FALL 24 EC516 Problem Set 04

Due: Sunday October 5 (Before 11:59pm)

You must submit your homework attempt on Blackboard Learn. For this purpose, you must convert your homework attempt to a pdf file and upload it at the corresponding homework assignment on Blackboard Learn.

Problem 4.1

Consider a digital FIR filter whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$$

(a) Draw a flowgraph for this filter. How many multiplications per output sample does this filter implementation (as represented by your flowgraph) require? What amount of memory access is needed per output sample for this filter implementation. Justify your answers.
(b) Write the algebraic expression for the system function $H(z)$ for this filter.
(c) Draw the magnitude of the frequency response of this filter.
(d) Is this filter stable? Justify your answer.

Problem 4.2

Consider a digital IIR filter whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n] = -0.125y[n-2] + 0.75y[n-1] + x[n]$$

(a) Draw a flowgraph for this filter. How many multiplications per output sample does this filter implementation (as represented by your flowgraph) require? What amount of memory retrieval is needed for this filter implementation. Justify your answers.
(b) Write the algebraic expression for the system function $H(z)$ for this filter.
(c) Give your reasoning for why this filter is stable.
(d) Determine all the values of ω for which it is guaranteed that $H(e^{j\omega}) = 0$.

Problem 4.3

Consider a *causal* digital filter whose input $x[n]$and output $y[n]$ are related through the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] - \frac{1}{4}x[n-2]$$

(a) Draw the flowgraph corresponding to the above difference equation.
(b) Show that the same filter can be implemented using a non-recursive difference equation (HINT: Figure out the System Function $H(z)$ for the filter, simplify it by cancelling any common factors between the numerator and denominator, and go back to the difference equation in the time domain).
(c) Draw a flowgraph corresponding to the non-recursive difference equation.
(d) How does the computational complexity of the flowgraph you obtained in part (a) of this problem compare to the computational complexity of the flowgraph you obtained in part (c) of this problem?

Problem 4.4

Consider a *second order section* with the system function:

$$H(z) = \frac{1}{(1 - 0.9e^{\frac{j\pi}{4}}z^{-1})(1 - 0.9e^{-\frac{j\pi}{4}}z^{-1})}$$

(a) Determine a difference equation relating the input and output of this second order section?

(b) Is it true that if the input signal to this filter is real-valued, the output signal from this section is also guaranteed to be real? Explain your answer.

(c) Draw a Direct Form flowgraph for this second order section and specify the number of multiplications per output sample by stored coefficients and the number memory retrievals per output sample for this implementation. Are the stored coefficients all real valued? Are the intermediate signals in the flowgraph also guaranteed to be real as long as the input signal to the second order section is real? Explain your answers.

(d) Sketch the pole-zero plot for the system function for this second order section.

(e) Draw by hand an approximation to the magnitude of the frequency response of this second order section. Is the magnitude of the frequency response zero at any frequency?

Experiential DSP Exercise 04

Approximating an Ideal Low Pass Filter by a FIR filter

In this assignment, we will investigate one way of approximating an ideal low-pass filter as a FIR filter in MATLAB. Our goal is to examine FIR filters with 9-point and a 65-point impulse responses as approximations to an ideal lowpass filter and then to compare their frequency responses. The impulse response of an ideal low-pass filter is the sinc function which, as we know, is the impulse response of a filter that is not practically implementable as a FIR or IIR filter. If the ideal lowpass filter has cutoff frequency $\omega_c = \frac{\pi}{2}\,\mathrm{rad/sec}$ (or equivalently, $f_c = \frac{\omega_c}{2\pi} = 0.25\ Hz$), its impulse response is $\frac{\sin(\frac{\pi n}{2})}{\pi n}$. We can approximate this ideal impulse response by a N-point impulse response of a FIR filter by shifting the ideal impulse response by $(N-1)/2$ and multiplying it by the "window" specified as: $u[n] - u[n-N]$.

1) Set `N = 9, fc = 0.25, n = 0:N-1, alpha = (N-1)/2`.
2) Create the 9-point FIR impulse response using the MATLAB sinc function: `h = sinc(2*fc(n-alpha))`.
3) To ensure that the gain of the frequency response of the FIR filter is 0 dB gain at $\omega = 0$, we normalize the impulse response: `h_normalized = h/sum(h)`. Please explain why this normalization ensures 0 dB gain at $\omega = 0$.
4) Set `N_fft = 512` and create `H = fft(h_normalized, N_fft)`. As we learned in Assignment 02, here we are taking 512 uniform samples of the frequency response from $\omega = 0$ to $\omega = 2\pi$.
5) Set `f = (0:N_fft - 1) / (N_ftt)`. Plot the magnitude of the frequency response in decibels (dB) scale using `plot(f, 20*log10(abs(H)))`. Save the figure as Figure 1.
6) Repeat steps 1-5 for `N = 65` and save the final figure as Figure 2.
7) Compare Figure 1 and Figure 2 and explain the differences. Pay particular attention to how "sloppy" the transition is from the passband of the filter (0 dB gain) to the stopband of the filter.
8) For each of the two filters (9-point FIR filter and the 65-point FIR filter), how many multiplications are required per output sample if each filter is implemented in direct form? Explain how you arrived at your answers.