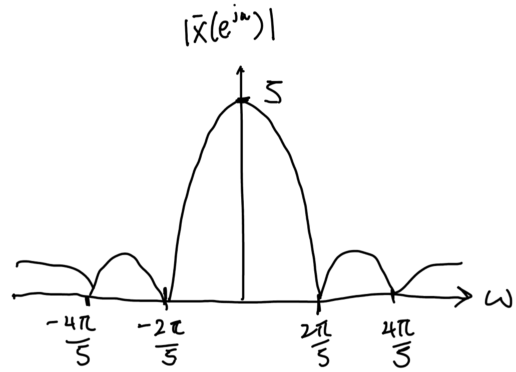
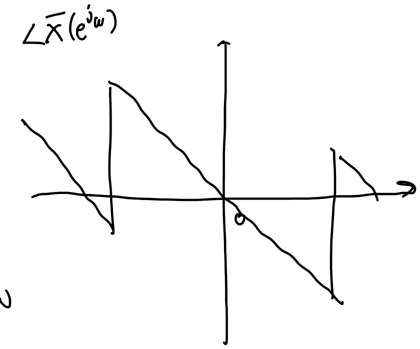
Problem 1.

A) $x[n] = u[n] - u[n-5]$

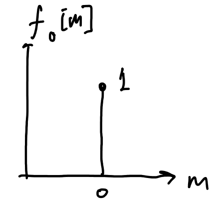
a) $\bar{X}(e^{j\omega}) = e^{-j\omega 4/2} \cdot \dfrac{\sin(5\omega/2)}{\sin(\omega/2)}$





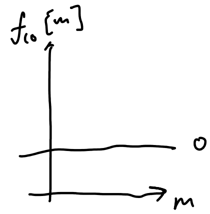
b) $f_n[m] = w[m]\,x[n+m]$
$= \delta[m]\,x[n+m]$

$f_0[m] = \underbrace{\delta[0]}_{1}\,x[m] =$

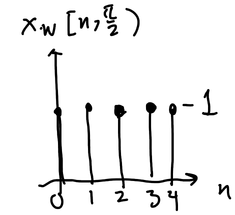


$f_{-1}[m] = w[m]\,x[-1+m]$
$= \delta[m]\,x[-1+m]$ $=$

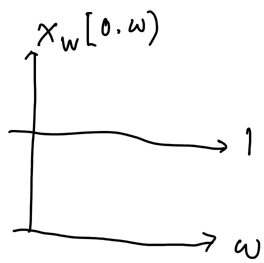
$f_{10}[m] = \delta[m]\,x[m+10]$ $=$



c) $\bar{X}_w[n, \frac{\pi}{2}) = x[n] =$

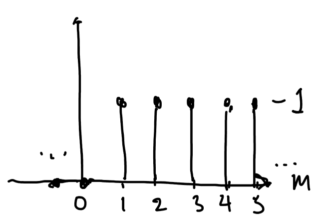


d) $\bar{X}_w[0, \omega) = x[0] = 1$

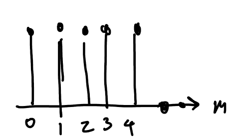


IIB. $f_n[m] = x[n+m]$

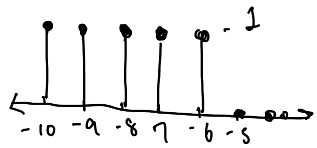
a) $f_{-1}[m] = x[m-1] =$



$f_0[m] = x[n] =$

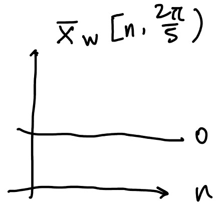


$f_{10}[m] = x[10+m]$

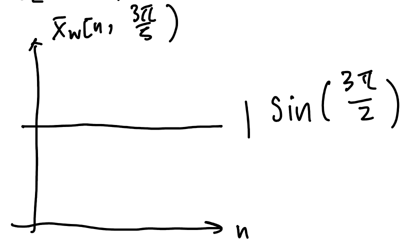


b) $X_w[n,\omega) = e^{j\omega n}\,\bar{X}(e^{j\omega})$

$\bar{X}_w[n, \frac{2\pi}{5}) =$

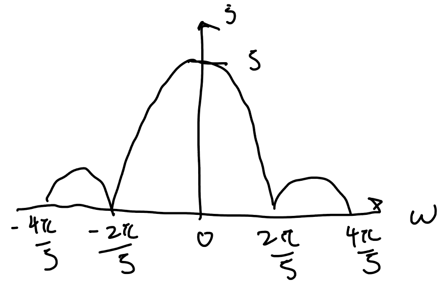


c) $\bar{X}_w[n, \frac{3\pi}{5})$

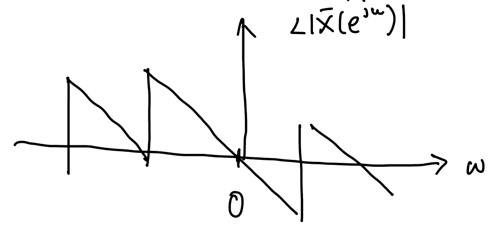


d) $X_w[0, \omega) = \bar{X}(e^{j\omega})$:

$|\bar{X}(e^{j\omega})|$



$\angle|\bar{X}(e^{j\omega})|$



11.2 a) TDFT is an output of a filter system. For a fixed ω,

The TDFT $\bar{X}_w[n, \omega)$ is obtained by : $x[n] * h_w[n]$

$h_w[n] \to w[-n]e^{j\omega n}$

Because the operation is convolution with a fixed filter $h_w[n]$

$x[n]$ can be mapped as the input to the output $\bar{X}_w[n, \omega)$

which is an LTI system.

For any time-invariant system, shift in the input would cause a shift to the output

b) TDFT is the DFT of $f_n[m] = x[n+m]\,w[m]$

$$\bar{X}_w[n, \omega)) = \sum_{m=-\infty}^{\infty} f_n[m]\, e^{-j\omega m}$$

The signal $x[n]$ is real, the window $w[n]$ is also real, therefore their produce $f_n[m]$ is also a real-valued sequence

Real-valued DTFT exhibits conjugate symmetry: $\vec{H}(\omega) = H^*(-\omega)$

which would apply to $\bar{X}_w[n, \omega) = \bar{X}^*_n[n, -\omega)$

c) Let $w[n] = \delta[n]$, $X_w[n, \omega) = x[n]$ $= \delta[n]$, $Y[n, \omega) = y[n]$ $= \delta[n-1]$

$\text{TDFT}\{x * y\} = x[n] \cdot y[n]$

$= \delta[n] \cdot \delta[n-1] = 0$ which is not true

11.3)

$$H(z) = G \Big/ 1+\sum_{k=1}^{10} a_k z^{-k} \qquad H\left(e^{\frac{j2\pi k}{256}}\right) = G \Big/ \underbrace{1+\sum_{l=1}^{10} a_l e^{\left(-j\frac{2\pi k}{256}\right)l}}$$

let this be $d[n]$ $\longleftarrow$ $\begin{cases} 1, & l=0 \\ a_l, & 1 \le l \le 10 \\ 0 & \text{otherwise} \end{cases}$

$$D[k] = \sum_{n=0}^{255} d[n] e^{-j\frac{2\pi k}{256}n}$$, $d[n]$ is 0 for $n>10$

$$\Rightarrow 1+\sum_{n=1}^{10} a_n e^{-j\frac{2\pi k}{256}n} \longrightarrow H\left(e^{j\frac{2\pi k}{256}}\right) = \frac{G}{D[k]}$$

11.4)

$$\begin{tikzcd}
y[n] \arrow[r] \arrow[d] & \text{Cepstrum} \arrow[r, "{\hat{y}[n]}"] & \text{Window} \arrow[r] \arrow[d] & {} \arrow[r] & \text{Inverse Cep} \arrow[r] & \alpha x[n] \\
x[n] * h[n] & & {w[n] = u[-n]} & {\hat{x}[n] + \log\alpha\,\delta[n]} \arrow[u] & &
\end{tikzcd}$$

$x[n]$ ↓ max, $h[n]$ ↓ min

Convolution in time = Addition in cepstral $\rightarrow \hat{y}[n] = \hat{x}[n] + \hat{h}[n]$

$\hat{x}[n]$: 0 for $n<0$; $\hat{h}[n]$: 0 for $n>0$

let $\hat{k}[n] = \hat{y}[n] \cdot w[n] = (\hat{x}[n] + \hat{h}[n]) \cdot u[-n]$

$\rightarrow \hat{k}[n]$: for
$n<0$: $u[-n]=1$, $\hat{h}[n]=0$, $\hat{k}[n] = \hat{x}[n]$
$n>0$: $u[-n]=0$, $\hat{k}[n]=0$
$n=0$: $u[-n]=1$ $\hat{k}[n] = \hat{x}[0] + \hat{h}[0]$

$\Big\}$ $\hat{k}[n] = \hat{x}[n] + \hat{h}[0]\delta[n]$
$= \hat{x}[n] + \log\alpha\,\delta[n]$

$\log\alpha = \hat{h}[0]$ Inverse cep of $\hat{k}[n]$

$= \log(\bar{K}(z)) = \log(\bar{X}(z)) + C \rightarrow C = \hat{h}[0] = \log\alpha\,\delta[n]$

$\rightarrow \bar{K}(z) = \bar{X}(z) \cdot e^{C}$ $\rightarrow \underline{\alpha = e^{\hat{h}[0]}}$

$\rightarrow \hat{k}[n] = e^{C} \cdot x[n]$

$\underline{\text{output} = x[n] \cdot e^{\hat{h}[0]}}$

11.5)

a) $x_c[n] = -2\delta[n+2] + 4\delta[n+1]$

$x_c[-2] = -2$

$x_c[-1] = 4$ $\qquad\qquad$ $\bar{X}_c(z) = -2z^2 + 4z$

$x_c[n] = 0$ for $n \neq -2, 4$

checking zeros: $\bar{X}_c(z) = -2z^2 + 4z = -2z(z-2)$ $\quad z = 0, 2$ outside unit circle

within unit circle requires shifting

Let $y[n] = x_c[n-2]$, $\quad y[0] = -2, \; y[1] = 4$

$\bar{Y}(z) = z^{-2}\bar{X}_c(z) = z^{-2}(-2z^2 + 4z) = -2 + 4z^{-1}$

Scaling: should scale by -0.5

b) $x_b[n] = (0.5)^n u[n]$

$\hat{g}_b[n] + \hat{x}_b[n] = -\dfrac{(0.5)^n}{n} u[n-1]$ $\qquad\qquad$ $\log(\bar{X}_b(z)) = \log \dfrac{1}{1-0.5z^{-1}}$

$X_b(z) = \sum_{n=0}^{\infty} (0.5z^{-1})^n = \dfrac{1}{1-0.5z^{-1}}$ $\qquad\qquad$ $= -\log(1-0.5z^{-1})$

$= \sum_{n=1}^{\infty} \dfrac{(0.5)^n}{n} z^{-n}$

$\hat{x}_b[n] = \dfrac{(0.5)^n}{n} u[n-1]$, $\quad \hat{g}_b[n] + \hat{x}_b[n] = \hat{x}_b[n]$; $\quad \hat{g}_b[n] = 0 \rightarrow \log(\bar{G}_b(z)) = 0$

$\rightarrow \bar{G}_b(z) = e^0 = 1$

$\therefore\; g_b[n] = \delta[n]$