Problem 1.

a) $h[n] = (0.5)^n \sin\left(\frac{\pi n}{2}\right) u[n]$

$$= (0.5)^n \frac{e^{j\frac{\pi n}{2}} - e^{-j\frac{\pi n}{2}}}{2j} u[n] = \frac{(0.5)^n}{2j} \cdot j^n - (-j)^n u[n]$$

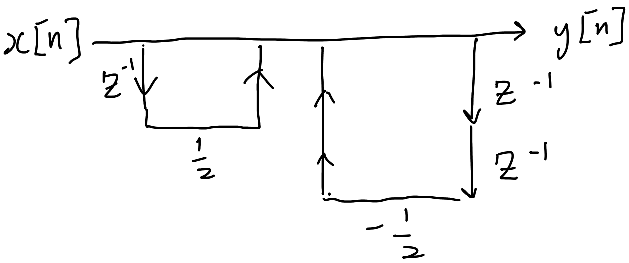
$$= \frac{1}{2j}\left[(0.5j)^n - (-0.5j)^n\right] u[n]$$

$$H(z) = \frac{1}{2j}\left[\frac{1}{1-0.5jz^{-1}} - \frac{1}{1+0.5jz^{-1}}\right]$$

$$= \frac{1}{2j}\left[\frac{(1+0.5jz^{-1}) - (1-0.5jz^{-1})}{1-(0.5jz^{-1})^2}\right] = \frac{1}{2j}\frac{jz^{-1}}{1+0.25z^{-2}} = \frac{0.5z^{-1}}{1+0.25z^{-2}} = \frac{Y(z)}{X(z)}$$

$$Y(z)(1+0.25z^{-2}) = 0.5z^{-1}X(z) \rightarrow y[n] + 0.25y[n-2] = 0.5x[n-1]$$

$$\rightarrow y[n] = 0.5x[n-1] - 0.25y[n-2]$$



b) $H(e^{jw}) = e^{-j2w}\left(\frac{\sin(\omega)}{\sin(\frac{\omega}{2})}\right)^4 \rightarrow [H'(e^{jw})]^4 \rightarrow$ multiplication in ω = convolution in time

$$\hookrightarrow = \frac{\sin(Nw/2)}{\sin(w/2)} \rightarrow N=2 = u[n] - u[n-2]$$

let this be $H'(e^{jw})$

$L = (2-1)+(2-1)+(2-1)+2 = 5$: 5 point DFT, answer = yes

c) $Y[n] = \sum_{k=0}^{3} x[k]x[n-k]$: $x[n]$ convolution w/ itself, $L_{Y[n]} = L_{x[n]} + (L_{x[n]}-1)$

$= 4+4-1 = 7$ points

$(X[k]_7)^2$ = 7-point $x[n]$ convolution, $L = 7$

$\Rightarrow$ Answer: $Y[k]_7 = (X[k]_7)^2$, yes

Problem 2.

a) $Q_w[n, \omega)$, $v[n] = u[n] - u[n-128]$, non-zero at $0 \le n \le 127$

$Q_v[n,k] = Q_v\left[2n, \frac{2\pi k}{256}\right] \to v[n]q[n+2] \to 0$ for $0 \le n \le 127$

$Q_v[2,k] = 0 \in [0 \le k \le 255]$, sequence $q[n] = 0$

$q[n] = 0$ for $n=2$ throug $n = 2+127 = 129$

$\to$ $\underline{q[n] = 0 \text{ for } 2 \le n \le 129}$

b) $g[n] = (0.5)^n \{u[n] - u[n-4]\}$

$A = \sum_{k=-\infty}^{\infty} h^2[k] =$ energy of $h[n]$

$g[n] = (0.5)^n$ for $0 \le n \le 3$

$g[0] = 1$, $g[1] = 0.5$, $g[2] = 0.25$, $g[3] = 0.125$

$A = 1^2 + 0.5^2 + 0.25^2 + 0.125^2$

$= 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{8}\right)^2 = \underline{\frac{85}{64}}$

c) Restricted model of C.C requires $\hat{X}(Z)$ has no zeros or poles on the unit circle ($|Z| = 1$)

$y[n] = (0.5)^n x[n] \leftrightarrow \hat{Y}(Z) = \hat{X}\left(\frac{Z}{0.5}\right) = \hat{X}(2Z)$

$\hookrightarrow$ moves zeros, Z_k to $\frac{Z_k}{2}$

if $Z_k = 2$, then Z_k new $= 1$

therefore no, not necessarily