HOMEWORK 11

This homework is due Wednesday, Dec. 10 before 11:59pm

Problem 11.1 (20 points)

(A) Consider the 5-point signal given as $x[n] = u[n] - u[n-5]$, where $u[n]$ is the unit step. Let $X_w[n,\omega)$ be the TDFT of $x[n]$ with respect to analysis window $w[n] = \delta[n]$. Also let $f_n[m] = w[m]x[n+m]$ denote the short-time section at each time *n* that is a signal as a function of *m*.

(a) Sketch magnitude and phase of $X(e^{j\omega})$.

(b) Sketch the short-time sections $f_{-1}[n], f_0[n]$, and $f_{10}[n]$. Justify your answers.

(c) Sketch $X_w[n,\frac{\pi}{2})$ as a function of *n*. Justify your answer

(d) Sketch $X_w[0,\omega)$ as a function of ω. Justify your answer.

(B) Consider the 5-point signal given as $x[n] = u[n] - u[n-5]$, where $u[n]$ is the unit step. Let $X_w[n,\omega)$ be the TDFT of $x[n]$ with respect to analysis window $w[n] = 1$ for all *n*.

(a) Sketch the short-time sections $f_{-1}[n], f_0[n]$, and $f_{10}[n]$. Justify your answers.

(b) Sketch $X_w[n,\frac{2\pi}{5})$ as a function of *n*. Justify your answer

(c) Sketch $X_w[n,\frac{3\pi}{5})$ as a function of *n*. Justify your answer

(d) Sketch $X_w[0,\omega)$ as a function of ω. Justify your answer.

Problem 11.2 (20 points)

a) Let $X_w[n,\omega)$ denote the TDFT of a signal $x[n]$ with respect to an analysis window $w[n]$. Use the filtering view of the TDFT to argue that the TDFT of $x[n-n_0]$ is $X_w[n-n_0,\omega)$. HINT: A filter is a time-invariant system.

b) Let $X_w[n,\omega)$ denote the TDFT of a real-valued signal $x[n]$ with respect to a real-valued analysis window $w[n]$. Use the Fourier transform view of the TDFT to argue that $X_w[n,\omega) = X_w^*[n,-\omega)$

c) Using the filtering view of the TDFT, construct a *counterexample* to show that the TDFT of the convolution of two signals is not necessarily equal to the product of their individual TDFTs.

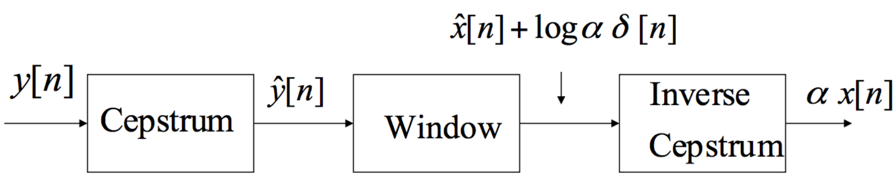
Problem 11.3 (20 points)

Let $g[n]$ be a 128-point signal whose Parametric Signal Model is given as $H(z) = \frac{G}{1+\sum_{k=1}^{10} a_k z^{-k}}$.

Show that the values of $H\left(e^{j\frac{2\pi k}{256}}\right)$ for $k = 0,1,\cdots,255$ can be calculated by taking the DFT of some signal (using, say, an FFT algorithm) and taking the reciprocal of those DFT values.

Problem 11.4 (20 points)

Consider the following situation:



In this situation, $y[n] = x[n] * h[n]$ and $h[n]$ is known to be minimum phase, while $x[n]$ is known to be maximum phase. The windowing operation uses $w[n] = u[-n]$ where $u[n]$ is the unit step.

Show that the final output after the inverse cepstrum operation is $\alpha x[n]$, where α is some constant.

Problem 11.5 (20 points)

(a) What shift and what amplitude scaling would you perform on the signal $x_c[n] = -2\delta[n+2] + 4\delta[n+1]$ such that the resulting signal has a z-transform that satisfies the restricted model for computing the complex cepstrum. *Justify your answer*.

(b) Suppose $x_b[n] = (0.5)^n u[n]$. Let $g_b[n]$ be a signal such that $\hat{g}_b[n] + \hat{x}_b[n] = -\frac{-(0.5)^n}{n} u[n-1]$, where $\hat{g}_b[n]$ and $\hat{x}_b[n]$ are the respective complex cepstra of $g_b[n]$and $x_b[n]$. Sketch $g_b[n]$. *Justify your answer*.

Experiential DSP 11 (100 points)

(A) Use MATLAB to record your voice while you say the word "sad" using a sampling rate of 16 KHz. Splice off the beginning portion from the recorded signal so that the resulting signal $x[n]$ has no silence preceding the sound of the "s" in "sad." Use MATLAB to plot the signal $x[n]$.

(B) Let $w[n] = u[n] - u[n-256]$ and let $f_n[m] = w[m]x[n+m]$.

  (a) Use MATLAB to calculate and plot $f_0[m]$ for $0 \leq m < 256$.

  (b) Determine an integer value n_0 such that $f_{n_0}[m]$ falls in a portion of $x[n_0 + m]$ during which the "a" sound is being spoken during "sad." Use MATLAB to calculate and plot $f_{n_0}[m]$ for $0 \leq m < 256$. You should observe that this signal has a somewhat periodic structure.

  (c) Plot (using MATLAB) the magnitude of the 512-point DFT of $f_{n_0}[m]$ you observed in the previous part. How is this plot consistent with the fact that the corresponding short-time section has a somewhat periodic structure.