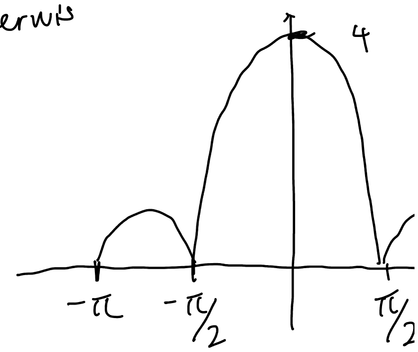
Problem 1.

$$X[k]_N = \begin{cases} X\left(e^{\frac{j2\pi k}{N}}\right) & ; \quad 0 \le k < N \\ 0 & ; \quad \text{otherwis} \end{cases}$$

$x[n] = u[n] - u$



a) $\bar{X}(e^{j\omega}) = \sum_{n=0}^{3} e^{-j\omega n} = e^{-j\frac{3\omega}{2}} \frac{\sin(2\omega)}{\sin(\omega/2)}$

$|\bar{X}(e^{j\omega})| = \left|\frac{\sin(2\omega)}{\sin(\omega/2)}\right|$

b) $\omega_k = \frac{2\pi k}{4} = \frac{k\pi}{2}$ for $k = 0, 1, 2, 3$

$\rightarrow X[k]_4 = 0$ for $\underline{k = 1, 2, 3}$

c) $\omega_k = \frac{2\pi k}{8} = \frac{k\pi}{4}$

$\sin(2\omega_k) = 0$

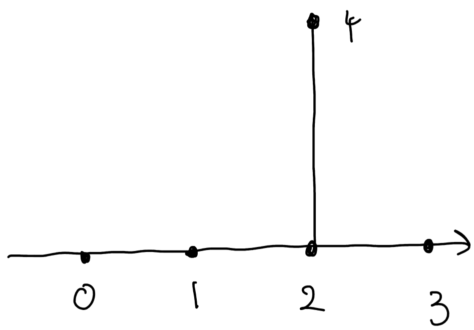
$\hookrightarrow \omega = \frac{\pi}{2}$

$\rightarrow k =$ multiple

not 4

$k = \underline{2, 6}$

d) $\bar{Y}(e^{j\omega}) = \bar{X}(e^{j(\omega - \pi)})$



Problem 2

a) $Q[k_0]_{256} = Q(e^{-j\pi/2})$

let $\omega_{k_0} = -\frac{\pi}{2}$, $\frac{2\pi k_0}{256} = -\frac{\pi}{2} \rightarrow k_0 = -\frac{\pi}{2} \cdot \frac{256}{2\pi} =$

$k \in [0, 256) \Rightarrow k = 256 - 64 = \underline{192}$

Problem 3

a) $P(e^{j\omega})$ sampling every $\frac{2\pi}{N}$ $\quad T_{New} = \frac{2\pi}{\left(\frac{2\pi}{N}\right)} = \underline{N}$

b) $r[n] = q[n]\cdot(u[n]-u[n-N]) = \sum_{m=-\infty}^{\infty} p[n-mN]\cdot(u[n]-u[n-N])$

for $n \in [0, N-1]$, $\sum_{m=-\infty}^{\infty} p[n-mN] \neq 0$ for $m=0$ where $\rightsquigarrow p[n]$

$m \neq 0$, $p[n-mN]$ out of $[0, N-1]$

$\therefore r[n] = p[n]\cdot\left(u[n]-u[n-N]\right) = p[n]$

c) sampling every $\frac{2\pi}{N-1}$, $T_{New} = \frac{2\pi}{(2\pi/N-1)} = \underline{N-1}$

d) $r[0] = q[0] = \sum_{m=-\infty}^{\infty} p[0-m(N-1)] = p[0] + p[N-1]$

$\rightarrow m=0:\ p[0-0] \rightarrow p[0]$

$m=1:\ p[0-(N-1)] \rightarrow p[-N+1]$

$m=-1:\ p[0-(-1)(N-1)] = p[N-1]$

$\rightarrow p[N-1]$

for $n<0$; $p[n]=0$

e) Sampling every $\frac{2\pi}{N-2}$, $T_{New} = N-2$

f) $r[n] = q[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-2)]$

$n=0$

$m=0:\ p[0]$

$m=-1:\ p[N-2]$

$r[0] = p[0] + p[N-2]$

$n=1$

$m=0:\ p[1]$

$m=-1:\ p[1+(N-2)] = p[N-1]$

$r[1] = p[1] + p[N-1]$