EC516 HW04 Solutions

Problem 4.1

(a) Flowgraph is shown below. This implementation requires 4 retrievals and 4 additions per output sample. No multiplication is needed since all the coefficients are 1's.

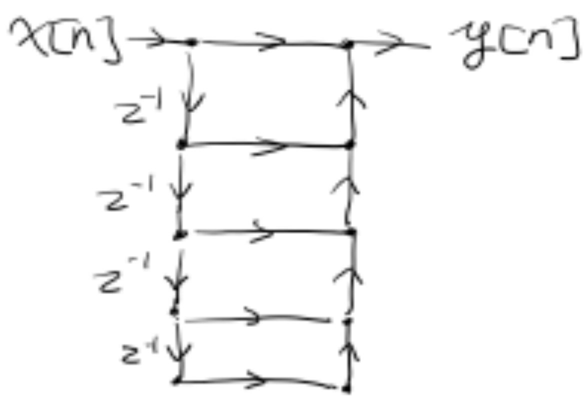


Figure 1: FIR flowgraph

(b)

$$
\begin{aligned}
Y(z) &= X(z) + z^{-1}X(z) + z^{-2}X(z) + z^{-3}X(z) + z^{-4}X(z) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} \\
&= \sum_{n=0}^{4} z^{-n}
\end{aligned}
$$

(c) We first substitute $z = e^{j\omega}$ to obtain the frequency response.

$$
\begin{aligned}
H(e^{j\omega}) &= \sum_{n=0}^{4} e^{-j\omega n} \\
&= \frac{1 - e^{j5\omega}}{1 - e^{j\omega}} \\
&= \frac{e^{j5\omega/2}(e^{-j5\omega/2} - e^{j5\omega/2})}{e^{j\omega/2}(e^{-j\omega/2} - e^{j\omega/2})} \\
&= e^{j2\omega}\frac{-2j\sin(5\omega/2)}{-2j\sin(\omega/2)} \\
&= e^{j2\omega}\frac{\sin(5\omega/2)}{\sin(\omega/2)}
\end{aligned}
$$

The magnitude of the frequency response is

$$
\begin{aligned}
|H(e^{j\omega})| &= \left| e^{j2\omega} \frac{\sin(5\omega/2)}{\sin(\omega/2)} \right| \\
&= \frac{\sin(5\omega/2)}{\sin(\omega/2)}
\end{aligned}
$$

$|H(e^{j\omega})|$ is drawn in the figure below.

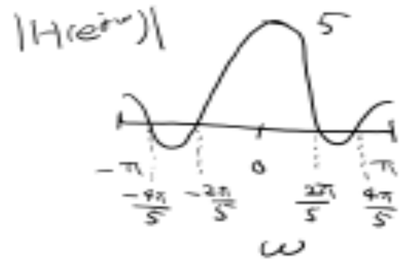


Figure 2: Magnitude of $H(e^{j\omega})$

(d) The filter is a FIR filter with finite coefficients. It is stable.

Problem 4.2

(a) Flowgraph is shown below. This implementation requires 2 retrievals, 2 multiplication, and 2 addition per output sample.

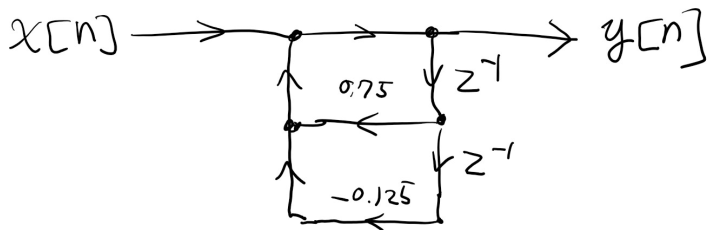


Figure 3: IIR flowgraph

(b)

$$
\begin{aligned}
Y(z) &= -0.125z^{-2}Y(z) + 0.75z^{-1}Y(z) + X(z) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{1}{0.125z^{-2} - 0.75z^{-1} + 1} \\
&= \frac{8z^2}{1 - 6z + 8z^2} \\
&= \frac{8z^2}{(1-2z)(1-4z)}
\end{aligned}
$$

(c) The poles of the filter are $z = \frac{1}{2}$ and $z = \frac{1}{4}$. Since all poles are inside the unit circle on the complex plane, this filter is stable.

(d) This filter has 2 zeros at $z = 0$. However since there are not on the unit circle of the complex plane, $|H(e^{j\omega})|$ is non-zero everywhere.

Problem 4.3

(a) The flowgraph is shown below.

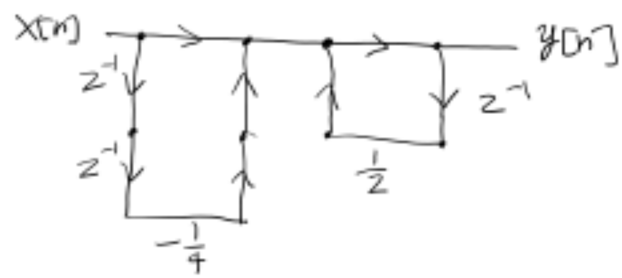


Figure 4: Recursive implementation of FIR

(b)

$$
\begin{aligned}
Y(z) - \frac{1}{2}z^{-1}Y(z) &= X(z) - \frac{1}{4}z^{-2}X(z) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{1 - \frac{1}{4}z^{-2}}{1 - \frac{1}{2}z^{-1}} \\
&= \frac{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)}{1 - \frac{1}{2}z^{-1}} \\
&= 1 + \frac{1}{2}z^{-1} \\
Y(z) &= X(z) + \frac{1}{2}z^{-1}X(z) \\
y[n] &= x[n] + \frac{1}{2}x[n-1]
\end{aligned}
$$

(c) The new flowgraph is shown below.

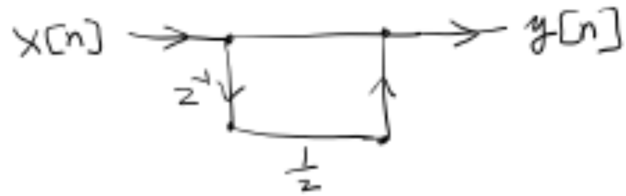


Figure 5: Non-Recursive implementation of FIR

(d) The implementation in (a) requires 2 retrievals, 2 multiplications, and 2 additions. While the implementation in (c) requires 1 retrieval, 1 multiplication, and 1 addition.

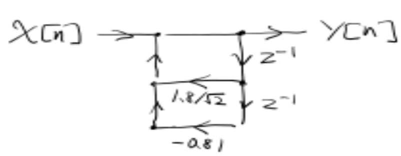
<u>Problem 4.4</u>

(a) Since $H(z) = Y(z)/X(z)$,

$$
\begin{aligned}
(1 - 0.9e^{\frac{j\pi}{4}}z^{-1})(1 - 0.9e^{-\frac{j\pi}{4}}z^{-1})Y(z) &= X(z) \\
(1 - 1.8\cos(\pi/4)z^{-1} + 0.81z^{-2})Y(z) &= X(z) \\
y[n] - 1.8\cos(\pi/4)y[n-1] + 0.81y[n-2] &= x[n] \\
y[n] &= (1.8/\sqrt{2})y[n-1] - 0.81y[n-2] + x[n]
\end{aligned}
$$

(b) All coefficients are real. Real valued input will generate real valued output signal.

(c) Since it's FIR, Direct II and Direct I is equivalent.

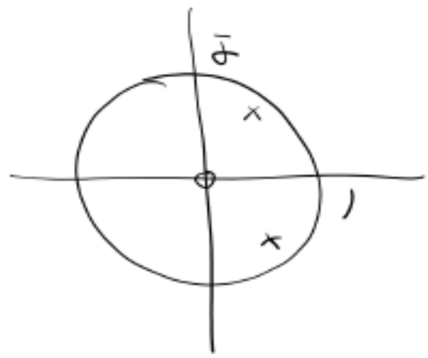


Problem 4.4(c): Direct II Flowgraph

(d) We multiply both numerator and denominator with z's to get rid of z^{-1}'s.

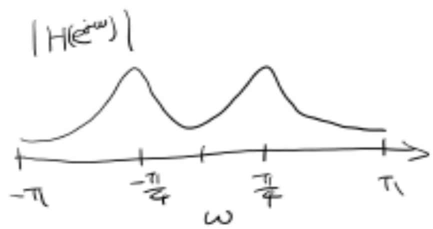
$$
\frac{1}{(1 - 0.9e^{\frac{j\pi}{4}}z^{-1})(1 - 0.9e^{-\frac{j\pi}{4}}z^{-1})} = \frac{z^2}{(z - 0.9e^{\frac{j\pi}{4}})(z - 0.9e^{-\frac{j\pi}{4}})}
$$

There are poles at $z = 0.9e^{\pm\frac{j\pi}{4}}$ and 2 zeros at $z = 0$.



Problem 4.4(d): Poles (x) and Zeros (o) on a complex plane

(e) Recall that $|\frac{ab}{c}| = \frac{|a||b|}{|c|}$ for complex numbers a, b, c. Using the zero-pole plot, we can get a sense of the frequency response.



Problem 4.4(e): Approximate $|H(e^{j\omega})|$

Since the zeros do not exist on the unit circle, there are no zeros in $H(e^{j\omega})$.