

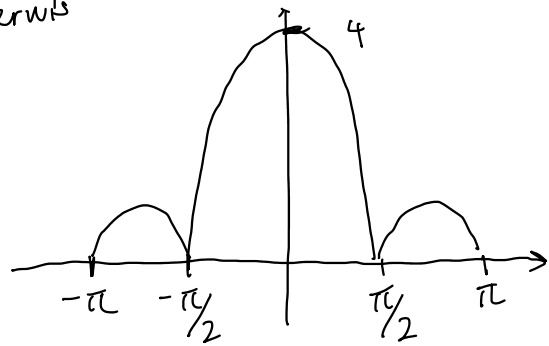
Problem 1.

$$x[k]_N = \begin{cases} X(e^{j\frac{2\pi k}{N}}) & ; \quad 0 \leq k < N \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$x[n] = u[n] - u[n-4]$$

$$a) \bar{X}(e^{j\omega}) = \sum_{n=0}^3 e^{-j\omega n} = e^{-j\frac{3\omega}{2}} \frac{\sin(2\omega)}{\sin(\omega/2)}$$

$$|\bar{X}(e^{j\omega})| = \left| \frac{\sin(2\omega)}{\sin(\omega/2)} \right|$$



$$b) \omega_k = \frac{2\pi k}{4} = \frac{k\pi}{2} \text{ for } k=0,1,2,3$$

$$\Rightarrow X[k]_4 = 0 \text{ for } \underline{k=1,2,3}$$

$$c) \omega_k = \frac{2\pi k}{8} = \frac{k\pi}{4}, \quad k=0,1,\dots,7$$

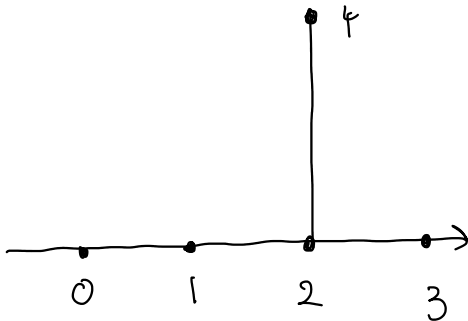
$$\sin(2\omega_k) = 0$$

$$\hookrightarrow \omega = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\Rightarrow k = \text{multiples of } 2, \text{ but not } 4$$

$$\underline{k=2,6}$$

$$d) \bar{Y}(e^{j\omega}) = \bar{X}(e^{j(\omega-\pi)})$$



Problem 2

$$a) Q[k_0]_{256} = Q(e^{-j\pi/2})$$

$$\text{let } \omega_{k_0} = -\frac{\pi}{2}, \quad \frac{2\pi k_0}{256} = -\frac{\pi}{2} \Rightarrow k_0 = -\frac{\pi}{2} \cdot \frac{256}{2\pi} = \frac{-256}{4} = -64$$

$$k \in [0, 256) \Rightarrow k = 256 - 64 = \underline{192}$$

$$b) r[n] = q[n-16]; \quad R[k]_{256} = Q[k]_{256} \cdot e^{-j\frac{2\pi k}{256} \times 16}$$

$$\Rightarrow \underline{|R[k]_{256}| = |Q[k]_{256}| \cdot \underbrace{|e^{-j\frac{2\pi k}{8}}|}_1}$$

Problem 3

a) $P(e^{j\omega})$ sampling every $\frac{2\pi}{N}$ $T_{\text{New}} = \frac{2\pi}{(\frac{2\pi}{N})} = N$

b) $r[n] = q[n] \cdot (u[n] - u[n-N]) = \sum_{m=-\infty}^{\infty} p[n-mN] \cdot (u[n] - u[n-N])$

for $n \in [0, N-1]$, $\sum_{m=-\infty}^{\infty} p[n-mN] \neq 0$ for $m=0$ where $\leadsto p[n]$

$\therefore r[n] = p[n] \cdot (u[n] - u[n-N]) = p[n]$ $m \neq 0$, $p[n-mN]$ out of $[0, N-1]$

c) sampling every $\frac{2\pi}{N-1}$, $T_{\text{New}} = \frac{2\pi}{(2\pi/(N-1))} = N-1$

d) $r[0] = q[0] = \sum_{m=-\infty}^{\infty} p[0-m(N-1)] = p[0] + p[N-1]$

$\rightarrow m=0: p[0-0] \rightarrow p[0]$

$m=1: p[0-(N-1)] \rightarrow p[-N+1]$

$m=-1: p[0-(-1)(N-1)] = p[N-1]$

$\rightarrow p[N-1]$

for $n < 0$; $p[n] = 0$

e) sampling every $\frac{2\pi}{N-2}$, $T_{\text{New}} = N-2$

f) $r[n] = q[n] = \sum_{m=-\infty}^{\infty} p[n-m(N-2)]$

$n=0$

$m=0: p[0]$

$m=-1: p[N-2]$

$r[0] = p[0] + p[N-2]$

$n=1$

$m=0: p[1]$

$m=-1: p[1+(N-2)] = p[N-1]$

$r[1] = p[1] + p[N-1]$