Problem 1.

a) $y[n] = -\left(\frac{1}{9}\right)y[n-2] + x[n] - \left(\frac{1}{2}\right)x[n-1]$

let $y[n] = h[n]$, $x[n] = \delta[n]$

$\rightarrow h[n] = -\left(\frac{1}{9}\right)h[n-2] + \delta[n] - \left(\frac{1}{2}\right)\delta[n-1]$

let $h[n<0] = 0$

$n=0$: $h[0] = -\left(\frac{1}{9}\right)h[-2] + \delta[0] - \left(\frac{1}{2}\right)\delta[-1] = -\left(\frac{1}{9}\right)0 + 1 + \left(\frac{1}{2}\right)0 \cdot 0 =$

$h[1] = -\left(\frac{1}{9}\right)h[-1] + \delta[1] - \left(\frac{1}{2}\right)\delta[0] = 0 + 0 - \frac{1}{2} = -\frac{1}{2}$

$h[2] = -\left(\frac{1}{9}\right)h[0] + \delta[2] - \left(\frac{1}{2}\right)\delta[1] = -\left(\frac{1}{9}\right) + 0 - 0 = -1/9$

$h[3] = -\left(\frac{1}{9}\right)h[1] + \delta[3] - \left(\frac{1}{2}\right)\delta[2] = \frac{1}{18} + 0 - 0 = \frac{1}{18}$

$h[4] = -\left(\frac{1}{9}\right)h[2] + \delta[4] - \left(\frac{1}{2}\right)\delta[3] = \frac{1}{81} + 0 - 0 = \frac{1}{81}$

$h[5] = -\left(\frac{1}{9}\right)h[3] + \delta[5] - \left(\frac{1}{2}\right)\delta[4] = -\frac{1}{162} + 0 - 0 = -1/11$

b) $H(e^{j\omega}) = 0$ at $\omega = \frac{2\pi}{8}k$ for $k = 2, 4, 6$

$= \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

FIR filter w/ zeros are $\omega = \underbrace{\pm \pi/2}_{\downarrow}$ and $\underbrace{\pi}_{\hookrightarrow (1+z^{-1})}$

$(1+z^{-2})$

$H(z) = (1+z^{-1})(1+z^{-2}) = 1 + z^{-1} + z^{-2} + z^{-3}$

$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$$H[k] = \sum_{n=0}^{3} e^{-j\frac{2\pi nk}{8}} = \sum_{n=0}^{3} \left(e^{-j\frac{\pi k}{4}}\right)^n$$

c) $y[n_1, n_2] = \underbrace{x[n_1, n_2]}$

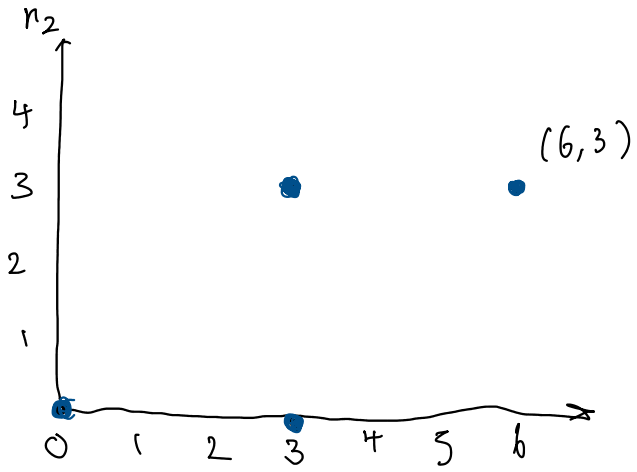
$$\sum_{k_1=0}^{3}\sum_{k_2=0}^{3} \delta[n_1-k_1,\ n_2-k_2]$$

$y[n_1, n_2] = 1$, let $m_1 = n_1 - n_2$

$m_2 = n_2 \cdot y[n_1, n_2] = 1$ when $x[m_1, m_2] = 1$

$0 \le m_2 \le 3$: $0 \le n_2$:

$0 \le m_1 \le 3$: $0 \le n_1 -$



Problem 2

a) (i) Yes, $N = 4$ DFT matrix: $W_4^{nk} = e^{-j\frac{2\pi}{4}nk} = e^{-j\pi/2\, nk}$

for $k = 0, 1, \ldots, 3$: $\{1, -j, -1, j\}$

(ii) No multiplication

b) $H(e^{j\omega}) = (\underbrace{1 + 0.5e^{-j\omega} + 0.25e^{-j2\omega}}_{\downarrow})^2$

let $P(e^{j\omega}) =$

$P[n] = \delta[n] + 0.5\delta[n-1] + 0.25\delta[n-2] \rightarrow$ length 3, n

$H(e^{j\omega}) = P(e^{j\omega}) \cdot P(e^{j\omega}) \Rightarrow h[n] = P[n] * P[n] \rightarrow$ length =