FALL 25 EC516 Homework 03

Due: Sunday September 28 (Before 11:59pm)

You must submit your homework attempt on Blackboard. For this purpose, you must convert your homework attempt to a single pdf file and upload it at the corresponding homework assignment on Blackboard.

Problem 2.1 (Fractional Digital Sampling)

Suppose we are interested in preserving analog frequencies up to 10Khz while sampling a real-valued analog signal $x(t)$ to produce a digital signal $x[n]$. Initially, we use analog sampling with a sampling frequency of 50 KHz and an analog antialiasing filter (with a flat gain of 1.0 up to 10Khz and a gradual tapering of its gain down to 0.0 at frequencies beyond 25KHz) to produce a signal $g[n]$.

(a) By what factor would you downsample the initial analog sampling result $g[n]$ to produce the desired signal $x[n]$?

(b) Explain how you would use operations of *sampling rate change* and ideal lowpass *digital filtering* to produce $x[n]$ from $g[n]$. You must specify the sampling rate changes that would be employed and the cutoff frequencies of any filters employed in your proposed solution.

Problem 2.2 (z-transform Basics)

PART: A

Let $x[n]$ be a discrete time signal with z-transform $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

(a) Show that $x[n - n_0]$ has z-transform $z^{-n_0}X(z)$

(b) Show that $x[-n]$ has z transform $X(z^{-1})$

(c) Show that $x^*[n]$ has z-transform $X^*(z^*)$

(d) Show that $x[n] * h[n]$ has z-transform $X(z)H(z)$

PART B:

Determine the z-transform of each of the following signals and the corresponding region of convergence in each case, the values of $|z|$ for which $X(z)$ converges. You may use $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$, the Finite Sum Formula and/or the Infinite Sum Formula.

(a) $x[n] = \delta[n - 3]$

(b) $x[n] = u[n] - u[n - 5]$

(c) $x[n] = (0.25)^n u[n]$

(d) $x[n] = (0.25)^{n-1} u[n - 1]$

(e) $x[n] = (0.25)^n u[n - 1]$

(f) $x[n] = (0.25)^n u[n] + (0.5)^n u[n]$

(g) $x[n] = (0.25)^n \cos(0.25\pi n) u[n]$

Problem 2.3 (Practical Filter Implementation)

Consider a digital filter with impulse response $h[n]$ which is zero for $n<0$ and whose input $x[n]$ and output $y[n]$ are related through the following difference equation:

$$y[n] = 0.25y[n-2] + x[n]$$

By noting that the impulse response $h[n]$ is the output of the digital filter when the input is $\delta[n]$, use the above difference equation to determine the numerical values of $h[0], h[1], h[2], h[3]$ & $h[1001]$. Justify your answers

Problem 2.4

PART A:

Calculate the convolution $y[n] = x[n] * h[n]$ in each of the following cases and *show your work*:

a) $x[n] = u[n] - u[n-5]$ and $h[n] = 0.5\delta[n-3]$

b) $x[n] = n\{u[n-1] - u[n-5]\}$ and $h[n] = 2\delta[n+3]$

c) $x[n] = u[n] - u[n-5]$ and $h[n] = u[n] - u[n-5]$

d) $x[n] = u[n] - u[n-5]$ and $h[n] = u[n] - u[n-3]$

e) $x[n] = u[n] - u[n-5]$ and $h[n] = u[n]$

PART B:

Sketch the phase of the DTFT of $x[n] = u[n] - u[n-4]$. *Justify your answer*.

PART C:

Sketch the magnitude and the phase of the DTFT of the signal $r[n] = \sum_{k=-\infty}^{\infty} x[k]x[n-k]$, where $x[n] = u[n] - u[n-6]$. *Show your work*.

PART D:

a) *Show* that if $x[n]$ has DTFT $X(e^{j\omega})$, then $x[n-n_0]$ has DTFT $e^{-j\omega_0}X(e^{j\omega})$

b) *Show* that a real and even $x[n]$ has a real and even DTFT $X(e^{j\omega})$.

c) *Show* that a real and odd $x[n]$ has an imaginary and odd DTFT $X(e^{j\omega})$.

Experiential DSP Exercise 03

A. Record and Load!

a. Open the MATLAB software.
b. Set these variables: `Fs = 16000; nBits = 16; nChannels = 1;`
c. Create an audio recorder object using the `recObj = audiorecorder (Fs, nBits, nChannels)` command and pass the defined variables in step b to it.
d. Use the `record(recObj)` command to start recording. In the recording, you need to say: **"Six students went to school".**
e. Write the command `pause`. With this command, you can stop recording by pressing any key on your keyboard.
f. Use the `stop(recObj)` command to stop recording.
g. Get the recorded data using by `getaudiodata(recObj)` command.
h. Save the recorded audio using the `audiowrite (filename, audioData, Fs);` choose `filename` as: **`audio.wav`.**
i. Read the audio and sampling rate using the `audioread` command. (Note that this command gives you **two** outputs, `x` and `F`)

B. Digital Upsampling

a. Set `U = 4`. Create `isr_x = upsample (x, U)`. This `upsample` Matlab command is badly named because it does **not** upsample a signal, rather it merely *increases the sampling rate* by a factor of 4 with three zero-valued samples inserted between each pair of consecutive samples of `x`. To obtain the digitally upsampled signal we will have to pass `isr_x` through an interpolation filter (see steps c and d below).
b. Create a stem plot of `x` and `isr_x` in two separate figures. Let's call the figures "Original" and "IncreasedSR" for future reference.
c. Load the given non-ideal interplolation filter `load ('filter1.mat', 'b');` Pass the IncreasedSR signal through the loaded filter: `upsampled_x = filter (b, 1, isr_x);`
d. Create a stem plot of `upsampled_x`. Let's call this figure "Upsampled".
e. Now comment on the differences between the "Original," "IncreasedSR," and "Upsampled" figures.

C. Digital Downsampling

a. Load the antialiasing filter: `load ('filter2.mat', 'b')`. Pass the upsampled signal `upsampled_x` from Part B above through this digital antialiasing filter: `filtered_upsampled_x = filter (b, 1, upsampled_x)`.
b. Set `D = 4`, create `downsampled_upsampled_x = downsample (filtered_upsampled_x, D)`. The `downsample` Matlab command is badly named because it does **not** downsample, rather it merely decreases the sampling rate without first applying a digital anti-aliasing filter.
c. Create a stem plot of `downsampled_upsampled_x`. Is it the same as "Original"? Explain why there might be differences.