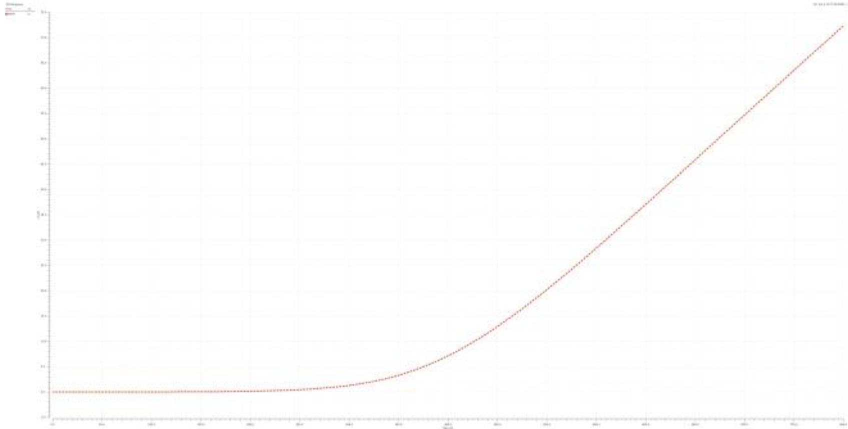
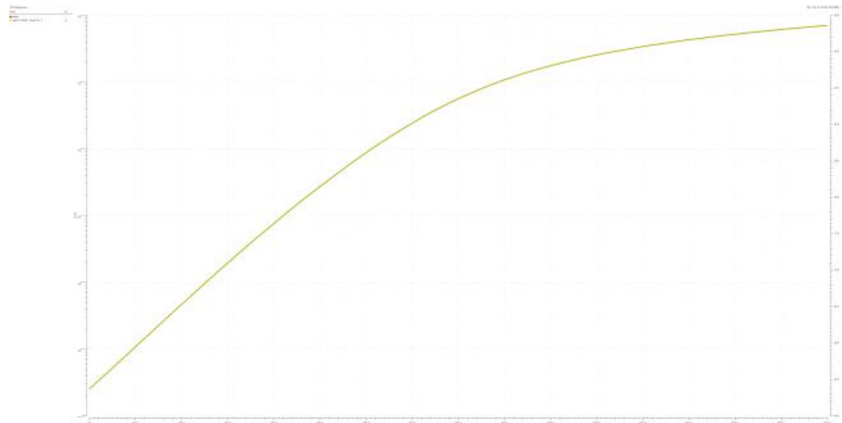
Problem 1

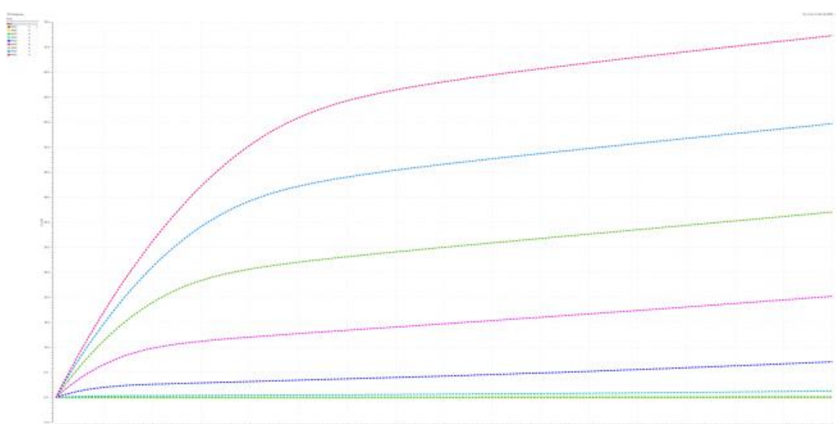
NMOS



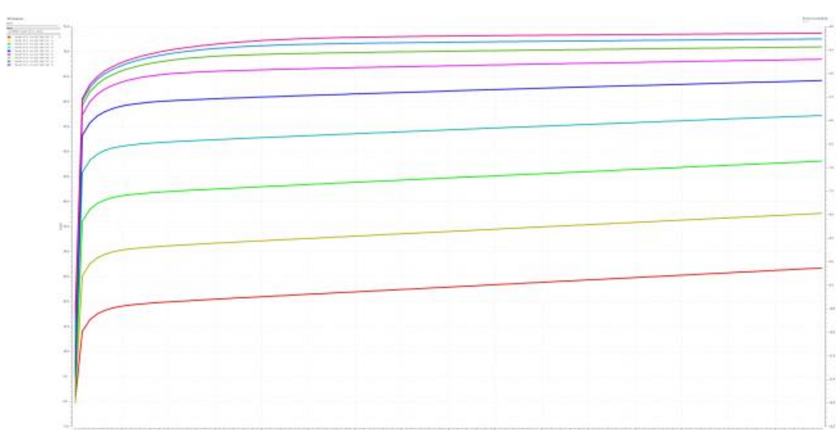
(i) I_{DS} vs. V_{GS} for $V_{DS} = 0.8V$



(ii) $\log(I_{DS})$ vs. V_{GS} for $V_{DS} = 0.8V$

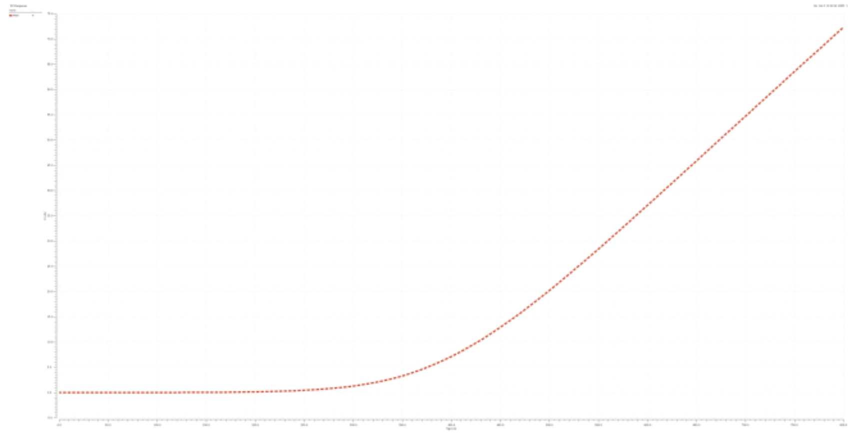


(iii) I_{DS} vs V_{DS}, Parametric Sweep V_{GS} from $0 \rightarrow 0.8V$ (V_{DD}) w/ 0.1V step size



(iv) $\log(I_{DS})$ vs V_{DS}, Parametric sweep V_{GS} from 0V to 0.8V (V_{DD}) w/ 0.1V step size

PMOS



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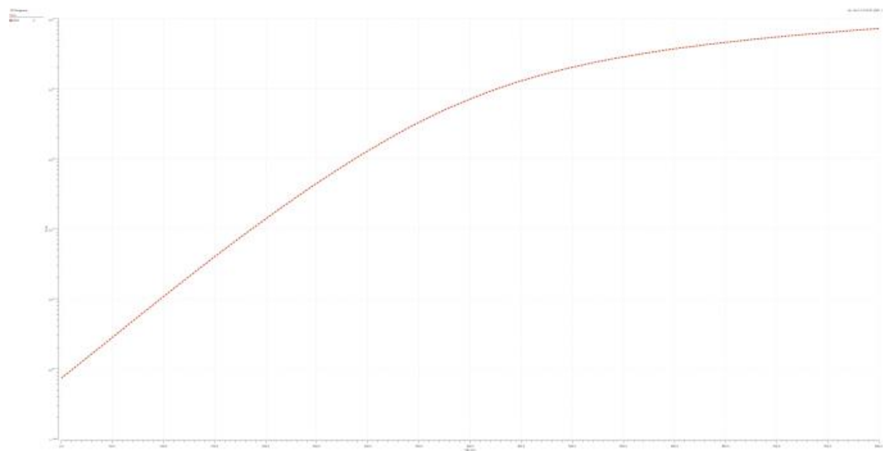
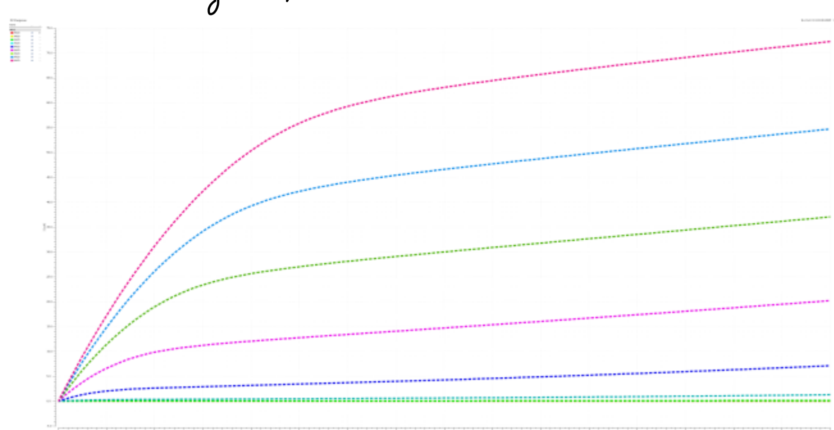
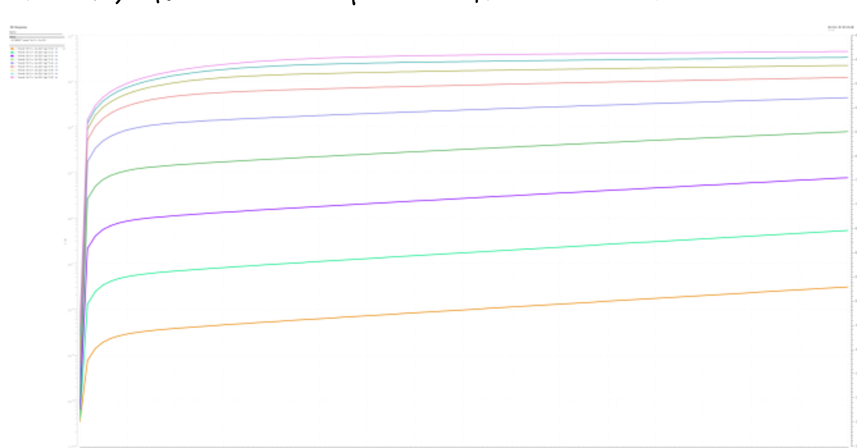


(ii) $\log(I_{DS})$ vs. V_{GS} for $V_{DS} = 0.8V$



(iii) I_{DS} vs V_{DS}, Parametric Sweep V_{GS} from $0 \rightarrow 0.8V$ (V_{DD}) w/ 0.1V step size



(iv) $\log(I_{DS})$ vs V_{DS}, Parametric sweep V_{GS} from 0V to 0.8V (V_{DD}) w/ 0.1V step size

Problem 2. (i) Finding α

$$I_{DS} \propto (V_{GS} - V_{TH}) V_{DS} - \frac{\alpha V_{DS}^2}{2}$$ in the triode region

To find α we can take the derivative of both sides which will give us:

$$\frac{\partial I_{DS}}{\partial V_{DS}} \propto (V_{GS} - V_{TH}) - \alpha V_{DS} \rightarrow \frac{\partial I_{DS}}{\partial V_{DS}} = C\left[(V_{GS} - V_{TH}) - \alpha V_{DS}\right] = C(V_{GS} - V_{TH}) - C\alpha V_{DS}$$

Taking a 2nd derivative: $\frac{\partial^2 I_{DS}}{\partial V_{DS}} = -C V_{DS}$; are the y-intercept $(V_{DS} = 0)$ of the first derivative:

$$\left.\frac{\partial I_{DS}}{\partial V_{gs}}\right|_{V_{DS}=0} = C(V_{GS} - V_{TH})$$

To eliminate C: * let m = slope

$$\frac{m}{\underbrace{\left(\left.\frac{\partial I_{DS}}{\partial V_{GS}}\right|_{V_{DS}=0}\right)}_{\text{let this be } G_{DS,0}}} = \frac{-C\alpha}{C\cdot(V_{GS} - V_{TH})} \rightarrow \alpha = -m \cdot \frac{V_{GS} - V_{TH}}{G_{DS,0}}$$

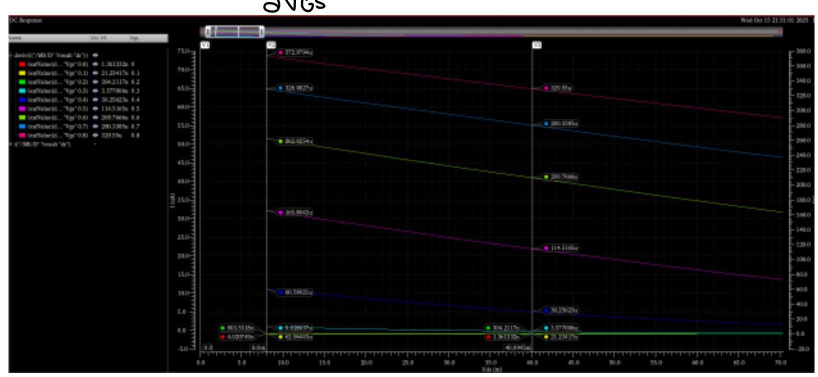
$$G_{DS,0} = G_{DS,1} - m \cdot V_{DS1}$$

$$m = \frac{G_{DS,2} - G_{DS,1}}{V_{DS,2} - V_{DS,1}}$$

using the first derivative plot of I_{DS} vs. V_{DS} $\left(\frac{\partial I_{DS}}{\partial V_{GS}} \text{ vs. } V_{DS}\right)$ and $V_{TH} = 0.3804\,V$

$V_{GS} = 0.8\,V$

$\frac{\partial I_{DS}}{\partial V_{GS}}$ vs. V_{DS} w/ V_{GS} sweep



For our case:

$G_{DS,1} = 372.9794u$, $G_{DS,2} = 329.55u$

$V_{DS,1} = 0.008V$, $V_{DS,2} = 0.040094V$

Plug in our values:

$$m = \frac{(329.55 - 372.9794)\times 10^{-6}\ S}{0.040094 - 0.008\quad V}$$

$$\approx -1.3532\times 10^{-3}\ \frac{S}{V}$$

$$G_{DS,0} = (372.9794u - m \times 0.08) = 0.3838\ mV$$

$$\alpha = -m \cdot \frac{V_{GS} - V_{TH}}{G_{DS,0}} = -(-1.3532\times 10^{-3}) \cdot \frac{(0.8 - 0.3804)}{0.3838\times 10^{-3}}$$

$$\rightarrow \underline{\alpha \approx 1.4794}\ //$$

Problem 2 (ii) Finding n, sub-threshold slope

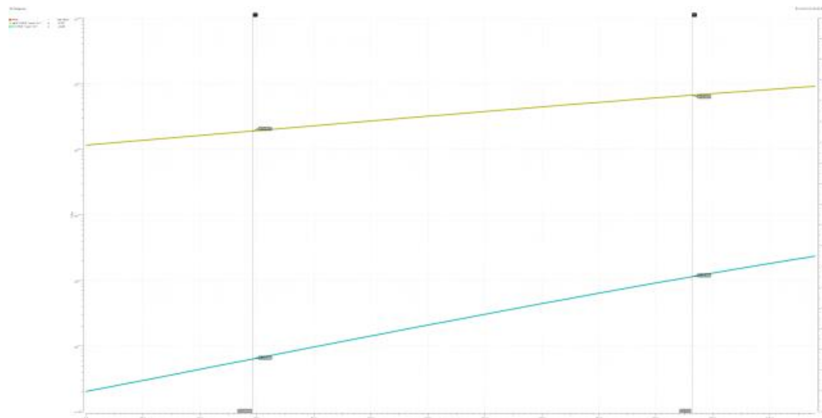
$I_{DS} \propto e^{\frac{V_{GS}}{n\phi_t}}$, using the same approach we took to find a from the previous question, we take the first derivative of $\ln(I_{DS})$ with respect to V_{GS} :

$$\underbrace{\frac{\partial}{\partial V_g} \ln(I_{DS})}_{\text{slope, } m} \propto \frac{1}{n\phi_t}$$

, and let $\phi_t = 25.8\,mV$ at $298\,K$.

$$\rightarrow \quad n = (m \cdot \phi_t)^{-1}$$

Taking two points of the plot to find the slope we have:



Plot of $\underbrace{\log(I_{DS}) \text{ vs } V_{GS}}_{\text{Green}}$ and $\underbrace{\ln(I_{DS}) \text{ vs } V_{GS}}_{\text{Blue}}$

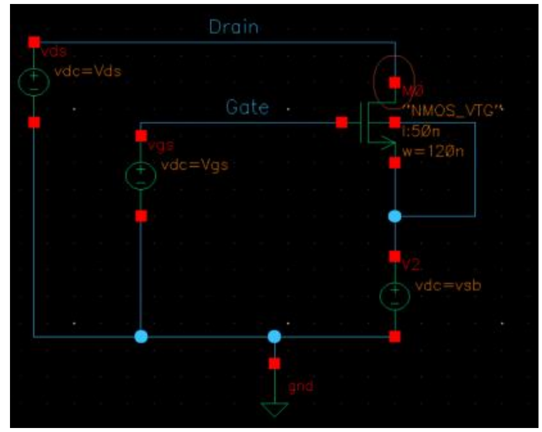
$$\frac{-15.489 - (-19.459)}{212.7 - 58.59668} \times \frac{1000}{mV} = 25.762 \approx 25.76$$

$$\therefore n = \frac{1}{m\phi_t} = (m \cdot \phi_t)^{-1} = (25.76 \times 0.0258) = 1.5045 \approx \underline{1.505}$$

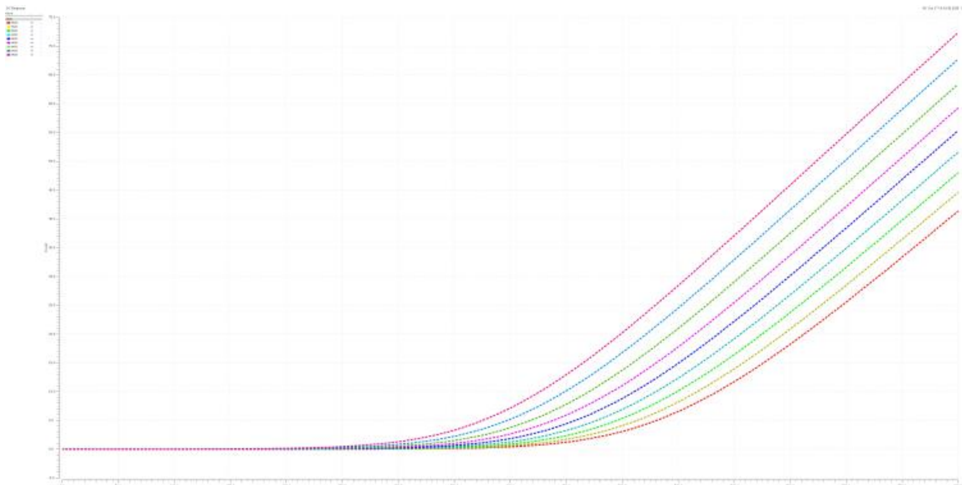
Problem 2 (iii) Finding γ, The body effect coefficient

$$V_T = V_{T_0} + \gamma \sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}$$

To solve this problem we must first rebuild the NMOS circuit to include V_{SB}, and parametric sweep it as a part of data collection to find different V_{TH}s.



NMOS Circuit w/ V_{SB}



I_{DS} vs V_{GS} parametrizing V_{SB}s,

	vsb	OP("/M0","vth") (V)
1	-800.0E-3	491.8E-3
2	-700.0E-3	476.4E-3
3	-600.0E-3	460.4E-3
4	-500.0E-3	443.9E-3
5	-400.0E-3	426.8E-3
6	-300.0E-3	409.0E-3
7	-200.0E-3	390.5E-3
8	-100.0E-3	371.1E-3
9	-138.8E-18	350.6E-3

V_{TH}s of different V_{SB}s,

Solving for a system of equations we would be able to find γ:

Assume: $\phi_F = 0.7V$

let $\Delta V_{TH} = V_{TH_i} - V_{TH_j}$ $\quad i = V_{SB} = 0.8V, \quad j = V_{SB} = 0.7V$

$$\Delta V_{TH} = \gamma \left[\sqrt{2\phi_F + 0.8} - \sqrt{2\phi_F + 0.7} \right] = 0.0154V$$

$$\rightarrow \gamma = \frac{\Delta V_{TH}}{\sqrt{2\phi_F + 0.8} - \sqrt{2\phi_F + 0.7}} = \frac{0.0154}{\sqrt{2.2} - \sqrt{2.1}} = 0.45\ V^{\frac{1}{2}}$$

2 (iv) Estimating Early Voltage, V_A

$$I_{DS} \propto \underbrace{(1+\lambda V_{DS})}_{\approx \left(1+\frac{V_{DS}}{V_A}\right)}, \quad I_{DS} = I_D'(1+\lambda V_{DS}), \quad I_D' = \frac{I_{DS}}{(1+\lambda V_{DS})}$$

If we take the first derivative of I_{DS} with respect to V_{DS} we have: $\frac{\partial I_{DS}}{\partial V_{DS}} = I_D' \lambda = \frac{I_{DS}}{(1+\lambda V_{DS})} \cdot \lambda$

Given that r_o is $\frac{1}{g_{ds}}$ or $\left(\frac{\partial I_{DS}}{\partial V_{DS}}\right)^{-1}$ we get:

$$r_o = \frac{1}{g_{ds}} = \frac{1+\lambda V_{DS}}{\lambda I_{DS}}, \quad \text{substitute: } \lambda = \frac{1}{V_A} \rightarrow r_o = \frac{1+\left(\frac{V_{DS}}{V_A}\right)}{\frac{I_{DS}}{V_A}} = \frac{V_A + V_{DS}}{I_{DS}}$$

$$\therefore V_A = (I_{DS} \cdot r_o) - V_{DS}$$

we use the plot of I_{DS} vs. V_{DS} with V_{GS} of 0.8V.

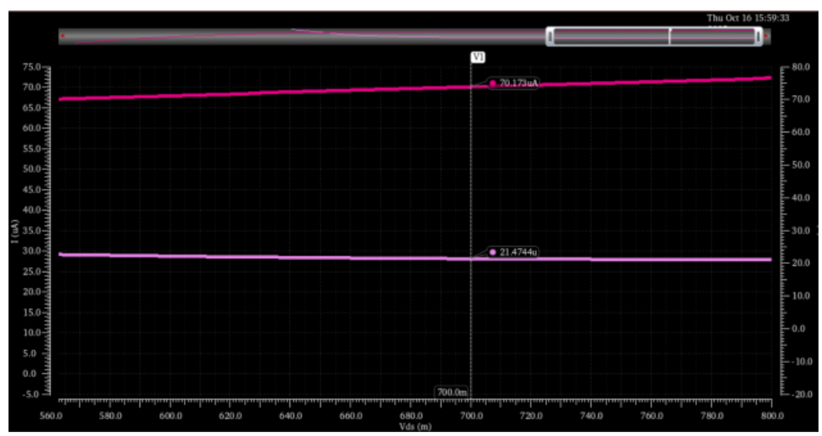
Take the reciprocal of its first derivative to get r_o.

At $V_{DS} = 0.7V$:

$g_{ds} = 21.4744\,u\,\Omega^{-1}$

$\rightarrow r_o \approx 46567.1\,\Omega$

$I_{DS} : 70.173\,uA$



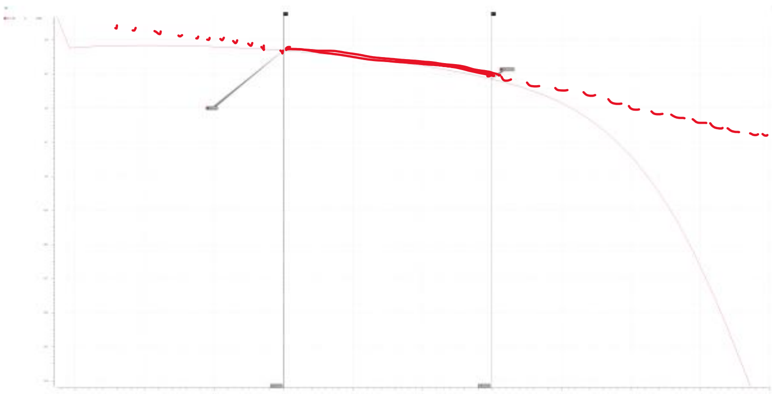
Plot of I_{DS} vs V_{DS} and ∂I_{DS} at $V_{GS} = 0.8V$

$$V_A \approx (I_{DS} \cdot r_o) - V_{DS}$$

$$\approx \underbrace{(70.173\times10^{-6}A \times 46567\,\Omega)}_{\approx 3.2678V} - 0.7V \approx \underline{2.5678\,V}$$

Problem 2 (v) Estimating specific current I_{spec}

plotting g_m/I_{DS} vs. I_{DS} in log-log space; we take and estimate 2 lines from the curve and find its intersection point.

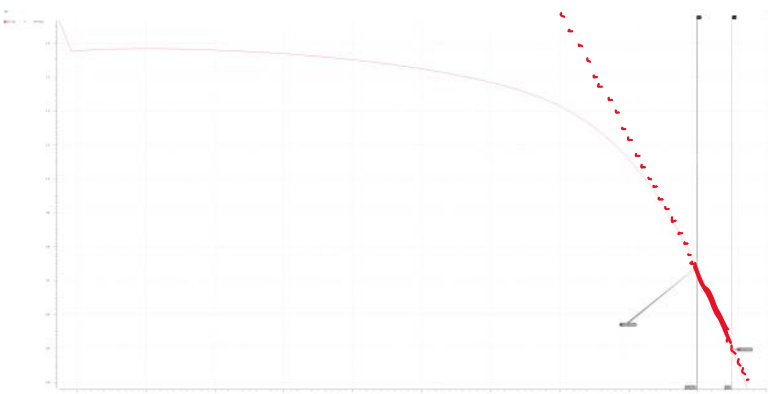


Points and equation:

Line 1: $\dfrac{1.286055 - 1.3696}{-6.006221 - (-7.501)}$

$= -0.055891205 32x$

Intercept $= \dfrac{y_1 + y_2}{2} = 1.3278275$



Line 2:

$\dfrac{0.497655 - 0.7397983}{-4.25 - (-4.50594)}$

$= -0.9460822849x$

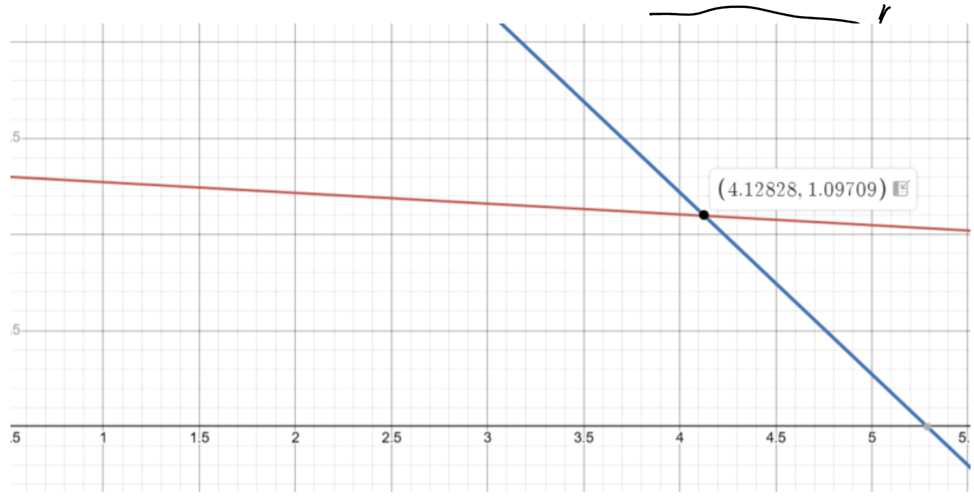
intercept: $y_2 + m(x_2) = 5.00248 5311$

Plots of $\log\left(\frac{g_m}{I_{DS}}\right)$ vs $\log(I_{DS})$

The point where both lines intersect: $(4.12828, 1.09709)$

$I_{spe} = 10^{x\text{-coord}}$

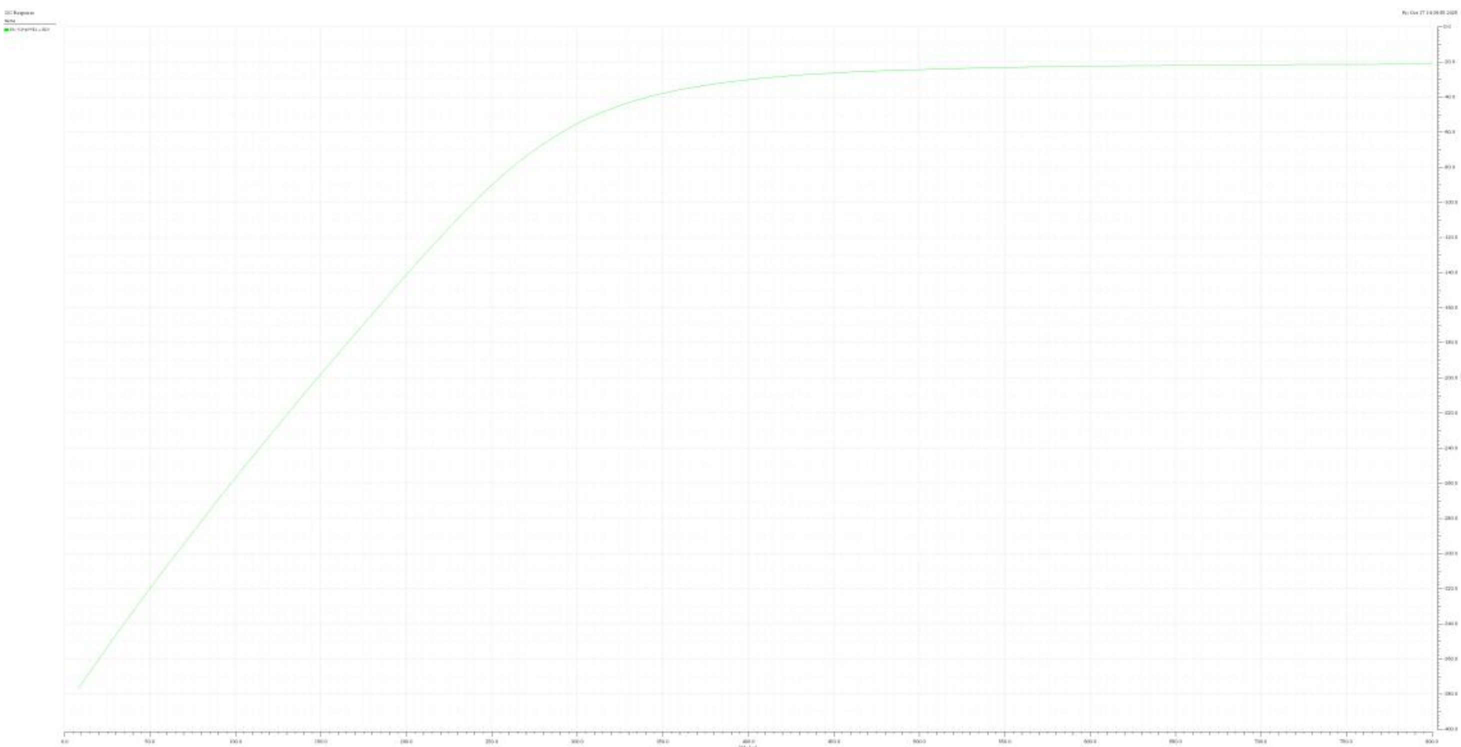
≈ 13436.3 A



Problem 2 (v): Transit frequency, f_T

$$f_T = \frac{g_m}{2\pi(C_{GS}+C_{GD})},$$

we use the calculator tool to get the constant value of capacitances, and derivative function to get gm.



f_T vs. V_{DS}