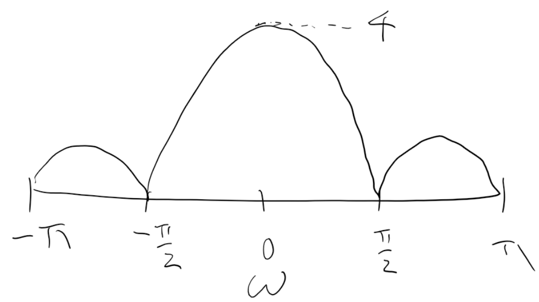
EC516 HW08 Solutions

Problem 08.1

(a)

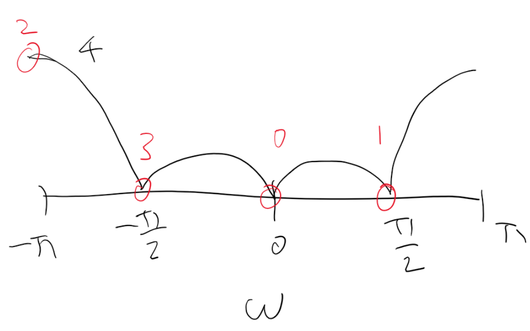
$$X(e^{j\omega}) = e^{-j\frac{3}{2}\omega}\frac{\sin(2\omega)}{\sin(\omega/2)}$$



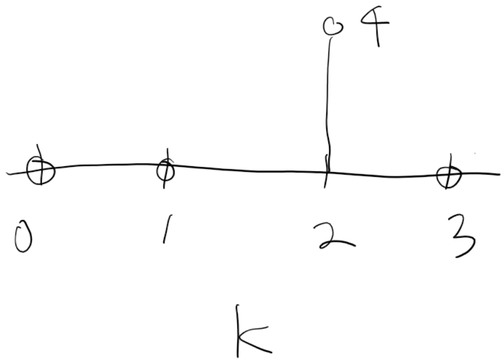
(b) Zero-crossings of $X(e^{j\omega})$ in $-\pi \leq \omega < \pi$ exist at $-\frac{\pi}{2}$, $-\pi$, $\frac{\pi}{2}$. These correspond to $k = 1$, $k = 2$, and $k = 3$.

(c) With the same logic, the zero crossings corresponds to $k = 2$, $k = 4$, and $k = 6$.

(d) $(-1)^n = e^{j\pi n}$. Multiplying $x[n]$ with $e^{j\pi n}$ will result in π frequency shift in the DTFT, as shown below.



$k = 0, 1, 2, 3$ of the 4-point DFT corresponds to DTFT at $\omega = 0, \frac{\pi}{2}, -\pi, -\frac{\pi}{2}$. Hence the 4-point DFT will look like below.



Problem 08.2

A(a) $Q[k]_{256} = Q\left(e^{j\frac{2\pi k}{256}}\right)$. Also $Q\left(e^{-j\frac{\pi}{2}}\right) = Q\left(e^{j\frac{3\pi}{2}}\right)$. Hence, $Q[k_0]_{128}$ becomes $Q\left(e^{-j\frac{\pi}{2}}\right)$ at $k_0 = 192$.

A(b) $R(e^{j\omega}) = e^{j16\omega}Q(e^{j\omega})$. Therefore, $R(e^{j\omega})$ and $Q(e^{j\omega})$ have the same magnitude.

Assuming that $q[n] = 0$ for $n < 0$ and $n \geq 128$, then $r[n] = 0$ for $n < 16$ and $n \geq 144$. Since shifting does not cause the signal to go out of bounds of the DFT ($n < 0$ or $n \geq 256$), both DFT will have the same magnitude.

Problem 08.3

(a)

$$
\begin{aligned}
Q[k]_N &= \sum_{n=-\infty}^{\infty} p[n] e^{-j\frac{2\pi k}{N}n} \\
&= \sum_{n=0}^{N} \sum_{m=-\infty}^{\infty} p[n-mN] e^{-j\frac{2\pi k}{N}(n-mN)} \\
&= \sum_{n=0}^{N} \underbrace{\sum_{m=-\infty}^{\infty} p[n-mN]}_{q[n]} e^{-j\frac{2\pi k}{N}n}
\end{aligned}
$$

$q[n]$ is periodic with N.

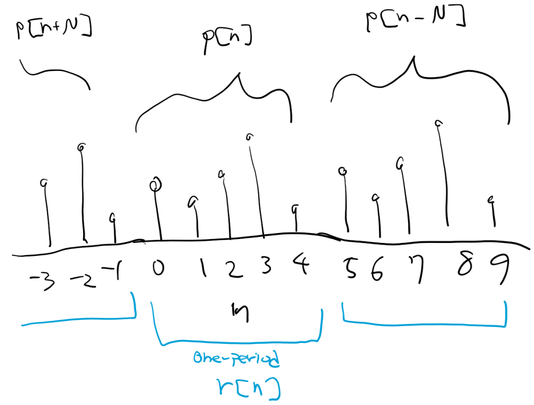
$$
\begin{aligned}
q[n+N] &= \sum_{m=-\infty}^{\infty} p[(n+N)-mN] \\
&= \sum_{m=-\infty}^{\infty} p[n-(m-1)N] \\
&= \sum_{m=-\infty}^{\infty} p[n-mN] \\
&= q[n]
\end{aligned}
$$

Same cannot be same for shift smaller than N and bigger than 1. Thus, the smallest period is N.

(b) Because $p[n]$ is an N-point signal,

$$
\begin{aligned}
q[n] &= \sum_{m=-\infty}^{\infty} p[n-mN] \\
&= p[n-mN] \quad \text{for } mN \le n < (m+1)N
\end{aligned}
$$

An example for $N = 5$ is shown below.



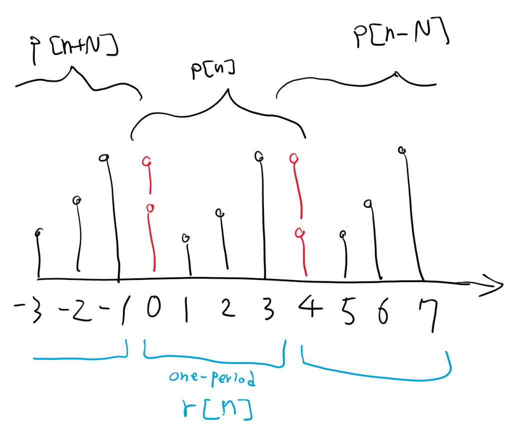
Since the $p[n]$ is an N-point signal and the shifted counterparts are also separated by N, there are no overlaps. Taking the first period of the signal would equal to $p[n]$.

(c)

$$Q[k]_{N-1} = \sum_{n=0}^{N-1} \underbrace{\sum_{m=-\infty}^{\infty} p[n - m(N-1)]}_{q[n]} e^{-j\frac{2\pi k}{N-1}n}$$

Using the same logic from part (a), we can see that the period of this signal is $N - 1$.

(d) An example of $q[n] = \sum_{k=-\infty}^{\infty} p[n - m(N-1)]$ when $N = 5$ is shown below.



Unlike previously, there are overlaps. Taking the first signal would give us

$$r[n] = \begin{cases} p[0] + p[N-1] & \text{for } n = 0 \\ p[n] & \text{for } 1 \leq n < N-1 \\ 0 & \text{otherwise} \end{cases}$$

(e)

$$Q[k]_{N-2} = \sum_{n=0}^{N-2} \underbrace{\sum_{m=-\infty}^{\infty} p[n - m(N-2)]}_{q[n]} e^{-j\frac{2\pi k}{N-2}n}$$

Smallest period is $N-2$.

(f) $q[n] = \sum_{k=-\infty}^{\infty} p[n - m(N-2)]$ will now cause 2 samples from $p[n]$ to be contaminated. Taking the first period will yield

$$r[n] = \begin{cases} p[0] + p[N-2] & \text{for } n = 0 \\ p[1] + p[N-1] & \text{for } n = 1 \\ p[n] & \text{for } 2 \leq n < N-2 \\ 0 & \text{otherwise} \end{cases}$$

Problem 08.4

(a) $\overline{X}[k]_2 = \sum_{n=0}^{1} x[n]\, e^{-j\frac{2\pi k}{2}n}$ for $k=0,1$

$\therefore \overline{X}[0]_2 = x[0] + x[1]$

and $\overline{X}[1]_2 = x[0] - x[1]$

(b) $\overline{X}[k]_6 = \overline{X}(e^{j\frac{2\pi k}{6}}) = \overline{X}(e^{j\pi})$ for $k=3$

$\downarrow$

$x[0] - x[1] + x[2] - x[3] + x[4] - x[5]$