EC516 HW01 Solutions

Problem 1.1 A)

$$
\begin{aligned}
0.5e^{j\omega t} + 0.5e^{-j\omega t} &= 0.5\left(\cos(\omega t) + j\sin(\omega t)\right) + 0.5\left(\cos(-\omega t) + j\sin(-\omega t)\right) \\
&= 0.5\left(\cos(\omega t) + j\sin(\omega t)\right) + 0.5\left(\cos(\omega t) - j\sin(\omega t)\right) \\
&= \cos(\omega t)
\end{aligned}
$$

B)

$$\begin{aligned}x(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left(\pi\delta(\omega-400\pi)+\pi\delta(\omega+400\pi)\right)e^{j\omega t}d\omega \\ &= \frac{1}{2}e^{j400\pi t}+\frac{1}{2}e^{-j400\pi t} \\ &= \cos(400\pi t)\end{aligned}$$

In order to make $x(t)$ odd, we must shift by $\pi/2$ radians or $1/800$ in t domain.

$$\sin(400\pi t) = \cos(400\pi t - \pi/2) = \cos(400\pi(t-1/800))$$

C) a) Yes. The function is a linear combination of sinusoids.
b) Check Figure 1.
c) Check Figure 1.
d) Check Figure 1.

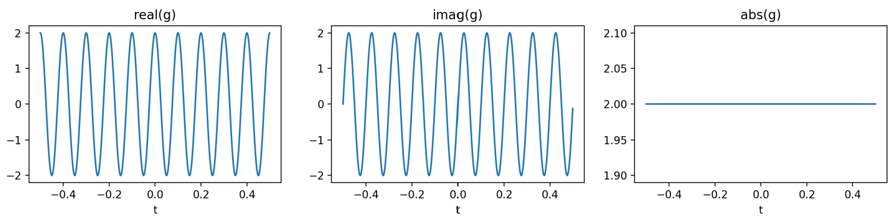


Figure 1: b)-d) real, imaginary, and absolute value of $g(t)$

<u>Problem 1.4</u> a) From finite sum formula for geometric series give us

$$\begin{aligned}T_n &= \sum_{n=0}^{N-1} a^n \\ T_n - aT_n &= \sum_{n=0}^{N-1} a^n - \sum_{n=0}^{N-1} a^{n+1} \\ (1-a)T_n &= 1-a^N \\ T_n &= \frac{1-a^N}{1-a}\end{aligned}$$

The proof is valid as long as $1-a \neq 0$.

b) Infinite sum formula is derived by assuming that $\lim_{N\to\infty} a^N = 0$.

$$\lim_{N\to\infty}\frac{1-a^N}{1-a} = \frac{1}{1-a}$$

This is true when $|a| < 1$.

Problem 1.2

Part(A) (a)

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j\omega t} dt$$
$$= e^{-j\omega t_0}$$

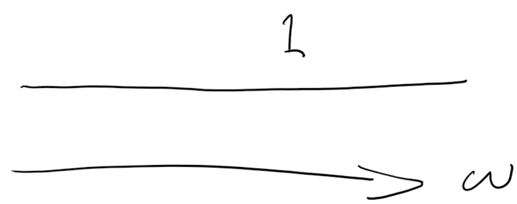


Figure 1: $|X(j\omega)|$ for Part (A) (a). It is a constant ($= 1$)

(b) $u(t+T) - u(t-T)$ is equivalent to rectangular function with length $2T$ centered at 0. Here, we derive that CTFT of rectangular function is a sinc function, but you are welcome to use the properties table.

$$\begin{aligned}
X(j\omega) &= \int_{-\infty}^{\infty} \left(u(t+T) - u(t-T)\right) e^{-j\omega t} dt \\
&= \int_{-T}^{T} e^{-j\omega t} dt \\
&= \left[\left. \frac{e^{-j\omega t}}{-j\omega} \right|_{t=-T}^{T} \right] \\
&= \frac{e^{-j\omega T} - e^{j\omega T}}{-j\omega} \\
&= \frac{-2j \sin(\omega T)}{-j\omega} \\
&= 2T \cdot \frac{\sin(\omega T)}{\omega T}
\end{aligned}$$

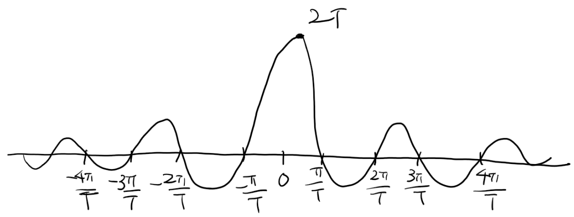


Figure 2: $|X(j\omega)|$ for Part(A) (b). It is a sinc function and becomes 0 at integer multiple of π/T.

Part(B) (a)

$$
\begin{aligned}
x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega - 100\pi)e^{j\omega t}dt\omega \\
&= e^{j100\pi t}
\end{aligned}
$$

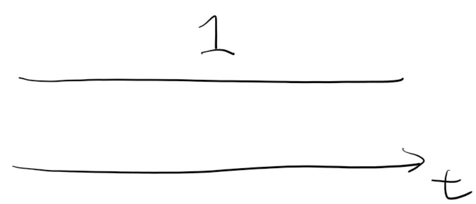
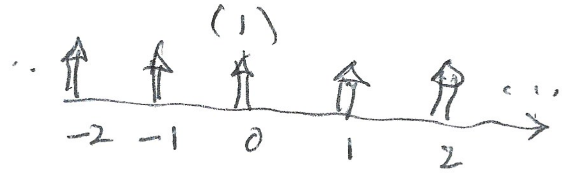


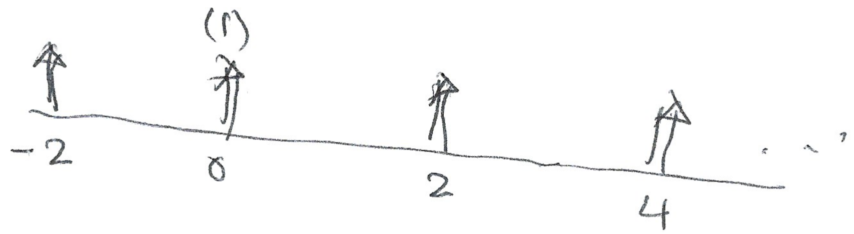
Figure 3: $|x(t)|$ for Part(B) (a). It is a constant $(= 1)$

(b) Inverse CTFT of a rectangular function is also a sinc function but divided by 2π.

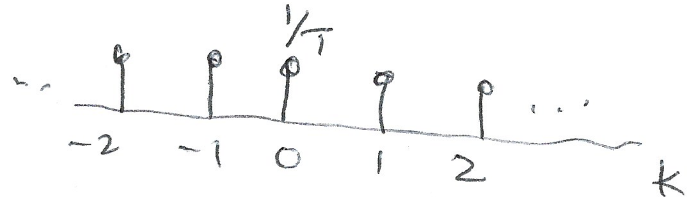
$$
\begin{aligned}
x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(u(\omega + 2\pi) - u(\omega - 2\pi)\right) e^{j\omega t}dt\omega \\
&= \frac{1}{2\pi}\int_{-2\pi}^{2\pi} e^{j\omega t}dt\omega \\
&= 2 \cdot \frac{sin(2\pi t)}{2\pi t}
\end{aligned}
$$

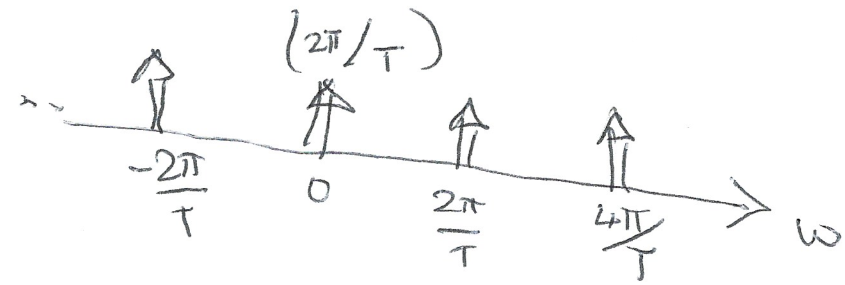
Problem 01.3

(a) 

(b) 

(c) $$a_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jk2\pi t/T}\, dt$$

$$= \frac{1}{T} =$$ 

(d) $M(j\omega) =$ 

(e) $$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * M(j\omega)$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j\left(\omega - \frac{2\pi k}{T}\right)\right)$$

= replicas of $X(\omega)$ shifted every $\frac{2\pi}{T}$ and scaled by $1/T$.