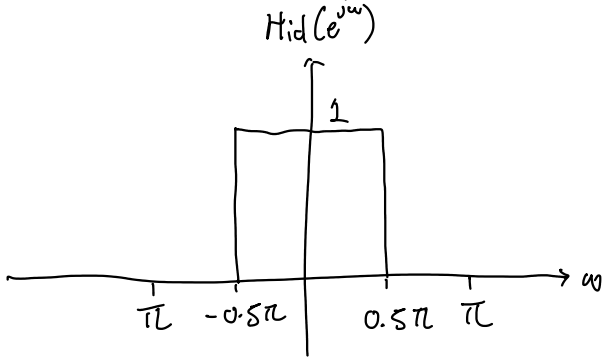
Problem 1.

$$h_{id} = \frac{\sin(0.5\pi n)}{\pi n}$$

a)

$$H_{id}(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le 0.5\pi \\ 0, & 0.5\pi < |\omega| \le \pi \end{cases}$$

$\omega_c = 0.5\pi$



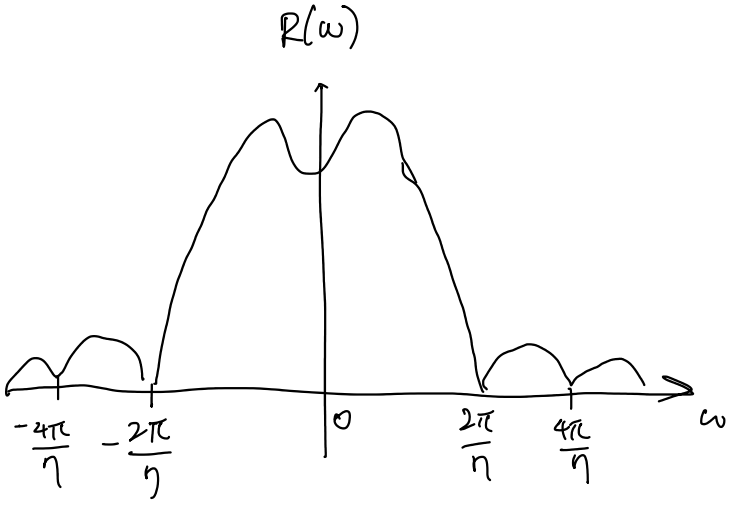
b) $h[n] = h_{id}[n-3]\, w[n]$, $w[n] = u[n] - u[n-7]$

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{id}(e^{j\theta})\, W(e^{j(\omega-\theta)})\, d\theta = \mathcal{F}\{h_{id}[n]\, w[n]\}$$

$$\rightarrow \mathcal{F}\{h[n]\} = H(e^{j\omega}) = e^{-j3\omega} \cdot \mathcal{F}\{h_{id}[n]\, w[n]\}$$

$$= e^{-j3\omega}\underbrace{\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} H_{id}(e^{j\theta})\, W(e^{j(\omega-\theta)})\, d\theta\right]}_{R(\omega)}$$

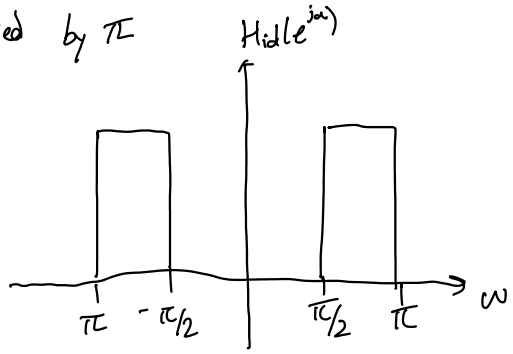
$$\Rightarrow \underline{H(e^{j\omega}) = e^{-j3\omega} R(\omega)}\ //$$



Problem 2. $h_{id}[n] = \dfrac{(-1)^n \sin(0.5\pi n)}{\pi n}$

a) $h_{id}[n] = (-1)^n \cdot \underbrace{h_{id_1}[n]}_{\text{from last question}}$

passband: shifted by π

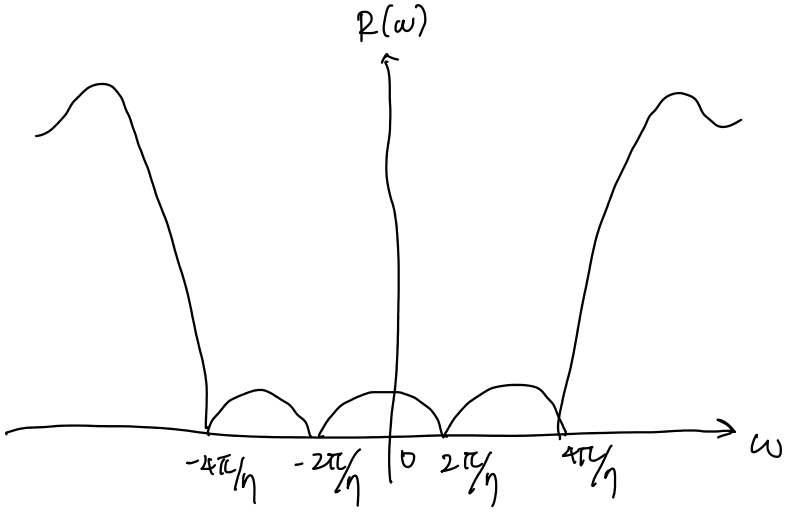


b)

$$H(e^{j\omega}) = e^{-j3\omega}\left[\frac{1}{2\pi}\int_{-2\pi}^{2\pi} H_{id}(e^{j\theta})\, W(e^{j(\omega-\theta)})\, d\theta\right]$$

$$H_{id}(e^{j\omega}) = \begin{cases} 1, & 0.5\pi < |\omega| \le \pi \\ 0, & |\omega| \le 0.5\pi \end{cases}$$

$$R(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{id}(e^{j\theta})\, W(e^{j(\omega-\theta)})\, d\theta$$



Problem 3.

$$w[n] = \sum_{k=-\infty}^{\infty} \{u[k] - u[k-4]\}\{u[n-k] - u[n-k-4]\}$$

$$h[n] = h_{id}[n-3]\, w[n]$$

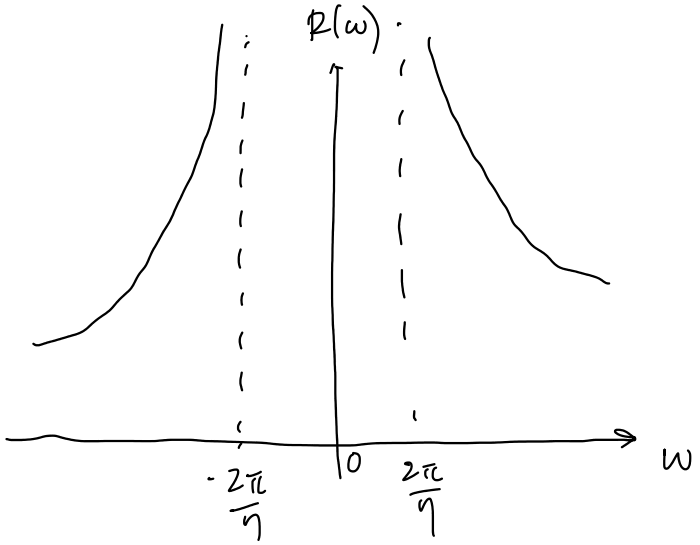
$$W(e^{j\omega}) = \left[\mathcal{F}\{u[n] - u[n-4]\}\right]^2 = \left[\frac{\sin(2\omega)}{\sin(\omega/2)}\, e^{-j\omega\overbrace{(4-1)/2}^{3}}\right]^2$$

$$R_w(\omega) = \mathrm{Re}\left(W(e^{j\omega})\right) = \left[\frac{\sin(2\omega)}{\sin(\omega/2)}\right]^2$$

a) $H_{id}(e^{j\omega})$ same as problem 2

b) $h[n] = h_{id}[n-3]\, w[n] \rightarrow H(e^{j\omega}) = e^{-j3\omega} \cdot \mathcal{F}\{h_{id}[n]\, w[n]\}$

$$H(e^{j\omega}) = R(\omega)\, e^{-j3\omega}$$



Part A

- Length of impulse is 18

Part B

- Final estimated order (n) is 21
- Final length of impulse response is 22

Part C

- Final estimated order (n) is 39
- Final length of impulse response is 40