HW 12: FALL 2025 EC516 PRACTICE FINAL EXAM

*Closed Book; Total Time=**60 minutes**; No collaboration with anyone; No electronics (such as cell phones, calculators etc.*

HW12 is optional to submit. If you wish to submit it to make up for a homework assignment you did not submit, you may submit HW 12 by Wednesday December 10 before 11:59pm. Please note that this homework also serves as a Practice Final Exam. Also the Final Exam formula sheet is at the end of this homework.

Problem 1 (50 points)

(a) (20 points) (10 points) Sketch a flowgraph for the IIR filter that has impulse response $h[n] = (0.5)^n \sin\left(\frac{\pi n}{2}\right) u[n]$. *Justify your answer.*

(b) Consider a digital filter ***F*** with frequency response $H\left(e^{j\omega}\right) = e^{-j2\omega}\left(\frac{\sin(\omega)}{\sin\left(\frac{\omega}{2}\right)}\right)^4$. Is it possible to take the 5-point DFT of the impulse response of the filter ***F***? *Justify your answer.*

(c) (20 points) Let $x[n]$ be a 4-point signal with 7-point DFT $X[k]_7$. Furthermore, let the signal $y[n]$ be given as $y[n] = \sum_{k=0}^{3} x[k]x[n-k]$. Would it be correct to say that $Y[k]_7 = (X[k]_7)^2$? *Justify your answer.*

Problem 2 (50 Points)

(a) (20 Points) Let $Q_w[n,\omega)$ be the TDFT of $q[n]$ with respect to analysis window $v[n] = u[n] - u[n-128]$. The discrete TDFT of $q[n]$ is computed as $Q_v[n,k] = Q_v[2n, \frac{2\pi k}{256})$. If it is observed that $Q_v[2,k] = 0$ for $0 \le k \le 255$, for what values of n is it possible to determine the numerical values of the signal $q[n]$. *Justify your answer.*

(b) (20 Points) Let $g[n] = (0.5)^n\{u[n] - u[n-4]\}$. Let the 2-pole PSM model of $g[n]$ be denoted by $h[n]$. Determine the numerical value of $A = \sum_{k=-\infty}^{\infty} h^2[k]$. *Justify your answer.*

(c) (10 Points) If $x[n]$ is a signal that satisfies the restricted model for determining the complex cepstrum, does the signal $(0.5)^n x[n]$ satisfy the restricted model for determining its complex cepstrum? *Justify your answer.*

EC516 FORMULA SHEET FINAL EXAM EC516 FALL 2025

Unit Step

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases}$$

Unit Impulse

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

Complex Exponentials and Sinusoids

$$e^{j\omega n} = \cos(\omega n) + j\sin(\omega n) \quad \cos(\omega n) = (1/2)(e^{j\omega n} + e^{-j\omega n}) \quad \sin(\omega n) = (1/2j)(e^{j\omega n} - e^{-j\omega n})$$

CTFT: $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$; **Inverse CTFT**: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$

Basic CTFT Properties:

$x(t - t_0) \Leftrightarrow e^{-j\omega t_0}X(j\omega)$ $\quad x(-t) \Leftrightarrow X(-j\omega)$ $\quad x(t) * h(t) \Leftrightarrow X(j\omega)H(j\omega)$

Common CTFT Pairs

$e^{j\omega_0 t} \Leftrightarrow 2\pi\delta(\omega - \omega_0)$ $\quad \delta(t - t_0) \Leftrightarrow e^{-j\omega t_0}$ $\quad u(t+T) - u(t-T) \Leftrightarrow 2\sin(\omega T)/\omega$

$\sin(\omega_0 t)/\pi t \Leftrightarrow u(\omega + \omega_0) - u(\omega - \omega_0)$ $\quad \sum_{k=-\infty}^{\infty}\delta(t - kT) \Leftrightarrow \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - (2\pi k/T))$

DTFT: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$; **Inverse DTFT**: $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$

DT Convolution : $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ **CT Convolution** : $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

FSF : $\sum_{k=0}^{N-1}\alpha^k = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & ;\alpha \neq 1 \\ N & ;\alpha = 1 \end{cases}$ **ISF** : $\sum_{k=0}^{\infty}\alpha^k = \frac{1}{1-\alpha} \quad ;|\alpha| < 1$

Basic DTFT Properties:

$x[n - n_0] \Leftrightarrow e^{-j\omega n_0}X(e^{j\omega})$ $\quad e^{j\omega_0 n}x[n] \Leftrightarrow X(e^{j(\omega-\omega_0)})$ $\quad x^*[n] \Leftrightarrow X^*(e^{-j\omega})$

$x[-n] \Leftrightarrow X(e^{-j\omega})$ $\quad x[n] * h[n] \Leftrightarrow X(e^{j\omega})H(e^{j\omega})$

Common DTFT Pairs

$e^{j\omega_0 n} \Leftrightarrow 2\pi\sum_{k=-\infty}^{\infty}\delta(\omega - k\omega_0)$ $\quad \delta[n - n_0] \Leftrightarrow e^{-j\omega n_0}$ $\quad u[n] - u[n-N] \Leftrightarrow \frac{\sin(\omega N/2)}{\sin(\omega/2)}e^{-j\omega(N-1)/2}$

$$\frac{\sin\omega_0 n}{\pi n} \Leftrightarrow \begin{cases} 1 & 0 \leq |\omega| \leq \omega_0 \\ 0 & \omega_0 < |\omega| \leq \pi \end{cases}$$

z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Properties of z-transform:

$x[n - n_0] \Leftrightarrow z^{-n_0}X(z)$ $\quad x[-n] \Leftrightarrow X(z^{-1})$ $\quad x^*[n] \Leftrightarrow X^*(z^*)$

$x[n] * h[n] \Leftrightarrow X(z) \times H(z)$

Basic z-transform pair: Decaying Exponential $a^n u[n] \Leftrightarrow \frac{1}{1 - az^{-1}}$ ROC: $|z| > |a|$

Difference Equation

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{m=0}^{M} b_m x[n-m]$$

Sampling Rate Increase by M:

$$y[n] = \begin{cases} x[n/M] & for \quad n = kM \\ 0 & otherwise \end{cases} \qquad Y(e^{j\omega}) = X(e^{j\omega M})$$

Sampling Rate Decrease by M:

$$y[n] = x[nM] \qquad Y(e^{j\omega})\frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\frac{\omega}{M}-\frac{2\pi k}{M})})$$

Digital Upsampling by factor of M:
Increase sampling rate by M and then apply interpolation filter with cutoff π/M
Digital Downsampling by factor of M:
Apply antialiasing filter with cutoff π/M and then decrease sampling rate by M.
Linear Phase FIR Filters:

Type I: Odd Length, Symmetric — Type II: Even Length, Symmetric
Type III: Odd Length, Anti-symmetric — Type IV: Even Length, Anti-symmetric

FIR Design - Order of KW Method: $N = (-10\log(\delta_{min}^2) - 8)/2.285\Delta\omega$
FIR Design - Order of PM Method: $N = (-10\log(\delta_p\delta_s) - 13)/2.324\Delta\omega$
N-Point Signal: $h[n] = 0$ for $n<0$ and for $n>N-1$

DFT: $X[k]_N = \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$ $\qquad 0 \le k \le N-1$ and zero for $k < 0$ and $k > N-1$

Inverse N-point DFT: $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]_N e^{j2\pi kn/N} \qquad 0 \le n \le N-1$

2-D DTFT:$X(e^{j\omega_1}, e^{j\omega_2}) = \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} x[n_1, n_2]e^{-j(\omega_1 n_1 + \omega_2 n_2)}$
2-D Convolution:$x[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} h[k,p]x[n-k, m-p]$
Radix-2 FFT Computation: $2N log_2 N$ real multiplications
Time-Dependent Fourier Transform

$$X_w[n,\omega) = \sum_{m=-\infty}^{\infty} w[m]x[n+m]e^{-j\omega m} = x[n] * w[-n]e^{j\omega n} \qquad X_w[n,k] = X_w[nL, 2\pi k/M)$$

PSM Model: $\dfrac{G}{1+\sum_{k=1}^{P} a_k z^{-k}}$;**Indirect Least Squares**: Minimize $E_I = \sum_{n=0}^{\infty}(x[n] + \sum_{k=1}^{P} a_k x[n-k])^2$

Properties of PSM Model: Guaranteed stable and autocorrelation of model matches autocorrelation of data.

Cepstrum: If $\hat{x}[n]$ is complex cepstrum of $x[n]$, then $\hat{x}[n]$ has the z-transform $\hat{X}(z) = \log(X(z))$.

Restricted z-transform model for defining a complex cepstrum:

$$X(z) = G\frac{\prod_{k=1}^{M_i}(1-a_k z^{-1})\prod_{l=1}^{M_o}(1-b_l z)}{\prod_{r=1}^{N_i}(1-c_r z^{-1})\prod_{s=1}^{N_o}(1-d_s z)}$$

where $G > 0, \quad |a_k| < 1, \quad |b_l| < 1, \quad |c_r| < 1, \quad |d_s| < 1$.

Series Expansion of Natural Logarithm: $\log(1-x) = -\sum_{n=1}^{\infty}\frac{x^n}{n}$

Complex Cepstrum of $\delta[n-n_0]$: $-n_0\frac{\cos(\pi n)}{n}$ for $n \neq 0$ and 0 otherwise.