# Problem 1.

A) $x[n] = u[n] - u[n-5]$

a) $\bar{X}(e^{j\omega}) = e^{-j\omega \, 4/2} \cdot \dfrac{\sin(5\omega/2)}{\sin(\omega/2)}$

$|\bar{X}(e^{j\omega})|$


$\angle \bar{X}(e^{j\omega})$

b) $f_n[m] = w[m]\, x[n+m]$
$\qquad = \delta[m]\, x[n+m]$

$f_{-1}[m] = w[m]\, x[-1+m]$
$\qquad = \delta[m]\, x[-1+m]$ $=$

$f_{-1}[m]$

$f_{10}[m] = \delta[m]\, x[m+10]$ $=$

$f_{10}[m]$

$f_0[m] = \delta[0]\, x[m]$ $=$
$\qquad\quad \underset{\underset{1}{\sim}}{}$

$f_0[m]$

c) $\bar{X}_w\left[n, \frac{\pi}{2}\right) = x[n] \;=$

$X_w\left[n, \frac{\pi}{2}\right)$

d) $\bar{X}_w[0, \omega) = x[0] = 1$

$X_w[0, \omega)$
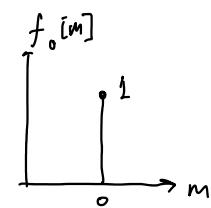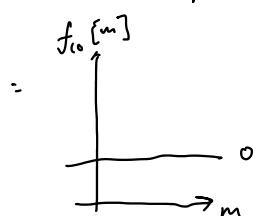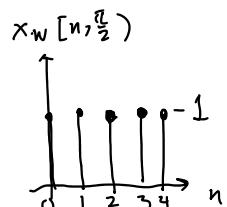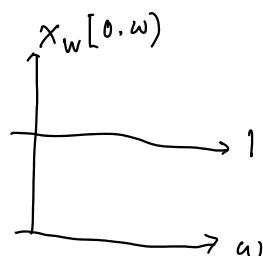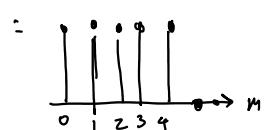
IIB. $f_n[m] = x[n+m]$
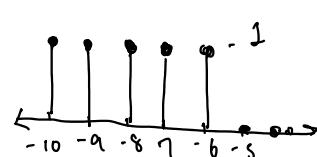
a) $f_{-1}[m] = x[m-1];$
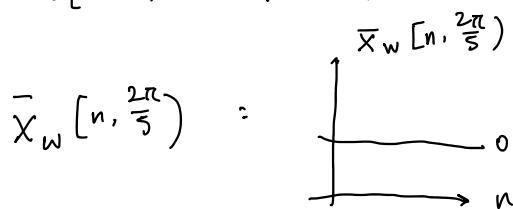
$f_0[m] = x[m]$ $=$
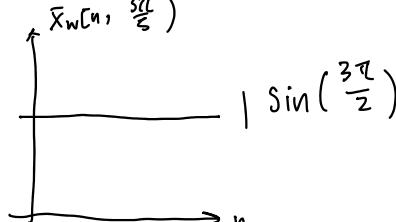
$f_{10}[m] = x[10+m]$

b) $X_w[n, \omega) = e^{j\omega n}\, \bar{X}(e^{j\omega})$

$\bar{X}_w\left[n, \frac{2\pi}{5}\right) \;=$

$\bar{X}_w\left[n, \frac{2\pi}{5}\right)$

c) $\bar{X}_w\left[n, \frac{3\pi}{5}\right)$

$\bar{X}_w\left[n, \frac{3\pi}{5}\right)$

$\left|\sin\left(\frac{3\pi}{2}\right)\right|$
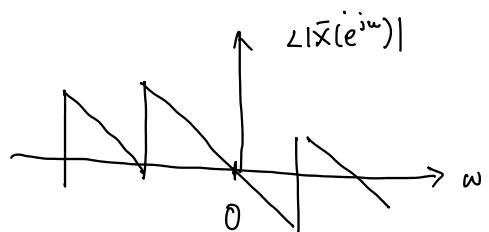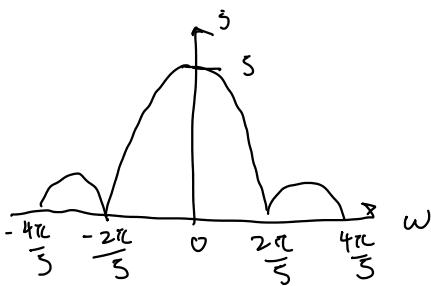
d) $X_w[0, \omega] = \bar{X}(e^{j\omega})$:

$|\bar{X}(e^{j\omega})|$

$\angle \bar{X}(e^{j\omega})|$

11.2 a) TDFT is an output of a filter system. For a fixed ω,
The TDFT $\bar{X}_w[n, \omega)$ is obtained by : $x[n] * h_w[n]$
$\hookrightarrow w[-n]e^{j\omega n}$

Because the operation is convolution with a fixed filter $h_w[n]$
$x[n]$ can be mapped as the input to the output $\bar{X}_w[n, \omega)$
which is an LTI system.

For any time-invariant system, shift in the input would cause a shift
to the output

b) TDFT is the DFT of $f_n[m] = x[n+m] w[m]$

$\bar{X}_w[n, \omega)) = \sum_{m=-\infty}^{\infty} f_n[m] e^{-j\omega m}$

The signal $x[n]$ is real, the window $w[n]$ is also real, therefore their
produce $f_n[m]$ is also a real-valued sequence

Real-valued DTFT exhibits conjugate symmetry: $\vec{H}(\omega) = H^*(-\omega)$
which would apply to $\bar{X}_w[n, \omega) = \bar{X}^*_n[n, -\omega)$

c) Let $w[n] = \delta[n]$, $X_w[n, \omega) = x[n]$, $Y[n, \omega) = y[n]$
$= \delta[n]$ $= \delta[n-1]$

TDFT $\{x * y\} = x[n] \cdot y[n]$

$= \delta[n] \cdot \delta[n-1] = 0$ which is not true

11.3)

$$H(z) = G \Big/ 1 + \sum_{k=1}^{10} a_k z^{-k} \qquad H\left(e^{\frac{j2\pi k}{256}}\right) = G \Big/ 1 + \sum_{\ell=1}^{10} a_\ell \, e^{\left(j\frac{2\pi k}{256}\right)\ell}$$

let this be $d[n]$ ← $\begin{cases} 1, & \ell = 0 \\ a_\ell, & 1 \le \ell \le 10 \\ 0 & \text{otherwise} \end{cases}$
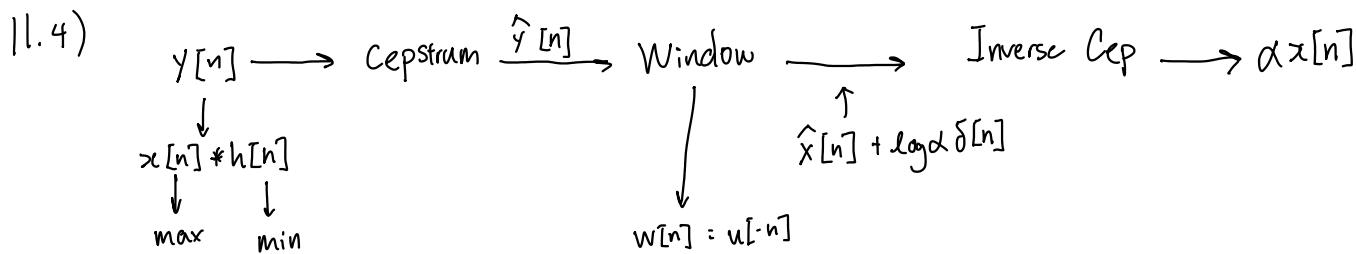
$$D[k] = \sum_{n=0}^{255} d[n]\, e^{-j\frac{2\pi k}{256} n} \quad,\quad d[n] \text{ is } 0 \text{ for } n>10$$

$$\Rightarrow \quad 1 + \sum_{n=1}^{10} a_n e^{-j\frac{2\pi k}{256} n} \longrightarrow H\left(e^{j\frac{2\pi k}{256}}\right) = \frac{G}{D[k]}$$

11.4)

$$y[n] \longrightarrow \text{Cepstrum} \xrightarrow{\hat{y}[n]} \text{Window} \longrightarrow \text{Inverse Cep} \longrightarrow \alpha x[n]$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow \qquad\qquad \uparrow$$

$$x[n]*h[n] \qquad\qquad\qquad\qquad\qquad \hat{x}[n] + \log\alpha\,\delta[n]$$

$$\downarrow \qquad \downarrow \qquad\qquad\qquad w[n] = u[-n]$$

$$\text{max} \quad \text{min}$$

Convolution in time = Addition in cepstral → $\hat{y}[n] = \hat{x}[n] + \hat{h}[n]$

$\downarrow$ 0 for $n<0$ $\quad\searrow$ 0 for $n>0$

let $\hat{k}[n] = \hat{y}[n] \cdot w[n] = (\hat{x}[n] + \hat{h}[n]) \cdot u[-n]$

$\rightarrow \hat{k}[n]$: for $n<0$: $u[-n]=1$, $\hat{h}[n]=0$, $\hat{k}[n] = \hat{x}[n]$

$\qquad\qquad n>0$: $u[-n]=0$, $\hat{k}[n]=0$

$\qquad\qquad n=0$: $u[-n]=1$ $\quad \hat{k}[n] = \hat{x}[0] + \hat{h}[0]$

$\Big\}$ $\hat{k}[n] = \hat{x}[n] + \hat{h}[0]\,\delta[n]$

$\qquad\qquad = \hat{x}[n] + \log\alpha\,\delta[n]$

$$\log\alpha = \hat{h}[0] \qquad \text{Inverse Cep of } \hat{k}[n]$$

$$= \quad \log\left(\overline{K}(z)\right) = \log(\overline{X}(z)) + C$$

$\searrow C = \hat{h}[0] = \log\alpha\,\delta[n]$

$$\rightarrow \overline{K}(z) = \overline{X}(z) \cdot e^C$$

$\rightarrow \alpha = e^{\hat{h}[0]}$

$$\rightarrow \hat{k}[n] = e^C \cdot x[n]$$

$$\underline{\text{output} = x[n] \cdot e^{\hat{h}[0]}}$$

11.5)

a) $\quad x_c[n] = -2\delta[n+2] + 4\delta[n+1]$

$\quad x_c[-2] = -2$

$\quad x_c[-1] = 4 \qquad\qquad\qquad \bar{X}_c(Z) = -2z^2 + 4z$

$\quad x_c[n] = 0 \quad$ for $\quad n \neq -2, 4$

checking zeros: $\bar{X}_c(Z) = -2z^2 + 4z = -2z(z-2) \quad z = 0, 2$ ↘ outside unit circle

↙ within unit circle
requires shifting

Let $y[n] = x_c[n-2], \quad y[0] = -2, \; y[1] = 4$

$\bar{Y}(Z) = Z^{-2}\bar{X}_c(Z) = z^{-2}(-2z^2 + 4z) = -2 + 4z^{-1}$

Scaling: should scale by -0.5

b) $x_b[n] = (0.5)^n u[n]$

$\hat{g}_b[n] + \hat{x}_b[n] = -\dfrac{(0.5)^n}{n} u[n-1]$

$X_b(Z) = \displaystyle\sum_{n=0}^{\infty} (0.5z^{-1})^n = \dfrac{1}{1-0.5z^{-1}}$

$\log(\bar{X}_b(Z)) = \log \dfrac{1}{1-0.5z^{-1}}$
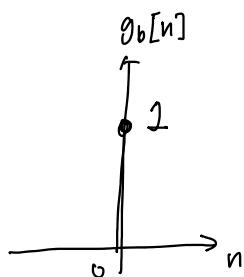
$\qquad\qquad = -\log(1-0.5z^{-1})$

$\qquad\qquad = \displaystyle\sum_{n=1}^{\infty} \dfrac{(0.5)^n}{n} z^{-n}$

$\hat{x}_b[n] = \dfrac{(0.5)^n}{n} u[n-1], \quad \hat{g}_b[n] + \hat{x}_b[n] = \hat{x}_b[n]; \quad \hat{g}_b[n] = 0 \rightarrow \log(\bar{G}_b(Z)) = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \rightarrow \bar{G}_b(Z) = e^0 = 1$

$g_b[n]$

$\therefore \; g_b[n] = \delta[n]$

```matlab
% Part A
[y, Fs] = audioread('sad.wav');
N = length(y);
t = (0:N-1) / Fs;

t_start = 0.6011787;
t_end = 1.1004;

idx_start = round(t_start * Fs);
idx_end   = round(t_end * Fs);

if idx_start < 1
    idx_start = 1;
end

spliced = y(idx_start : idx_end);

% Plot the spliced signal
figure;
plot(spliced);
title('Spliced Signal x[n]');
xlabel('Samples (n)');
grid on;
```
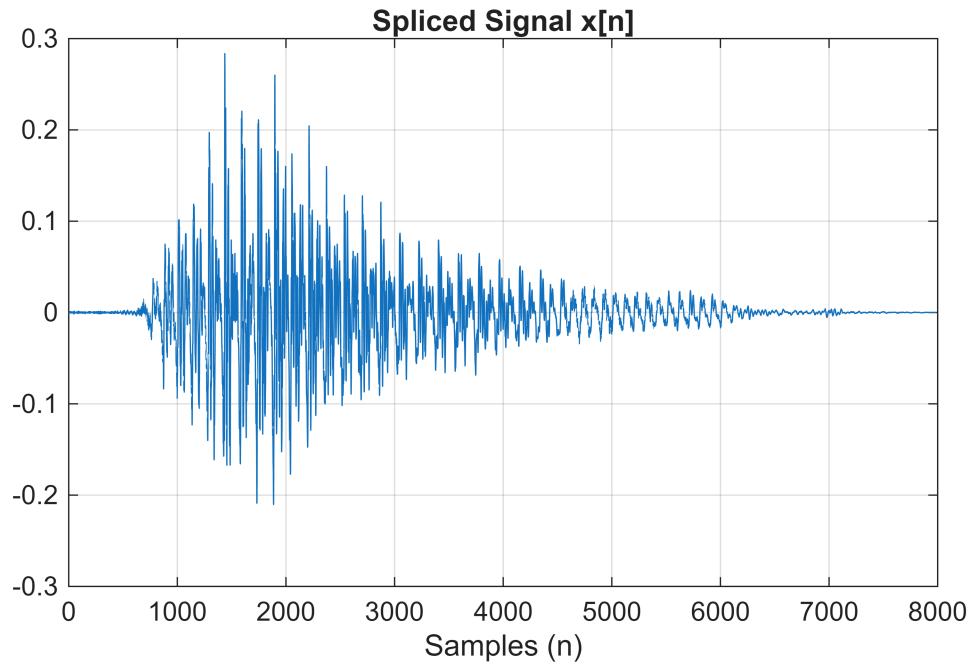


```matlab
% Part B
% a)

M = 256;
m = 0:M-1;
```
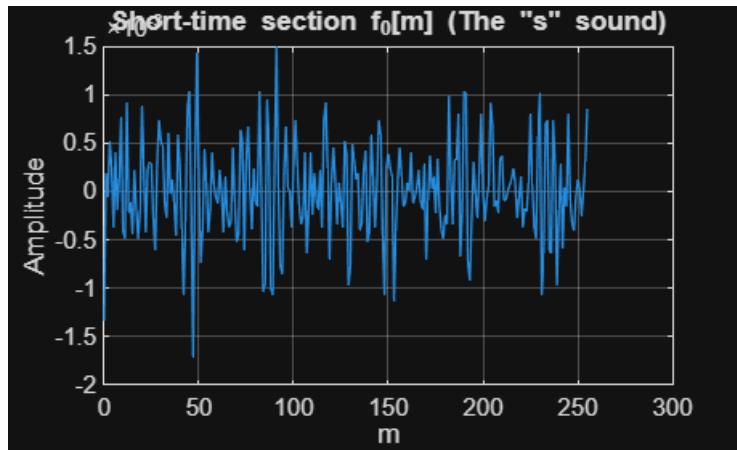
```matlab
f0 = spliced(1:M);

% Plot
figure;
plot(m, f0);
title('Short-time section f_{0}[m] (The "s" sound)');
xlabel('m'); ylabel('Amplitude');
grid on;
```


Short-time section f₀[m] (The "s" sound)
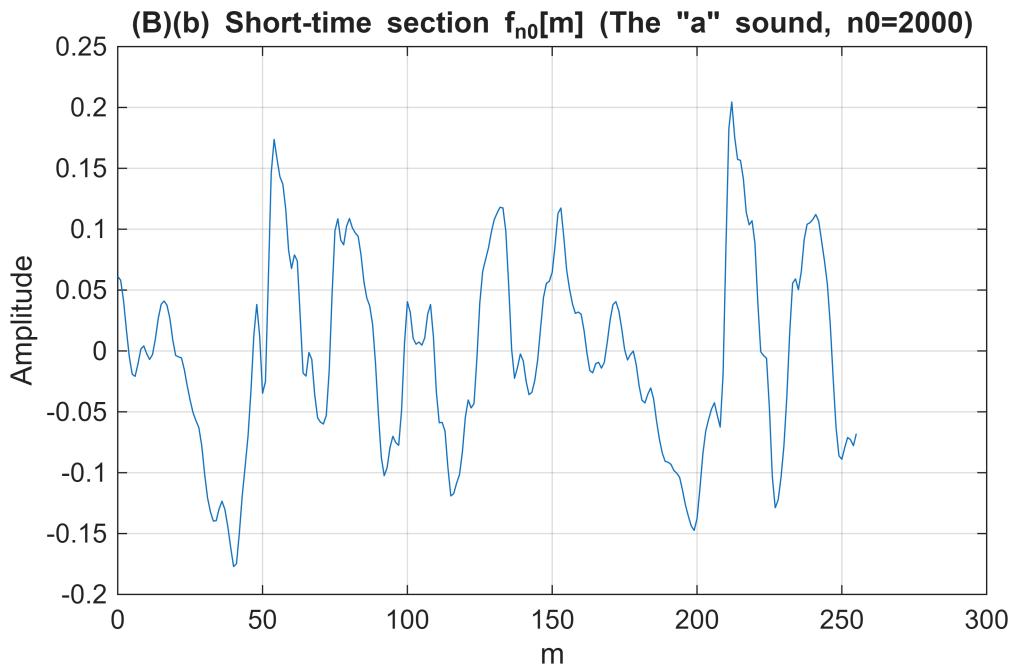
```matlab
% Part B
% b)
n0 = 2000;
f_n0 = spliced(n0+1 : n0+M);

figure;
plot(m, f_n0);
title(['(B)(b) Short-time section f_{n0}[m] (The "a" sound, n0=', num2str(n0),
')']);
xlabel('m'); ylabel('Amplitude');
grid on;
```

(B)(b) Short-time section $f_{n0}[m]$ (The "a" sound, n0=2000)


```
% Part B
% c)

N_fft = 512;
X_k = fft(f_n0, N_fft);

k = 0:N_fft-1;
freq_axis = k * (Fs / N_fft);

half_N = N_fft/2;

figure;
plot(freq_axis(1:half_N), abs(X_k(1:half_N)));
title('(B)(c) Magnitude of 512-point DFT of f_{n0}[m]');
xlabel('Frequency (Hz)'); ylabel('Magnitude');
grid on;
```

(B)(c) Magnitude of 512-point DFT of $f_{n0}[m]$