Problem 1

a) $\bar{X}[k]_N = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n}$ for $0 \le k \le N-1$

$N=3$; $W_{3\times3}$:

$$\begin{bmatrix} X[0]_3 \\ X[1]_3 \\ X[2]_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{2\pi}{3}} \end{bmatrix}}_{W_{3\times3}} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \end{bmatrix}$$

b) $(W_{3\times3}^T)^* = W_{3\times3}^{-1}$, $[W_{3\times3}^T]^*_{k,n} = W_3^{-kn} = e^{j\frac{2\pi kn}{3}}$

$$W^{-1}_{3\times3} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^2 & W_3 \\ 1 & W_3 & W_3^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{4\pi}{3}} & e^{-j\frac{2\pi}{3}} \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-j\frac{4\pi}{3}} \end{bmatrix}$$

c) $W_{3\times3} * (W_{3\times3}^T)^* = N \cdot I_N$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

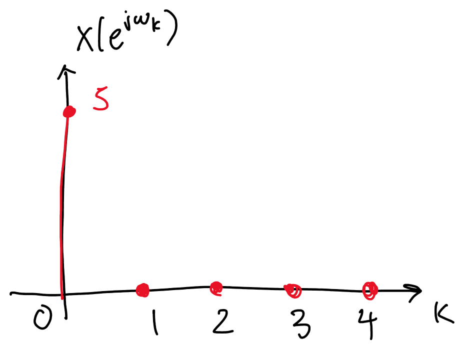
d) $W_5^k = e^{-j\frac{2\pi k}{5}}$

$$W_{5\times5} : \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & W_5 & W_5^2 & W_5^3 & W_5^4 \\ 1 & W_5^2 & W_5^4 & W_5^6 & W_5^8 \\ 1 & W_5^3 & W_5^6 & W_5^9 & W_5^{12} \\ 1 & W_5^4 & W_5^8 & W_5^{12} & W_5^{16} \end{bmatrix}$$

$W_5^A = W_5^{A(\text{mod } 5)}$

Distinct elements: $W_5^0, W_5, W_5^2, W_5^3, W_5^4$

e) $x[n] = u[n] - u[n-5]$



Problem 2 $\quad p[n] = u[n] - u[n-2] \quad b[n] = \sum_{k=0}^{3} p[n-4k] \quad q[n] = \delta[n] - 0.25\,\delta[n-2]$

a) $P[k]_4 = P(e^{j\frac{2\pi k}{4}})$

$P(e^{j\omega}) = \sum_{n=0}^{1} e^{-j\omega n} = 1 + e^{-j\omega} \rightarrow P[0]_4 = 1 + e^{-j(0)} = 1+1 = 2$

$P[2]_4 = P(e^{j\pi}) = 1 + e^{-j\pi} = 1+(-1) = 0$

b) $b[n] = p[n] + p[n-4] + p[n-8] + p[n-12]$

$B(e^{j\omega}) = P(e^{j\omega}) + P(e^{j\omega})e^{-j4\omega} + P(e^{j\omega})e^{-j8\omega} + P(e^{j\omega})e^{-j12\omega}$

$= P(e^{j\omega})\left(1 + e^{-j4\omega} + e^{-j8\omega} + e^{-j12\omega}\right)$

$B[7]_{16}$, $\omega = \frac{2\pi(7)}{16} = \frac{7\pi}{8}$: $\sum_{m=0}^{3}\left(e^{-j\frac{4}{8}7\pi}\right) = \sum_{m=0}^{3}\underbrace{\left(e^{-j\frac{7\pi}{2}}\right)}_{j}^{m} = 1 + j + j^2 + j^3 = \underline{0}$

$B[8]_{16}$; $\omega = \pi$

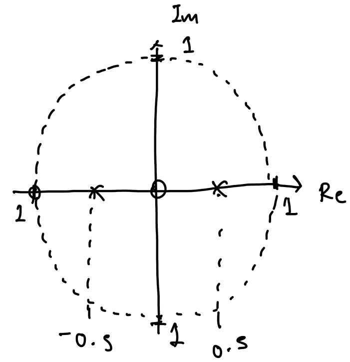
$\sum_{m=0}^{3}\underbrace{\left(e^{-j4\pi}\right)}_{1}^{m} = 4$

$P(e^{j\pi}) = 1 + e^{-j\pi} = 0$

$B[8]_{16} = 0 \cdot 4 = \underline{0}$

c) $H(e^{j\frac{2\pi k}{128}}) = \frac{P[k]_{128}}{Q[k]_{128}}$; $P(z) = Z\{p[n]\} = 1 + z^{-1}$

$Q(z) = Z\{q[n]\} = 1 - \frac{z^{-2}}{4}$

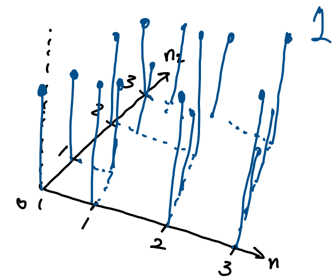


$H(z) = \frac{1+z^{-1}}{1-\frac{z^{-2}}{4}} = \frac{z(z+1)}{z^2-\frac{1}{4}}$ $\longrightarrow$ $z = -1$ or 0

$\longrightarrow$ $p = \pm \frac{1}{2}$

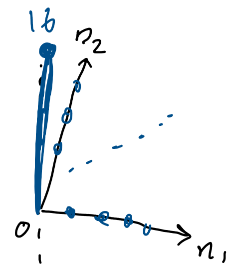
Problem 3

a) $x[n_1, n_2] = \underbrace{\{u(n_1) - u(n_1 - 4)\}}_{1 \text{ for } 0 \le n_1 \le 3} \underbrace{\{u(n_2) - u(n_2 - 4)\}}_{1 \text{ for } 0 \le n_2 \le 3}$



b) $\bar{X}[k_1, k_2] = \bar{X}[k_1] \cdot \bar{X}[k_2] = 16$ for $k_1 = k_2 = 0$

$\bar{X}[0] = 4$, $\bar{X}[k] = 0$ for $k = 1, 2, 3$



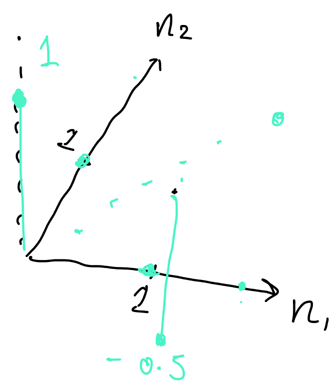
c) Given that the DFT is $\bar{X}_1[k] \cdot \bar{X}_2[k]$ it is separable

d) Yes, again one strong stem at the origin but zeros everywhere else.

Problem 4

a) $H(z_1, z_2) = 1 - 0.5 z_1^{-1} z_2^{-1}$ $\quad H(z_1, z_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} h[n_1, n_2] z_1^{-n_1} z_2^{-n_2}$

↳ $\underbrace{h[0,0]}_{1} \cdot z_1^0 z_2^0 + \underbrace{h[1,1]}_{-0.5} \cdot z_1^{-1} z_2^{-1}$



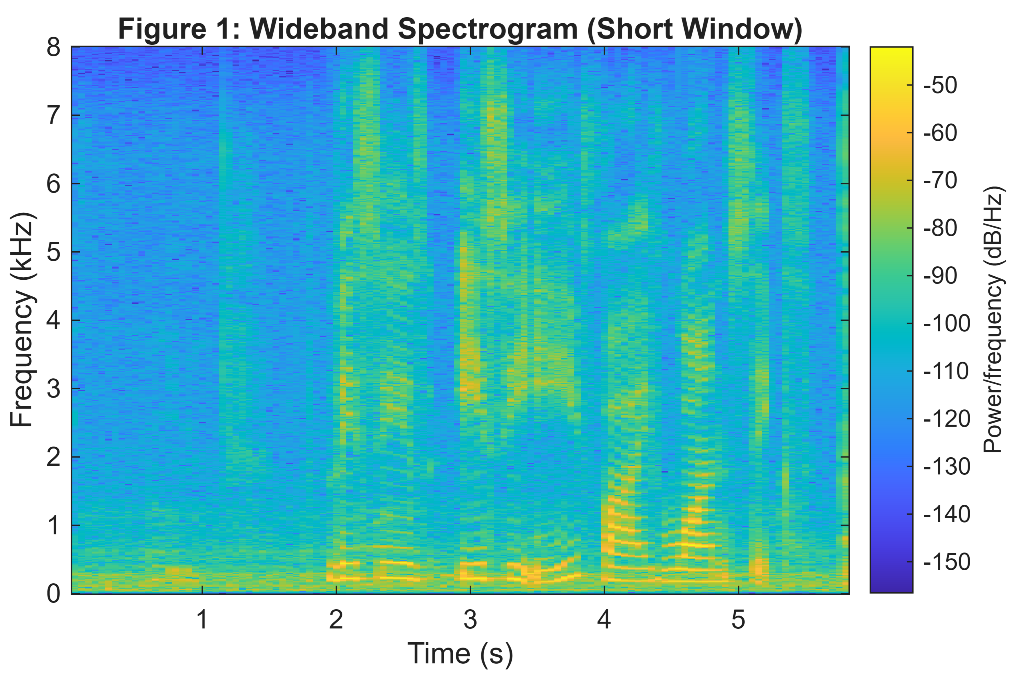
b) $$\sum_{k=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} h[n_1-k,\ n_2-p]\, e^{(j\omega_1 k + \omega_2 p)} = 0$$

$$= e^{j(\omega_1 n_1 + \omega_2 n_2)} \cdot \underbrace{H(e^{j\omega_1}, e^{j\omega_2})}_{\text{2D DTFT}}, \quad H(z_1, z_2) = 1 - 0.5 z_1^{-1} z_2^{-1}$$

$$\hookrightarrow 1 - 0.5 e^{-j(\omega_1+\omega_2)}$$

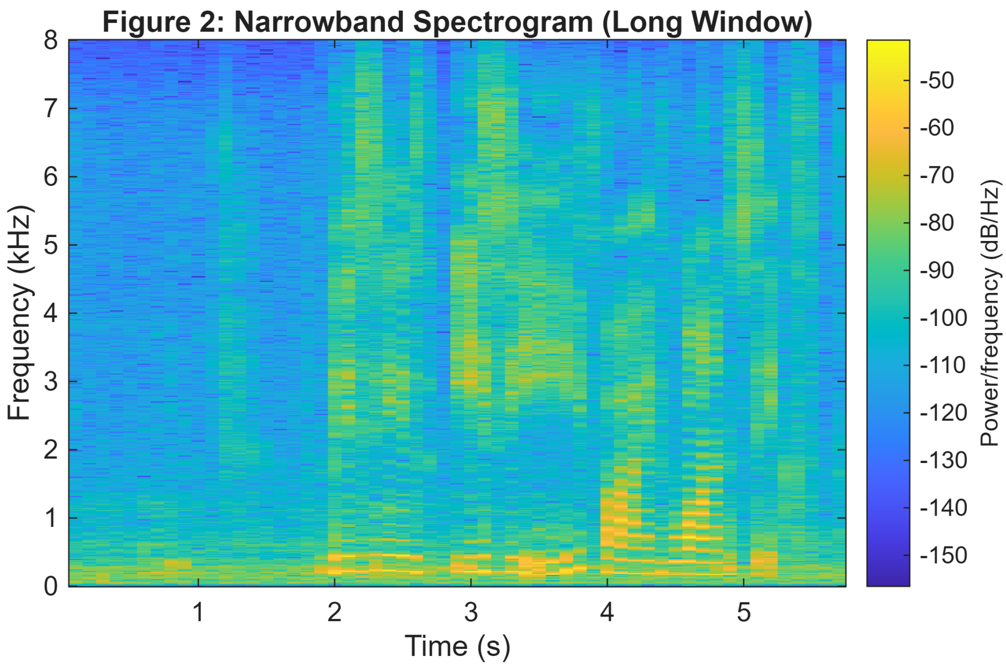
For $1 - 0.5 e^{-j(\omega_1+\omega_2)} = 0 \rightarrow e^{-j(\omega_1+\omega_2)} = 2$ $\therefore$ impossible

```
% Experiential DSP Assignment 9

% Reading the file
[y, Fs] = audioread('ece516.wav');

% Setting nfft Size
nfft = 1024;

% Setting window length
win_len_ms = 100;

% Compute number of samples
win_len_samp = round(win_len_ms * Fs / 1000);

% Designing the window
win = hamming(win_len_samp);

% Number of overlapping samples
noverlap = round (win_len_samp * 0.5);

% Plotting wide band spectrogram
figure(1)
spectrogram (y, win, noverlap, nfft, Fs, 'yaxis')
title('Figure 1: Wideband Spectrogram (Short Window)');
```




```
% Step 9 but with win_len_ms = 200
% Setting window length
win_len_ms_2 = 200;

% Compute number of samples
```

```
win_len_samp_2 = round(win_len_ms_2 * Fs / 1000);

% Designing the window
win_2 = hamming(win_len_samp_2);

% Number of overlapping samples
noverlap_2 = round (win_len_samp_2 * 0.5);

% Plotting wide band spectrogram
figure(2)
spectrogram (y, win_2, noverlap_2, nfft, Fs, 'yaxis')
title('Figure 2: Narrowband Spectrogram (Long Window)');
```




```
%{
The wide band spectrogram (short window) has sharper vertical lines compared to the
narrowband spectrogram (long window)
The frequency resolution is clearer on the narrowband than the wide band
spectrogram.
%}
```