\omega + 0.5\pi - 2\pi k) + \pi\delta(\omega - 0.5\pi - 2\pi k)\}$

B) Suppose that the input $x[n]$ to a digital circuit is related to the output $y[n]$ of that digital circuit through the following difference equation:

$$y[n] = 0.5y[n-1] + x[n]$$

Determine the mathematical relationship between $Y(e^{j\omega})$ and $X(e^{j\omega})$.

C) If $x[n]$ is a real-valued signal with $X(e^{j0.25\pi}) = 1 + j$, use $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ to determine the value of $X(e^{-j0.25\pi})$. This result shows that any algorithm for determining the DTFT of a real-valued signal need not directly calculate the DTFT for negative frequencies.

Problem 2.4 (Digital Sampling)

Suppose the discrete-time signal $x[n]$ is a real-valued signal. We perform digital downsampling by a factor of 3 on $x[n]$ to obtain the signal $y[n]$. The digital downsampling uses a digital anti-aliasing filter with input $x[n]$ and output $g[n]$. The signal $g[n]$ then undergoes a *sampling rate change* that reduces the sampling rate by a factor of 3.

(a) Draw and label the ideal frequency response of the digital antialiasing that should be used in the digital sampling process. *Justify your answer*.
(b) Is the ideal digital antialiasing filter causal? Justify your answer.
(c) Are there any values of ω for which $G(e^{j\omega})$ is guaranteed to be zero regardless of what $X(e^{j\omega})$ looks like? *Justify your answer*.
(d) Determine the values of the constants α, β, and γ such that

$$3Y(e^{j\omega}) = G(e^{j(\omega-\alpha)}) + G(e^{j(\omega-\beta)}) + G(e^{j(\omega-\gamma)})$$

Justify your answer.
(e) Is the signal $g[n]$ guaranteed to be reconstructable from the signal $y[n]$? Justify your answer.
(f) Is the signal $x[n]$ guaranteed to be reconstructable from the signal $g[n]$? Justify your answer.

Experiential DSP Exercise 02

Part A: Plot and Observe!

1. Place `audio.wav` from HW02 assignment in your current MATLAB folder (check with `pwd` or use the Current Folder pane).
2. Read the audio and sampling rate using the `audioread` command. (Note that this command gives you **two** outputs, `x` and `F`.)
3. Following the instructions of Assignment 1, plot the waveform of the audio signal.
4. Observe that the audio signal is roughly periodic. What approximately is the period?

Part B: DTFT in MATLAB

In this part, we want to learn how the `fft` command in MATLAB works and how to analyze its output.

1. The `fft` command takes two inputs. `x`, which is the signal that you get from the previous part, and `n`, which is the number of samples. The `fft` command computes `n` uniformly spaced samples every $\frac{2\pi}{n}$ from $\omega = 0$ to $\omega = 2\pi$ of the discrete-time Fourier transform (DTFT).
2. Set `nfft_1 = 512` and `nfft_2 = 1024.`
3. Get `X_1 = fft(x, nfft_1)` and Get `X_2 = fft(x, nfft_2).`
4. Create two stem plots: `stem(20*log10(abs(X_1))` and `stem(20*log10(abs(X_2)).` Save these figures as `figure1` and `figure2`, respectively. Note that the x-axis in these figures represents the frequency, which here spans from 0 to 2π.
5. In each figure, find the middle point and explain how the portion to the left of the middle point and the portion to the right of the middle point are related to each other.
6. Observe the two saved figures and determine how the samples in `figure1` are just a *subset* of the samples in `figure2`. Specify how the index of a sample in `figure1` maps to the corresponding index in `figure2`.