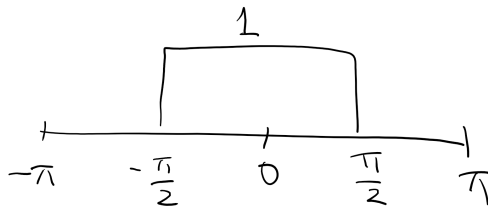


EC516 HW06 Solutions

Problem 6.1

(a)

$$H_{id}(e^{j\omega}) = \text{rect}\left(\frac{\omega}{\pi}\right) = \begin{cases} 1 & |\omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq |\omega| \leq \pi \end{cases}$$



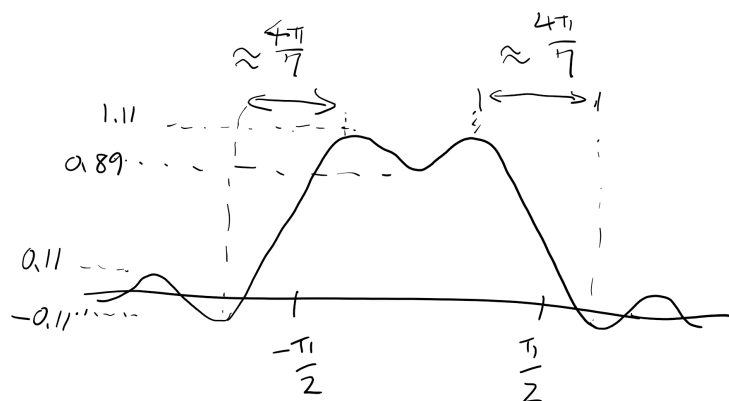
(b) DTFT of $w[n]$ is

$$W(e^{j\omega}) = e^{-j3\omega} \frac{\sin(7\omega/2)}{\sin(\omega/2)}$$

Multiplications in time domain corresponds to convolution in frequency domain. Recall that $\text{DTFT}\{h_{id}[n-3]\} = e^{-j3\omega} H_{id}(e^{j\omega})$.

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j3\theta} e^{-j3(\omega-\theta)} \frac{\sin(7(\omega-\theta)/2)}{\sin((\omega-\theta)/2)} d\theta \\ &= e^{-j3\omega} \cdot \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{\sin(7(\omega-\theta)/2)}{\sin((\omega-\theta)/2)} d\theta \\ &= R(\omega) e^{-j3\omega} \end{aligned}$$

Transition bandwidth of $H(e^{j\omega})$ will roughly equal to the width of mainlobe of $W(e^{j\omega})$.



Problem 6.2

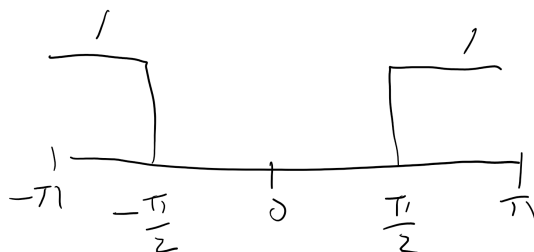
- (a) Notice that $(-1)^n = e^{j\pi n}$. DTFT of $e^{j\pi n}$ is

$$\text{DTFT}\{e^{j\pi n}\} = \delta(\omega - \pi) \quad \text{for } -\pi < \omega \leq \pi$$

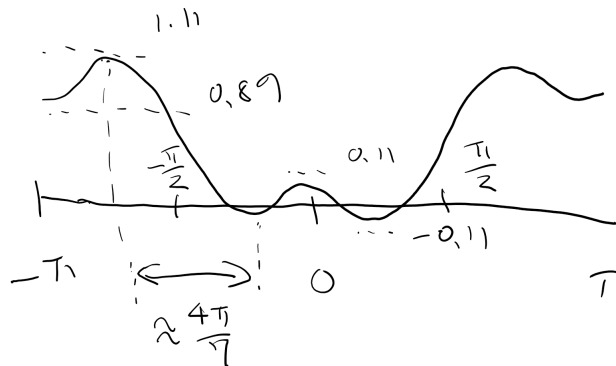
Since $(-1)^n$ is being multiplied in time domain, we perform convolution in frequency domain.

$$\begin{aligned} \text{DTFT} \left\{ (-1)^n \cdot \frac{\sin(0.5\pi n)}{\pi n} \right\} &= \delta(\omega - \pi) * \text{rect} \left(\frac{\omega}{\pi} \right) \\ &= \begin{cases} 1 & \frac{\pi}{2} < |\omega| \leq \pi \\ 0 & |\omega| \leq \frac{\pi}{2} \end{cases} \end{aligned}$$

Contrary to previous problem, this is a high pass filter.



- (b) $H(e^{j\omega})$ can still be written in the form $R(\omega)e^{j3\omega}$.



Problem 6.3

- (a) $H_{id}(e^{j\omega})$ is the same.
- (b) $w[n]$ is a triangular function (convolved with rectangle with rectangle). Its DTFT is simply the square of sinc function.

$$\begin{aligned}
 W(e^{j\omega}) &= \left(e^{-j3\omega/2} \frac{\sin(4\omega/2)}{\sin(\omega/2)} \right)^2 \\
 &= e^{-j3\omega} \frac{\sin^2(4\omega/2)}{\sin^2(\omega/2)}
 \end{aligned}$$

The phase shift would be the same as before.

The width of the mainlobe in $W(e^{j\omega})$ is π hence the transition width will approximately be π . Since $W(e^{j\omega})$ will have smaller sidelobes, $H(e^{j\omega})$ will have smaller ripples.

