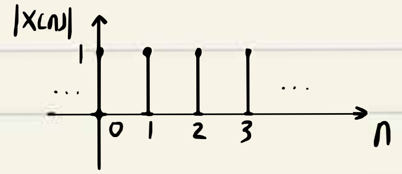
Problem 2.1

A) (a) Yes. The fundamental period $N = \frac{2\pi}{0.375\pi} \cdot k = \frac{2\pi}{0.375\pi} \cdot 3 = 16$ (k is a positive number such that N is the smallest positive number)

(b) $|x(n)| = |e^{j0.375\pi n}| = 1$



(c) $x(8+16k) = e^{j0.375\pi(8+16k)} = e^{j3\pi} = -1 \quad (k \in \mathbb{Z})$

B) $$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} \delta[n-3] e^{-jwn}$$

$$= e^{-j3w}$$

C) a) $$X(e^{jw}) = \sum_{n=-2}^{2} e^{-jwn}$$

$$= \frac{e^{j2w}(1-e^{-j5w})}{1-e^{-jw}}$$

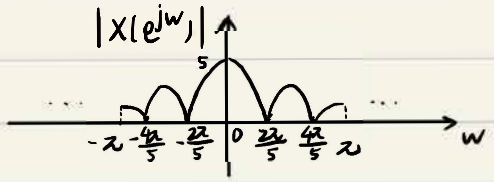
$$= \frac{e^{j2w} e^{-j\frac{5}{2}w} (e^{j\frac{5}{2}w} - e^{-j\frac{5}{2}w})}{e^{-j\frac{1}{2}w}(e^{j\frac{1}{2}w} - e^{-j\frac{1}{2}w})}$$

$$= \frac{\sin(\frac{5}{2}w)}{\sin(\frac{1}{2}w)}$$

$|X(e^{jw})| = \left|\frac{\sin(\frac{5}{2}w)}{\sin(\frac{1}{2}w)}\right|$ (Period $T = 2\pi$)

zero crossings: $w = \frac{2k\pi}{5}$ ($k \in \mathbb{Z}$ and $\frac{k}{5} \notin \mathbb{Z}$)

$\frac{0}{0}$: $w = 2k\pi$ ($k \in \mathbb{Z}$) L' Hôpital's rule: $\lim_{w \to 2k\pi} |X(e^{jw})| = \lim_{w \to 2k\pi} 5 \left|\frac{\cos(\frac{5}{2}w)}{\cos(\frac{1}{2}w)}\right| = 5$



b) $$X(e^{jw}) = \sum_{n=0}^{4} e^{-jwn}$$

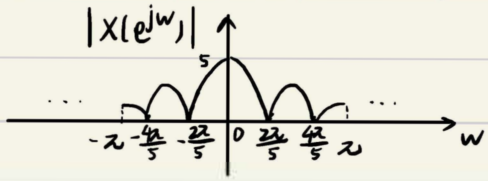
$$= \frac{1-e^{-j5w}}{1-e^{-jw}}$$

$$= e^{-j2w} \frac{\sin(\frac{5}{2}w)}{\sin(\frac{1}{2}w)}$$

$$|X(e^{jw})| = \left|e^{-j2w}\frac{\sin(\frac{5}{2}w)}{\sin(\frac{1}{2}w)}\right| = \left|\frac{\sin(\frac{5}{2}w)}{\sin(\frac{1}{2}w)}\right| \quad (\text{Period } T = 2\pi)$$

zero crossings: $w = \frac{2k\pi}{5}$ ($k \in \mathbb{Z}$ and $\frac{k}{5} \notin \mathbb{Z}$)

$\frac{0}{0}$: $w = 2k\pi$ ($k \in \mathbb{Z}$) L' Hôpital's rule: $\lim\limits_{w \to 2k\pi} |X(e^{jw})| = \lim\limits_{w\to 2k\pi} 5\left|\frac{\cos(\frac{5}{2}w)}{\cos(\frac{1}{2}w)}\right| = 5$



c) $X(e^{jw}) = \sum\limits_{n=0}^{3} e^{-jwn}$

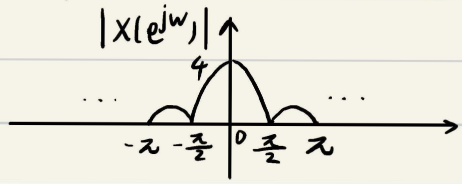
$$= \frac{1 - e^{-j4w}}{1 - e^{-jw}}$$

$$= e^{-j\frac{3}{2}w}\frac{\sin(2w)}{\sin(\frac{1}{2}w)}$$

$$|X(e^{jw})| = \left|e^{-j\frac{3}{2}w}\frac{\sin(2w)}{\sin(\frac{1}{2}w)}\right| = \left|\frac{\sin(2w)}{\sin(\frac{1}{2}w)}\right| \quad (\text{Period } T = 2\pi)$$

zero crossings: $w = \frac{k\pi}{2}$ ($k \in \mathbb{Z}$ and $\frac{k}{4} \notin \mathbb{Z}$)

$\frac{0}{0}$: $w = 2k\pi$ ($k \in \mathbb{Z}$) L' Hôpital's rule: $\lim\limits_{w \to 2k\pi} |X(e^{jw})| = \lim\limits_{w\to 2k\pi} 4\left|\frac{\cos(2w)}{\cos(\frac{1}{2}w)}\right| = 4$

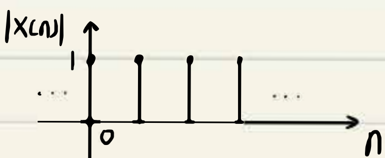


Problem 2.3

A) a) $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(w - 0.5\pi)e^{jwn}\,dw$

$$= e^{j0.5\pi n}$$

$|x(n)| = 1$

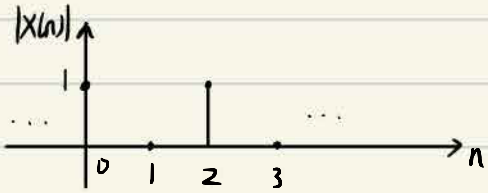


b) $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}[\pi\delta(w + 0.5\pi) + \pi\delta(w - 0.5\pi)]e^{jwn}\,dw$

$$= \frac{1}{2}(e^{j0.5\pi n} + e^{-j0.5\pi n})$$

$$= \cos 0.5\pi n$$

$|x(n)| = |\cos 0.5\pi n|$



B) $y[n] = 0.5y[n-1] + x[n]$

DTFT on both sides

$$Y(e^{jw}) = 0.5e^{-jw}Y(e^{jw}) + X(e^{jw})$$

$$Y(e^{jw}) = \frac{1}{1-0.5e^{-jw}} X(e^{jw})$$

c) if $x(n) \longleftrightarrow X(e^{jw})$

then $x^*(n) \longleftrightarrow X^*(e^{-jw})$

$\because$ $x(n)$ is real

$\therefore$ $x^*(n) = x(n)$

$\therefore$ $X(e^{jw}) = X^*(e^{-jw})$

$$X(e^{-j0.25\pi}) = X^*(e^{j0.25\pi}) = 1-j$$

[redacted]

Problem 2.2

(a) When $T = 0.0001$, sample rate is 20000π rads/s. 20000π in CTFT's ω-axis corresponds to 2π in DTFT's ω-axis. 10000π in CTFT's ω-axis corresponds to π in DTFT's ω-axis.

In the interval of $[-\pi, \pi]$, $|X(e^{j\omega}| = 0$ when ω is $-\pi$ or π.

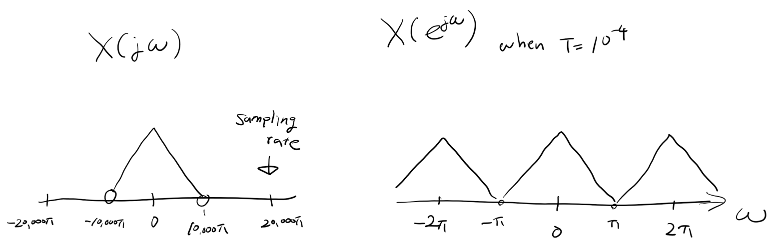


Figure 7: (left) CTFT of $x(t)$. (right) DTFT of $x[n]$ when $T = 0.0001$.

(b) When $T = 0.00005$, sample rate is 40000π rads/s. 40000π in CTFT's ω-axis corresponds to 2π in DTFT's ω-axis. 10000π in CTFT's ω-axis corresponds to $\frac{\pi}{2}$ in DTFT's ω-axis.

In the interval of $[-\pi, \pi]$, $|X(e^{j\omega})| = 0$ when ω is in $\left[-\pi, -\frac{\pi}{2}\right]$ or $\left[\frac{\pi}{2}, \pi\right]$.

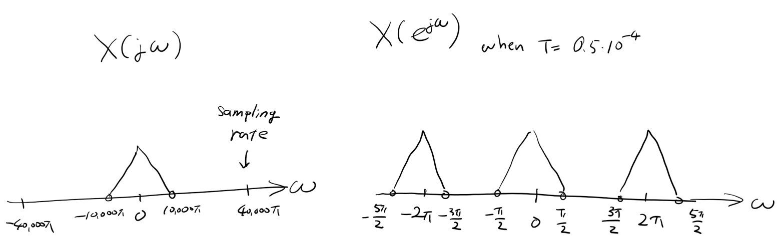


Figure 8: (left) CTFT of $x(t)$. (right) DTFT of $x[n]$ when $T = 0.00005$.

(c) When $T = 0.00001$, sample rate is 200000π rads/s. 200000π in CTFT's ω-axis corresponds to 2π in DTFT's ω-axis. 10000π in CTFT's ω-axis corresponds to $\frac{\pi}{10}$ in DTFT's ω-axis.

In the interval of $[-\pi, \pi]$, $|X(e^{j\omega})| = 0$ when ω is in $\left[-\pi, -\frac{\pi}{10}\right]$ or $\left[\frac{\pi}{10}, \pi\right]$.

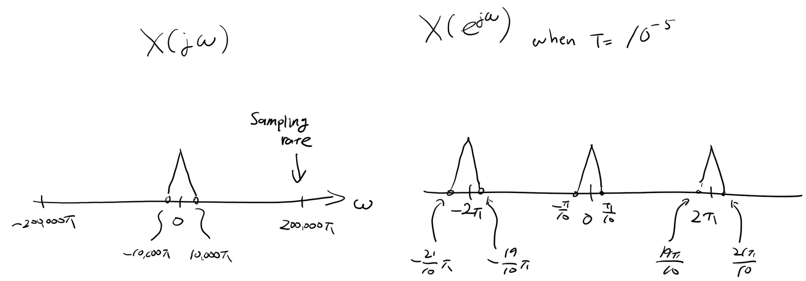


Figure 9: (left) CTFT of $x(t)$. (right) DTFT of $x[n]$ when $T = 0.00001$.

Problem 2.4

a) Box with cutoff frequency $\pi/3$.
b) No. Impulse response is sinc that is non-zero for negative times.
c) For $\frac{\pi}{3} < |\omega| \leq \pi$
d) $3Y\left(e^{j\omega}\right) = G\left(e^{j\left(\frac{\omega}{3}-0\right)}\right) + G\left(e^{j\left(\frac{\omega}{3}-\frac{2\pi}{3}\right)}\right) + G(e^{j\left(\frac{\omega}{3}-\frac{4\pi}{3}\right)})$
e) Yes, because there is no aliasing
f) No, the frequencies lost by the digital antialiasing filter cannot be recovered.