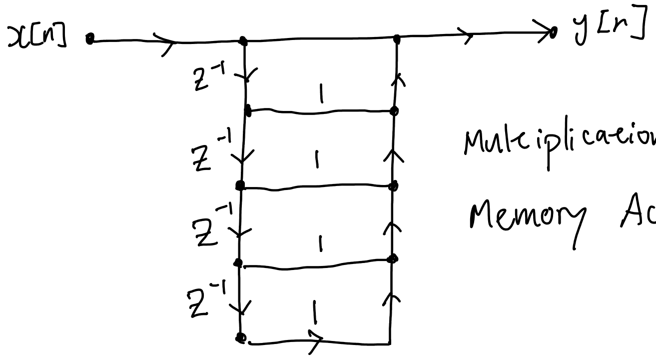
1. $y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$

a)



Multiplications = 0, all coefficients are 1

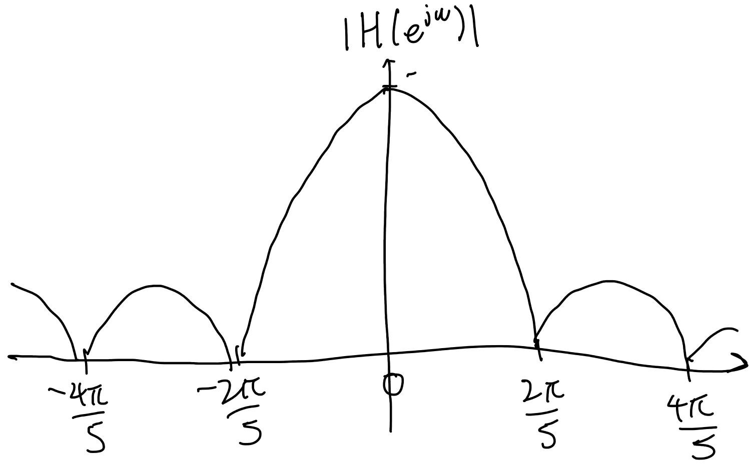
Memory Access = 5, $4 \times z^{-1}$ branches, one storage

b) $Y(z) = X(z) + z^{-1}X(z) + z^{-2}X(z) + z^{-3}X(z) + z^{-4}X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$$

c) $H(e^{j\omega}) = \sum_{n=0}^{4} e^{-j\omega n}$, $|H(e^{j\omega})| = \left| \frac{\sin(5\omega/2)}{\sin(\omega/2)} \right|$, $\frac{5\omega}{2} = k\pi$

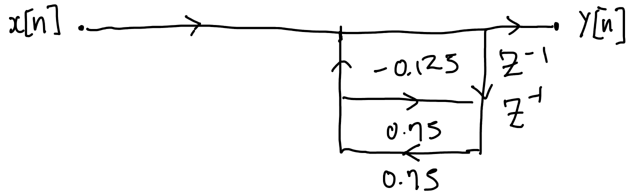
$\rightarrow \omega = \frac{2k\pi}{5}$



d) $H(z) = \frac{z^4 + z^3 + z^2 + z + 1}{z^4}$, pole is at $z = 0$, within the

unit circle ($|z| < 1$), filter is stable

2 a)



- 2 multiplications (−0.125, 0.75)
- 2 memory retrievals, 2 branches

b) $H(z) = \dfrac{\bar{Y}(z)}{\bar{X}(z)}$

$Y(z) = \bar{X}(z) + 0.75\bar{Y}(z)z^{-1} - 0.125\bar{Y}(z)z^{-2}$

$Y(z)(1 - 0.75z^{-1} + 0.125z^{-2}) = \bar{X}(z)$

$H(z) = \dfrac{1}{1 - 0.75z^{-1} + 0.125z^{-2}}$

c) set denominator of $H(z) = 0$

$1 - 0.75z^{-1} + 0.125z^{-2} = 0$

$\rightarrow z^2 - 0.75z + 0.125 = 0$

$(z-0.5)(z-0.25)$

$z = 0.5, 0.25$

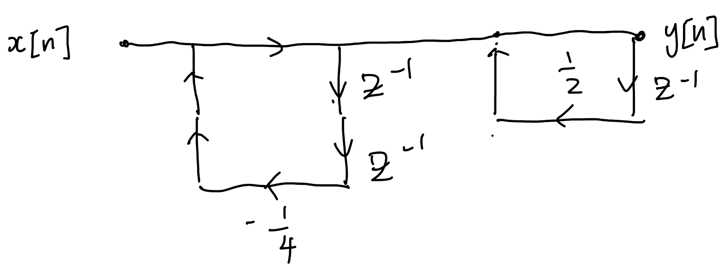
both values are $|z| < 1$, therefore stable

(in unit circle)

d) Since $H(z) \neq 0 \ \forall z$ that is finite

The same can be said for $H(e^{j\omega})$ therefore there are no ω that would produce a 0 value.

3. a) $y[n] = x[n] - \frac{1}{4}x[n-2] + \frac{1}{2}y[n-1]$

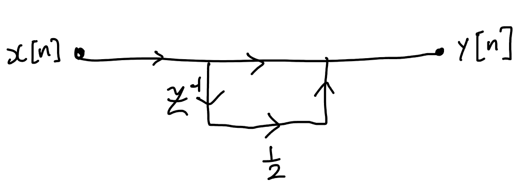


b) $\bar{Y}(z) - \frac{1}{2}\bar{Y}(z)z^{-1} = \bar{X}(z) - \frac{1}{4}\bar{X}(z)z^{-2}$

$H(z) = \dfrac{\bar{Y}(z)}{\bar{X}(z)} = \dfrac{1 - \frac{1}{4}z^{-2}}{\cancel{1 - \frac{1}{2}z^{-1}}}$

$(1 + \frac{1}{2}z^{-1})\cancel{(1 - \frac{1}{2}z^{-1})}$

c)



$\rightarrow H(z) = 1 + \frac{1}{2}z^{-1}$

$\bar{Y}(z) = \bar{X}(z) + \frac{1}{2}\bar{X}(z)z^{-1}$

$y[n] = x[n] + \frac{1}{2}x[n-1]$

d)
- Non-recursive is significantly simple
- Memory access and multiplications are both halved

4. a) $H(z) = \left[\underbrace{(1-0.9e^{j\pi/4}z^{-1})(1-0.9e^{-j\pi/4}z^{-1})}_{1-\underbrace{(0.9e^{j\pi/4}+0.9e^{-j\pi/4})}_{2\cdot\mathrm{Re}\{0.9e^{j\pi/4}\} = 2\cdot 0.9\cos\pi/4 = 1.8\cdot\frac{\sqrt{2}}{2}}z^{-1}+0.81z^{-2}}\right]^{-1}$ $\quad H(z) = \frac{\bar{Y}(z)}{\bar{X}(z)}, \quad \bar{Y}(z) = \bar{H}(z)\bar{X}(z)$

$H(z) = (1-0.9\sqrt{2}z^{-1}+0.81z^{-2})^{-1}$

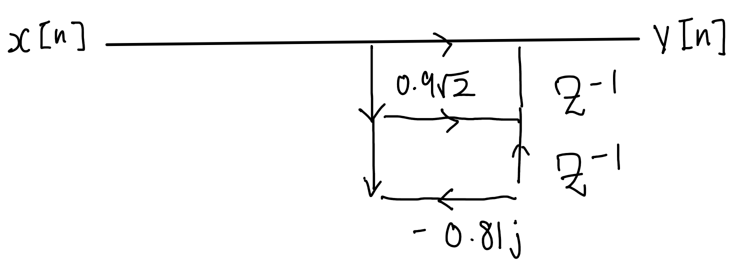
$\rightarrow \quad \bar{Y}(z)[1-0.9\sqrt{2}z^{-1}+0.81z^{-2}] = \bar{X}(z)$

$\bar{Y}(z) = 0.9\sqrt{2}\,\bar{Y}(z)z^{-1} - 0.81\,\bar{Y}(z)z^{-2} + \bar{X}(z)$

$\rightarrow \underline{Y[n] = 0.9\sqrt{2}\,y[n-1] - 0.81\,y[n-2] + x[n]}$ //

b) No, the coefficiems of the filter will determine whether the resulting output would be real-valued or complex.

c)

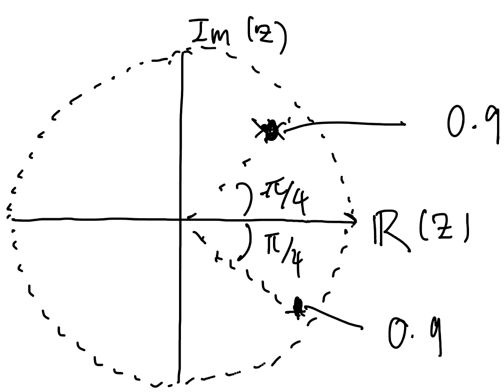


Multiplications: 2 complex multiplications = 4

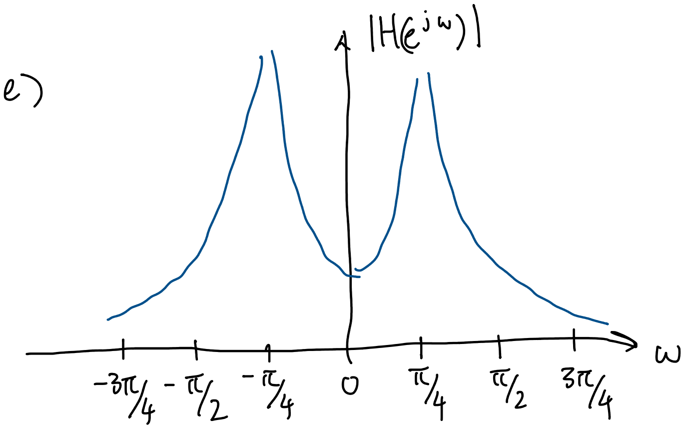
Memory : 2 × each access of y[n-1] and y[n-2] = 4

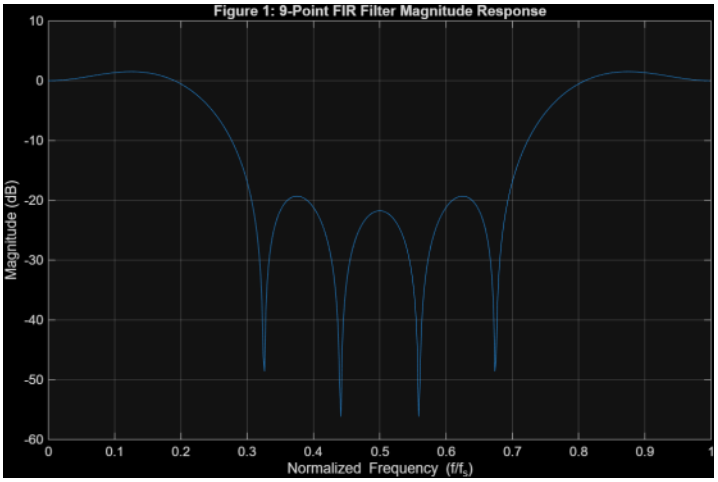
intermediate signals are not guranteed real

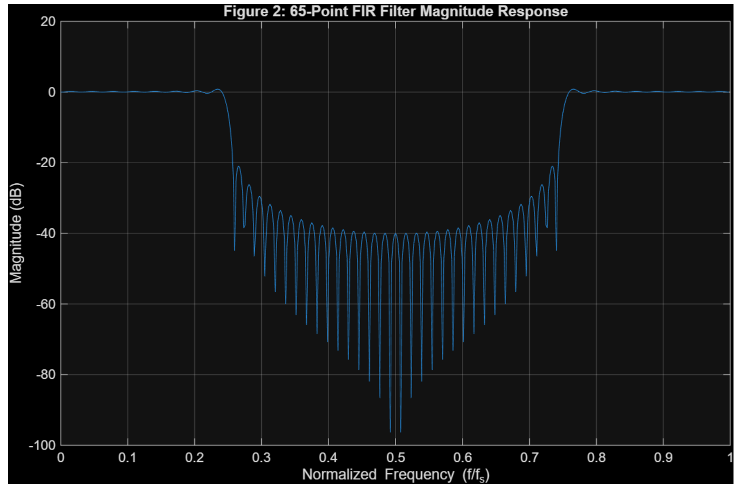
d)



e)







Part 3 - Normalization:

The normalization step as described in the exercise ensures that the filter has a gain of 0 dB at $\omega = 0$; ensuring that a constant signal as an input through the filter would result in a constant with unchanged amplitude. The new coefficients after normalization is 1, a linear gain of 1 equates to a logarithmic gain of 0 dB.

Comparing both figures, figure 2 shows faster gain transitions whereas figure 1 with the 9 point FIR shows a slow and wider transition between the gain drop from pass to stop band. The sides of the 65-point FIR also shows lower levels of gain compared to the 9-point FIR.

The multiplication or rather cost required for both filters alludes to their names, 9 points for 9 multiplications and 65 points for 65 multiplications.