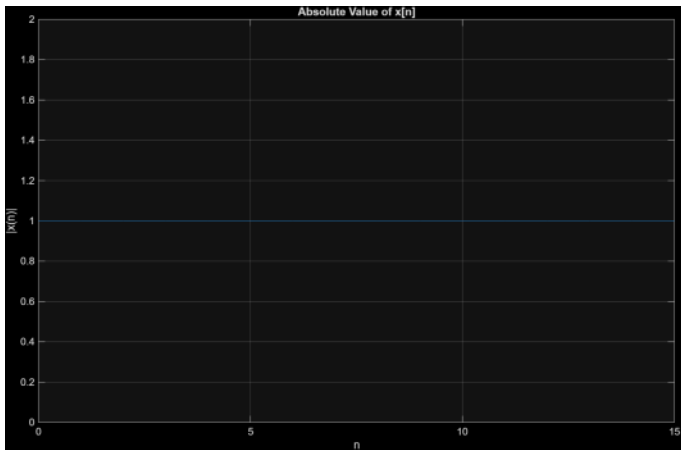
2.1 A) $X[n] = e^{j0.375\pi n}$

a) is $x[n]$ periodic?

$$\frac{0.375\cancel{\pi}}{2\cancel{\pi}} = \underbrace{\frac{k}{N}}_{\frac{k}{N} \in \mathbb{R}} = \frac{0.375}{8} = \frac{3/8}{2} = \frac{3}{16}, \quad \frac{3}{16} \in \mathbb{R}$$

$x[n]$ is periodic with fundamental period of 16

b)



c) $e^{j0.375\pi n} = -1, \quad e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1$

$\Rightarrow 0.375 n_0 \pi = m\pi$ for m is odd $\Rightarrow 3/8\, n_0 \cancel{\pi} = m\cancel{\pi}$

$\Rightarrow \frac{3}{8} = \frac{m}{n_0}$ for $m = 3,\ n_0 = 8 \Rightarrow n_0 = 8$

B) $x[n] = \delta[n-3]$

$$\bar{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$= e^{-j\omega 3}$, for $x[n] = 1$ iff $n = 3$, 0 for everywhere else

C) $\bar{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

a) $x[n] = u[n+2] - u[n-3] \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{everywhere else} \end{cases}$

$$\bar{X}(e^{j\omega}) = \sum_{n=-2}^{2} (1) e^{-j\omega n}$$

$$= e^{j2\omega} \frac{1 - (e^{-j\omega})^5}{1 - e^{-j\omega}} = e^{j2\omega} \frac{1 - e^{-j5\omega}}{1 - e^{-j\omega}} = \frac{\sin(5\omega/2)}{\sin(\omega/2)}$$

b) $x[n] = u[n] - u[n-5] \rightarrow \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{everywhere else} \end{cases}$

$$\bar{X}(e^{j\omega}) = \sum_{n=0}^{4} e^{-j\omega n}$$

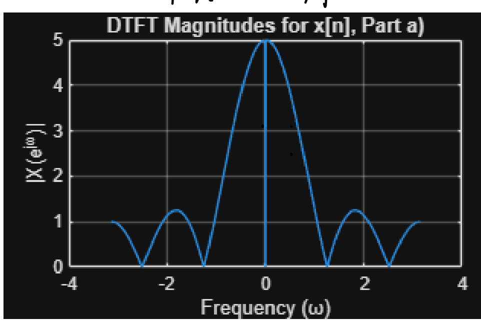
$$= \frac{1 - e^{-j5\omega}}{1 - e^{-j\omega}} = \frac{\sin(5\omega/2)}{\sin(\omega/2)} e^{-j2\omega}$$

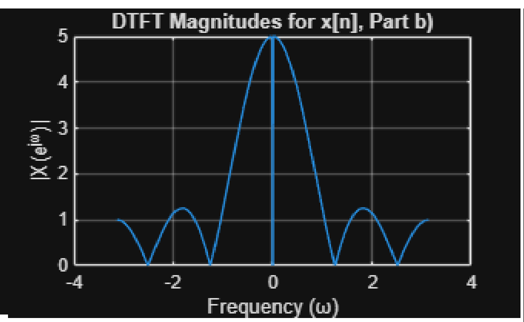
c) $x[n] = u[n] - u[n-4] \rightarrow \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{everywhere else} \end{cases}$

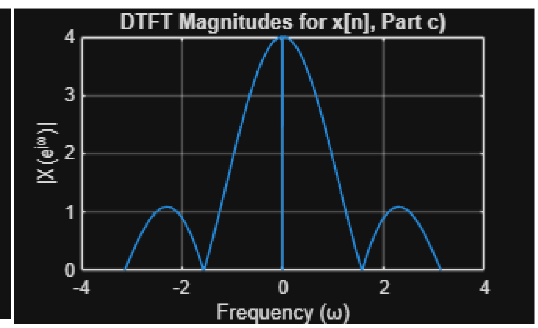
$$\bar{X}(e^{j\omega}) = \sum_{n=0}^{3} e^{-j\omega n}$$

$$= \frac{1 - e^{-j4\omega}}{1 - e^{-j\omega}} = \frac{\sin(2\omega)}{\sin(\omega/2)} e^{-j\,3/2\,\omega}$$

Plots of $|\bar{X}(e^{j\omega})|$







2.2) $|X(j\omega)| = 0$ for $|\omega| \ge 10000\,\pi$; $x[n] = x(nT)$

$\hookrightarrow \omega_{max}$

a) $T = 0.0001$ s

Sampling Frequency $= \frac{2\pi}{T} = \frac{2\pi}{0.0001} = 20\,000\,\pi = 2\,\omega_{max}$: Nyquist Theorem sampling rate

$X(e^{j\omega})$ is non-zero $\forall\, \omega \in [-\pi, \pi]$, not guranteed to be zero

b) $T = 0.00005$

Sampling Frequency $\omega_s = \frac{2\pi}{T} = \frac{2\pi}{0.00005} = 40000\pi$, over sampling

Original signal from $[-\pi, \pi]$ compressed to $[-0.5\pi, 0.5\pi]$

$[-10000\,\pi \cdot 0.00005,\ 10000\pi \cdot 0.00005]$

$X(e^{j\omega})$ is guranteed zero for all ω out of the ranges:

$$[-\pi, -0.5\pi) \quad (0.5\pi, \pi]$$

c) $T = 0.0001$

$$\omega_s = \frac{2\pi}{T} = 200\,000\pi >> 2\omega_{max}$$

The compressed signal will exist in range:

$$[-10000\pi \cdot 0.0001, 10000\pi \cdot 0.0001] = [-0.1\pi, 0.1\pi]$$

Therefore $X(e^{j\omega})$ is guranteed zero $\forall \omega$ outside of:

$$[-\pi, -0.1\pi), (0.1\pi, \pi]$$

2.3 A) $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \bar{X}(e^{j\omega}) e^{j\omega n} d\omega$

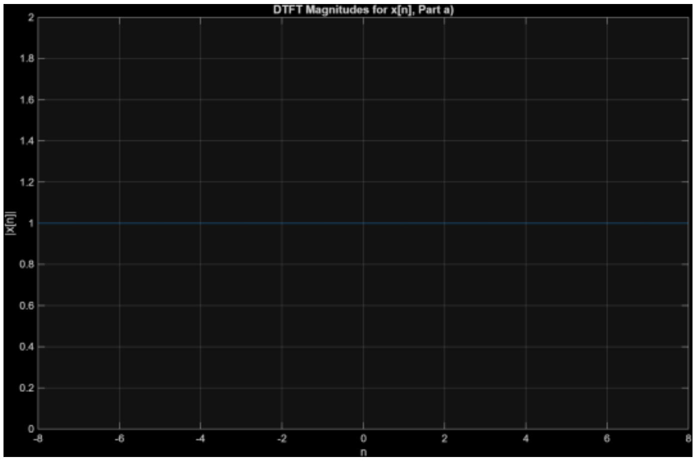
a) $$x[n] = \frac{1}{\cancel{2\pi}}\int_{-\pi}^{\pi}\left(\sum_{k=-\infty}^{\infty} \cancel{2\pi}\,\delta(\omega - 0.5\pi - 2\pi k)\right) e^{j\omega n} d\omega$$

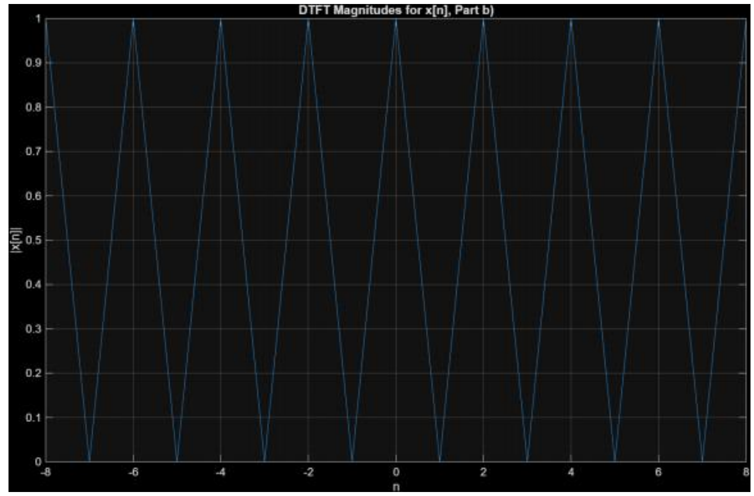
$$= \int_{-\pi}^{\pi} \delta(\omega - 0.5\pi) e^{j\omega n} d\omega = \underline{e^{j(0.5\pi)n}}$$

b) $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \{\pi\delta(\omega + 0.5\pi - 2\pi k) + \pi\delta(\omega - 0.5\pi - 2\pi k)\} e^{j\omega n} d\omega$$

$$= \frac{1}{2\cancel{\pi}}\Big(\underbrace{\int_{-\pi}^{\pi} \pi\delta(\omega + 0.5\pi) e^{j\omega n} d\omega}_{\cancel{\pi} e^{j(-0.5)n}} + \underbrace{\int_{-\pi}^{\pi} \pi\delta(\omega - 0.5\pi) e^{j\omega n} d\omega}_{\cancel{\pi} e^{j(0.5n)}}\Big)$$

$$= \frac{1}{2}\left(e^{j(-0.5n)} + e^{j(0.5n)}\right) \Rightarrow x[n] = \cos(0.5\pi n)$$





B) $y[n] = 0.5\, y[n-1] + x[n]$

$$\Rightarrow \mathcal{F}(y[n]) = 0.5\,\underbrace{\mathcal{F}(y[n-1])}_{Y(e^{j\omega})e^{-j\omega}} + \mathcal{F}(x[n]) \Rightarrow 0.5\, Y(e^{j\omega})e^{-j\omega} + X(e^{j\omega})$$

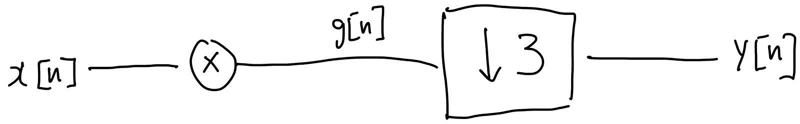
$$\Rightarrow (1 - 0.5e^{-j\omega})\, Y(e^{j\omega}) = X(e^{j\omega}) \Rightarrow \underline{Y(e^{j\omega}) = \frac{1}{1-0.5e^{-j\omega}} X(e^{j\omega})}$$

C) $$X(e^{-j0.25\pi}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{j0.25\pi n}$$

$$\Rightarrow X(e^{-j0.25\pi}) = X^*(e^{j0.25\pi})$$

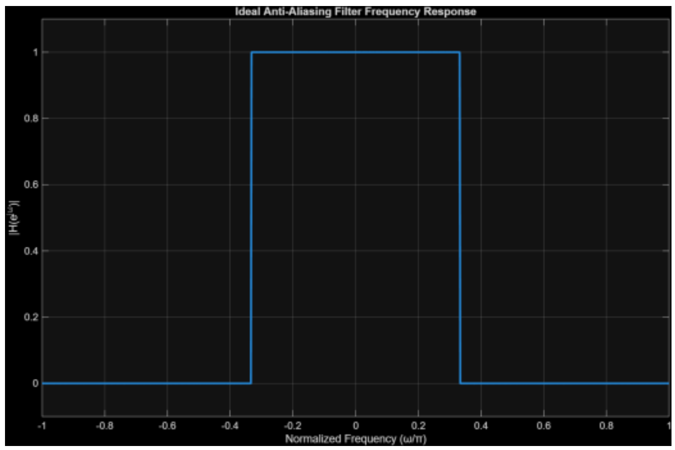
$$\Rightarrow (1+j)^* = \underline{1-j}$$

2.4



a) $Y(e^{j\omega}) = \frac{1}{M} \sum_{r=0}^{M-1} G(e^{j(\omega+2\pi r)/M})$; sampling frequency $|\omega| \le \pi/M$

$\Rightarrow \omega_c = \frac{\pi}{3}$



b) Not causal

The causal system's output is past and present input dependent.

The inverse DTFT of the ideal frequency response is a sinc function in the time domain, non-zero for ⊕ and ⊖ time values, meaning it has response for $n > 0$

c) Guranteed 0s in $G(e^{j\omega})$ for values of ω

$G(e^{j\omega})$ is a produce of $\bar{X}(e^{j\omega})$ and $H(e^{j\omega})$

for $|\omega| > \frac{\pi}{3}$, $\omega \in [-\pi/3, \pi/3] \neq 0$

d)

$$3Y(e^{j\omega}) = G(e^{j(\omega-\alpha)}) + G(e^{j(\omega-\beta)}) + G(e^{j(\omega-\gamma)})$$

Each $G(e^{j\omega})$ term = $Y(e^{j\omega})$, replicas from downsampling by 3

Replicas are shifted by multiples of $2\pi/3$

$$\alpha = 0, \quad \beta = 2\pi/3, \quad \gamma = 4\pi/3$$

e) $g[n]$ from $Y[n]$

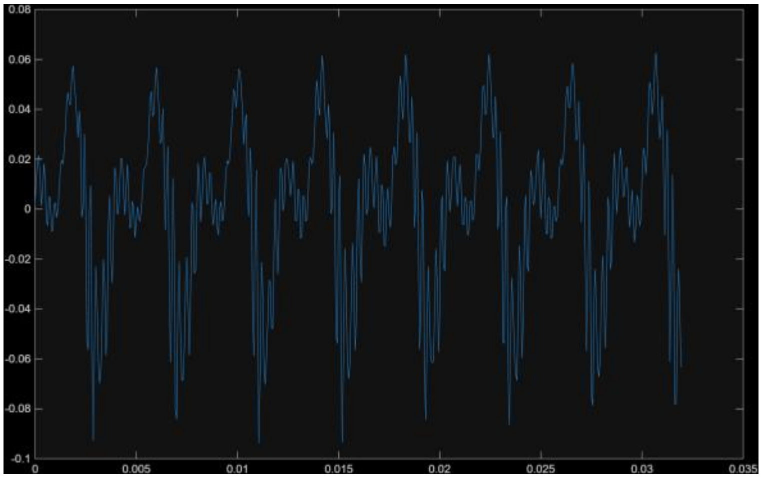
No, since the signal $g[n]$ underwent down sampling to get $Y[n]$, which would potentially had undergone loss of information

f) $x[n]$ from $g[n]$

No, $g[n]$ is the result of passing $X[n]$ through an anti-aliasing filter, the information outside of the sampling frequencies are all lost

Experiential DSP Exercise 2

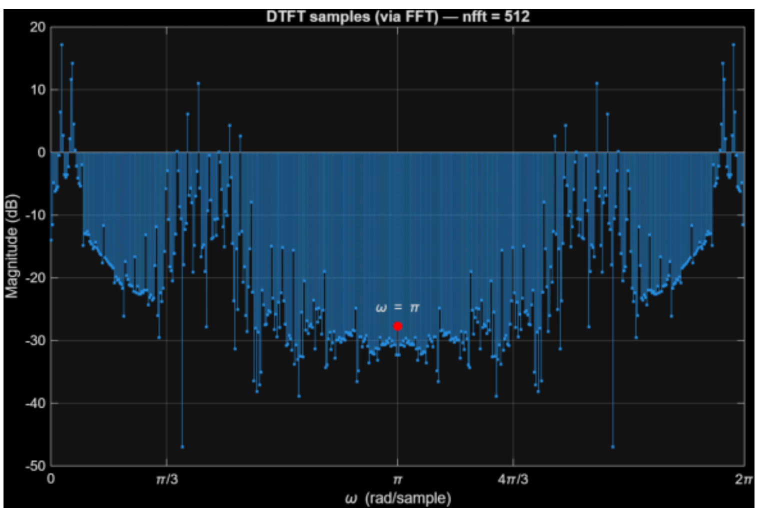
Part A)

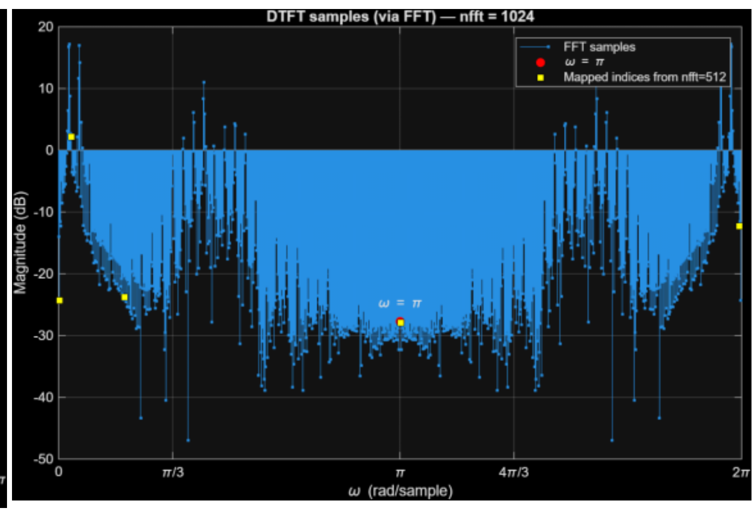


Using the findpeaks() command, finding the difference between 2 peaks and converting that difference from samples to seconds, the approaximate period peak to peak is 0.004125 seconds.

The total duration of the sample is 0.032 seconds.

Part B





5. The halves are mirror images of each othe, with the samples from 0 to π corresponding to the positive frequencies, π to 2π to the negative frequencies, mirroring that of the left half. The plotted magnitude symmetric at ω = π.
6. X2 is a higher resolution sampling of X1 with the same DTFT. The samples that are at x1 are scaled by the ratio nfft2/nfft1.

Example Mapping from k in X1 to k in X2

k_in_X1	**mapped_k_in_X2**
1	2
10	20
50	100
257	514
511	1022