

Task 4 - Channel Estimation

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# General

## Approvals and Dates

|  |  |
| --- | --- |
|  | **Approval Date** |
| A. E. Jones |  |

## Change Record

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Date** | **Version** | | **Author** | **Description of Changes** |
| Date | 0.01 | |  | Initial Draft |
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## Acronyms

|  |  |
| --- | --- |
| EPC | Evolved Packet Core |
| LTE | Long-Term Evolution |
|  |  |
|  |  |

## References

1. *[example](http://ipwuk-wss03/Shared%20Documents/Research%20and%20Development/LTE/Product%20Management/RRD/LTE%20RRD.xlsx) reference*

# Introduction

In the previous task the T-spaced model progressed to an over sampled T-spaced model, where the Timing Parameters for the ideal sampling were known. However in practise the receiver does not inherently know the ideal sampling points, one method of providing such information is through a channel estimate. In addition an estimate of the channels characteristics can also be obtained.

* Fading: The received signal varies in magnitude and phase in respect to time
* Multipath: Multiple received signals arrive at the antenna from multiple different routes. Considering two arrival paths, with similar magnitude reductions then the resulting fade is of a Rayleigh scenario.
* Doppler: Doppler occurs when the terminals are moving
* Time Shift: As the propagation distance changes, the timing of the received signal at the receiver will vary

# Reference Signal

A reference signal (pilot signal, midamble) is used to obtain information about the channel in order to provide the receiver with parameters to aid the demodulation process. Firstly the oversampled model shown in task 3 is required to be decimated back to a T-spaced model. A reference signal is added to the transmitted signal, where the signal can be introduced at the beginning middle or end of the transmitted burst. The reference signal is known to both the receiver and transmitter, therefore in order to synchronise the received signal, the correlation between the local replica (the reference signal at the receiver) and the transmitted reference signal can be exploited. The correlation between the two signals yields the timing parameters as well as an estimate for the amplitude and phase of the channel. The reference signal is generated using and m-sequence.

## Maximal Length Sequence

The maximal length sequence is used to generate a reference signal, where a primitive polynomial is used to define a Linear feedback shift register’s (LFSRs). The polynomial is determined from Galois field with distinct elements and is denoted by

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A irreducible polynomial cannot be expressed as the product of two or more polynomials of lower degrees over the same Galois field, then the polynomial is irreducible. For instance is an irreducible polynomial over , however is not irreducible over as = similarly . If a polynomial is both monic and irreducible it is a prime polynomial, where a monic polynomial for is denoted as:

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However is an extension of the basefield , with elements, given that is prime over due to its monic and irreducible properties. Then this polynomial can be used to produce by using the addition and multiplication tables for , where modulo addition and multiplication are used.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| + | 0 | 1 | X | X+1 |
| 0 | 0 | 1 | X | X+1 |
| 1 | 1 | 0 | X+1 | X |
| X | X | X+1 | 0 | 1 |
| X+1 | X+1 | X | 1 | 0 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| \* | 0 | 1 | X | X+1 |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | X | X+1 |
| X | 0 | X | X+1 | X |
| X+1 | 0 | X+1 | 1 | X |

Table -Galios Field for

For a given there are many prime polynomials of degree , as we wish to generate pseduo random binary values for the reference signal. Hence the same polynomial will generate the same pseduo random value at the transmitter and the receiver. Therefore it is desirable to generate such that its field is a primitive element (where a primitive element is a nonzero element of ) and thus generating a primitive polynomial.

The polynomial denoted by is implemented in the form of the LFSR shown in Figure 1, and therefore will produce a M sequence of pseudo random binary values where for a balance M-sequence the number of binary value 1’s is one greater than the number of binary 0’s generated by the M-sequence.

Figure - M-sequence

The binary values are then mapped to 1 for binary 1’s and -1 for binary 0’s and therefore producing the reference signal, this produces a signal with ideal autocorrelation.

## Linear Correlation

The discrete cross correlation between signals is denoted by

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The auto correlation can be determined by taking the time reversed complex conjugate of the reference signal convolved with the reference signal, this shown in Figure 2 and yields Figure 3 for the M-sequence described above.

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Figure -Linear Correlation

The implementation of linear correlation defines the point when the signal is matched or has the greatest correlation. Taking Figure 2 illustrates the time reversed complex conjugate convolved with , where the point in the sequence with the greatest magnitude shows the highest level of correlation. The auto correlated signal will have a magnitude of M for an M sequence where M is the length of the sequence. This yields Figure 3 for the polynomial, where full correlation occurs at the midpoint of the sliding linear correlator and hence has magnitude M at this point. This methodology can be used to synchronise signals allowing the received signal to be decimated back to a T-spaced model.



Figure - Linear Correlation of Pseudo Random Reference Signal

## Circular Correlation

Circular correlation is denoted as

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The transform pair is , the cyclic correlation of a signal can easily be obtained in the frequency domain, by using the Inverse Fast Fourier Transform of . To find the auto correlation of the reference signal generated by the M-sequence we correlate the signal with its self. Hence, it is often useful to carry out correlation in the frequency domain as the computation time is increased in the time domain. The time domain solution is shown in Figure 4, which can be implanted using modulo addition:

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Figure 4 shows that both and move in a circular manner where full correlation at the beginning of the sequence for a auto correlated function where the output is shown in Figure 5. Due the balanced nature of the M sequence i.e. for a balance M -sequence there must be one more binary value 1 than 0, the outcome for a mapped sequence of 1 and -1 respectively results in negative one for all uncorrelated points when correlated with itself. This can be seen if the M-sequence is substituted in for and for the second term in Figure 4.

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Figure - Circular Correlation



Figure - Circular Correlation of Pseudo Random Reference Signal

# System Model

The system model can now be amended to incorporate the reference signal and the channel estimate functions, where no inherent knowledge of the reference signal is known. In addition a channel can be added where the signal properties change as it propagates through the channel.

Binary Packet Stream

Symbol Mapping

Over Sampling

RRC Filter

AWGN

RRC Filter

Decimated Signal

Symbol Detector

Binary Packet Stream

Receiver

Transmitter

Channel

Pilot signal

Local Replica

Channel Estimate

Figure - System Model

## Pilot Signal

Circular convolution is the preferred correlation method if the reference signal is in a known position. Assuming the reference position is chosen to be a midamble, the receiver knows to perform cyclic correlation in the middle of the received signal. However to ensure sufficient correlation between the received signal and the local replica of the midamble signal a Cyclic Prefix is introduced. This is where the reference signal is appended with an extension of the reference signal. For the 15 symbol extension used in this report the final 15 bits of the reference signal are used for the cyclic extension. If the reference signal with the appended cyclic prefix in this report it will be denoted as the pilot signal. Then due to the cyclic properties of the pilot signal there is a degree of acceptable error when extracting the received pilot signal i.e. if the received signal was not perfectly aligned with the transmitted signal. Then Figure 6 demonstrates a 15 symbol cyclic prefix allowing an error of 15 symbols or more accurately 15 oversampled symbols, however in this example the signal is not oversampled. The first subplot shows the reference signal correlated with one to M symbols of the pilot signal where the pilot signal is of length M plus the cyclic prefix. Therefore the signal will be unaligned, where the signal is fully correlated at sixteenth symbol as expected. The second subplot simple show that given index from the first subplot the second subplot can be aligned and hence the receive can utilise this information to decimate back to a T-spaced model. It is important to note that the local replica is required to be oversample to the same rate as that of the message signal.



Figure -Circular Correlation with Channel Delay

## Channel Delay

The propagation distance between the receiver and transmitter can vary; as a result the timing of the received signal varies with time. The simulation has been adapted to incorporate a channel to work within the error boundaries of the cyclic prefix i.e. within the 15 symbol range. The channel delay can be shown using a quasi-static model, the transmitted signal is denoted as will therefore produces an output of , if a delay is introduced as a result of the channel then the resulting output signal . This can be implemented in matlab using a simple filter, where the delay in this case can span across 15 symbols. Accounting for the over sample rate, the delay filter has a length equal to the length of the 15 times oversampled symbols. The signal is shift by setting one element of the filter to unity. Consider the 4 times oversampled message.



Where a random delay spanning two symbols is introduced, the delay filter is defined as where denote the delay spread of the filter, a random integer is chosen where in this example ranges from 1 to 2, the channel delay is therefore:

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Where is set to one causing a time shift of unit power, enabling the synchronisation properties of the pilot signal to be tested. Figure 8 shows a four times over sampled QPSK data stream where the pilot signal is an M-sequence of length 63 with a 15 symbol cyclic prefix. The cyclic correlation between the local replica and the received reference signal shows that the channel has induced a nine symbol delay. The second subplot demonstrates that the received signal can be realigned and therefore allowing the signal to be decimated back to a T-spaced model.



Figure -Oversampled Signal with Channel Delay

The reference signal can be utilised further by providing estimates of the channels attenuation and phase, further more a noise estimate can be obtain which is feed into the LLR for soft decision decoding.

## Channel Estimate

By applying the pilot signal to the current over sampled simulation where a channel delay is randomly assigned for each physical layer burst and AWGN is added at the receiver the phase shift between the transmitter and receiver can be obtained allowing the I and Q components to be correctly identified.

### Normalisation

The first step is to ascertain a channel estimate; this is shown in Figure 9 for a four times oversample QPSK system. The first subplot shows the phase shift, the response of the filter and the noise of the system. The second subplot shows that the I and Q components for the received signal have been aligned with the I and Q components of the transmitted signal. In addition the signal has been normalised to unit power and therefore enabling an estimate of the spectral noise in the system. However before an estimate of the channel is obtained the received signal is decimated back to the original T-spaced model.



Figure - Channel Estimate where the signal is time varying and AWGN in added at the receiver

From the channel estimate the position of the reference signal is known, therefore the position of the cyclic prefix is also known. Figure 9 shows how the T-spaced symbols can be obtained from the received signal, where the number of symbols transmitted in each block is known. Therefore by taking Nth sample where N is the over sample rate from the known positions points for L number of symbols, the received signal is decimated back to the T-spaced model.

Front End Block

CP

Ref Sig

Back End Block

Figure - Received Signal Properties

### Noise Estimate

In addition the reference signal can provide a noise estimate, by decimating the received signal back to the T-spaced model and finding the difference between the T-spaced local replica and the T-space received reference signal. The residual power is therefore an estimate of the spectral noise density, normalising this value to take inconsideration the real and imaginary parts of the spectral noise density. The noise variance can be obtained which is then feed into the LLR.

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Figure 11 shows that the noise variance estimate is generally slightly higher than the measured noise variance. As expected for low SNR the noise variance is less accurate, the reason being is that the noise present at the receiver is only bandlimited by a RRC filter, therefore the out of band spectral components of the pulse shaped message is attenuated twice the amount of the out of band Additive White Gaussian Noise. Figure 12 subplot 1 shows the received signal and the AWGN present at the receiver where the variance of the AWGN generated at the receiver is the Nyquist rate times the noise spectral density. The received signal has passed through a RRC interpolation filter and therefore the out of band spectral content has been attenuated by approximately 70 dB. Both the received signal and AWGN pass through the second RRC interpolation filter where again the out of band spectral content is attenuated by another further 70 dB for the received signal however the AWGN is only attenuated by 70 dB. In practise the signals are not separated hence subplot two shows the combined received signal after the second RRC. It is important to note that the AWGN noise variance is now representative of the simulated noise variance due to the RRC filter. In addition the out of band spectral content has increased to that of the first RRC filter due to the AWGN.



Figure -Noise Variance vs. Noise Variance Estimate



Figure - 4 Times oversampled System, Noise Analysis

## Bit Error and Block Error performance in Channel Estimated Simulation

It can be seen in Figure 13 that the bit error rate and block error rate matches the associated theoretical values for an AWGN channel. Showing that the change in phase between the transmitted signal and received signal has been account for, for each I and Q component.



Figure -Theoretical and simulated performance in a time varying channel

# Rayleigh Fading

The received signal seen at the terminal receiver is made up of numerous attenuated, reflected, transmitted and diffracted version of the original transmitted signal. Currently the channel model only accounts for the received signal shift in phase in accordance with variations in propagation distance i.e. the time delay between the transmitted and received signals and the White Noise seen at the receiver. In reality multipath fading occurs where the received signal phase and magnitude changes in accordance to the environment. A Rayleigh Fading model is a good model for when there isn’t a dominant line of sight between the base station and the terminal; hence it is suited to built-up urban environments. The received signal can therefore be explained in terms of path loss and change in phase, as the user moves through an urban environment the electrical distance between the base station and terminal varies. As shown in the previous channel model, this results in a channel delay. In an urban environment the received signal is a summation of multiple paths, assuming two paths are present and arrive in phase the constructive interference will occur resulting in an increase in magnitude. However if the phase of the two paths are in anti phase and therefore 180 degrees out of phase then destructive interference will occur causing the magnitude to decrease.

## Channel Model

In order to implement a channel model it is important to verify the channel first, the current simulation model is has been proven against theoretical models. Therefore by implement a Rayleigh Fading Channel that conforms to the probability density function PDF for a Rayleigh random variable, the results obtained from the simulation will demonstrate how each modulation scheme performs in that environment. The Rayleigh random variable is denoted as:

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| --- | --- | --- |
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Where X represents the attenuation factor and Y represents the change in phase, both X and Y are Gaussian random variables each distributed in according to . The Rayleigh random variable is chi square variable and its PDF for z is therefore:

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The phase term is uniformly distributed between, assessing the joint probability of x and y the PDF for x is:

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| --- | --- | --- |
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Similarly the PDF for y is:

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As X and Y are independent random Gaussian variables the joint PDF:

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The joint probability for lies between , similarly for the joint probability lies between

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Converting from Cartesian from to polar form for where the area of integration in polar form is

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Hence the joint probability function is

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| --- | --- | --- |
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Where the variance using matlab the PDF for and generated for a Rayleigh Fading environment can be verified against the analytical PDF’s shown above. Figure 14 shows both PDF, where it can be seen that the simulated PDF closely matches the analytical PDF. Implementing this channel in the current simulation model will therefore give a good approximation of how each modulation scheme works in and urban environment where the terminal does not have a direct line of sight to the base station. The multipath environment to be simulated in the current matlab simulation is a flat fade; hence the multiple path has only one tap and is a good approximation for a low band width model.



Figure - PDF for Rayleigh Fading Variable

The theoretical Bit Errors for a Rayleigh Fade is as follows, where the bit error derivation for each modulation scheme is shown in the Appendix section 7:

|  |  |  |
| --- | --- | --- |
| BPSK |  |  |
| QPSK |  |  |
| 16-QAM |  |  |

This is yet another step to ensure that the simulation performs as expected in the new channel environment. Figure 15 shows a Rayleigh Fade where a physical layer burst has undergone constructive interference. In order to account for the phase rotation and increase in amplitude of the received signal the channel estimated can be used to obtain an estimate of the condition of the channel. Previously the channel estimate determined the shift in phase, to account for the change in phase the Cartesian value of the peak normalised channel estimate provides of the channel conditions. To equalise for the channel condition we apply the following formula

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| --- | --- | --- |
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Where the Rx is the physical layer burst and the peak channel estimate is as stated.



Figure - Constructive Interference

The effects the channel has on the received symbol constellation can be seen in Figure 16, where the first subplot show the received physical layer burst before the channel estimate has been taken into account. The second subplot shows the symbol constellation after the I and Q attenuation and phase adjust have been accounted for.



Figure

The Bit Error and Block error curves for a Rayleigh fade channel are shown in Figure 17. As expected the bit error rates have increased as a result of the Rayleigh Fading Channel. The simulated results closely match the theoretical Bit Error performance and hence validates the simulation model.



Figure - Bit Error and BLER for Rayleigh Fading Channel

# Diversity Reception

The Rayleigh Fading Channel has significantly degraded the performance; in order to achieve better bit error performance for a given EbNo, diversity gain can be achieved by exploiting the diversity between multiple channels. If we adjust the simulation model to account for R number of receiver branches, where each branch has a different receive antenna. The reason for using different antennas is to ensure that each antenna picks up a different path.

The fading process on R number of diversity channels created generated by each receive antenna is assumed to be mutually statistically independent. The signal is corrupted by an additive white Gaussian noise process with a PDF of at each receiver. Again the noise process is assumed to be mutually statistically independent. Due to the nature of the channel, the diversity between each path is exploited where the received signal can be denoted as:

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Where is the transmitted signal, denotes the number of transmitted signals and is the number of receive antennas and are the attenuation and phase factors for each R number of Diversity Channels. The channel experienced by each antenna is randomly varying with time. For R number of received antennas the received symbol is multiplied by where the mean and variance is of a Rayleigh Random Variable as shown in section 5.1 The Additive White Gaussian Noise introduced by each receiver is denoted by. Figure 18 depicts a communication system using maximum ratio combiner to exploit the diversity between channels.

Figure - MRC Block Diagram

The channel estimate for each receiver provides and an estimate of the channel (Rayleigh Random Variable in this case).

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Assuming that the channel only induces flat fading i.e. that there is only one multipath tap, equalisation is performed where the output of the RRC filter is multiplied by the complex conjugate of the channel gain .

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The simulated improvement for R number of diversity channels is shown in Figure 19, where it is clearly desirable to have as many receive antennas as possible. However the antennas may be separated physically (usually by approx 10 wavelengths) or may pick up different polarisation feeds from the same antenna.

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where c is the speed of light and therefore the wave length for a occupied bandwidth of 4.5MHz is 66.7m. Designing a mobile phone which can account for diversity paths of approximately 10 wavelengths apart of obtain different polarisation feed for the same antenna is a difficult feet due to the size consternates of modern mobile phones. Hence the model implemented for this task assumes two receive antennas.



Figure - Theoretical and Simulated Improvement for R number of Diversity Channels

From Equation 23 it is possible to assess the theoretical error probability of a receiver exploiting diversity. If the probability density function is denoted as whereas previously stated is o the form of a chi-square probability density function, if then the characteristic function of can be shown as:

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Where the signal to noise ratio is assumed to be identical for all channels and therefore is independent of the number of diversity channels . As stated prior the fading and noise seen at the receiver is mutually statistically independent, where the characteristic function of a chi squared disturbed variable has degrees of freedom and therefore the probability density function can be denoted as:

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As is assumed to a mean of one over R, the probability density function can therefore be evaluated in the following form:

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Where in the case is the probability of error for a QPSK modulation scheme

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This therefore reduces to:

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The theoretical and simulated bit error for a QPSK modulation scheme is shown in Figure 20, where it is clear that a significant improvement can be seen in the energy per bit for a given BER.



Figure - The Effects of Diversity in a Rayleigh Fading Channel



Figure -Estimate Noise Variance vs. Noise Variance MRC

# Appendix

There error probability for a BSPK and QPSK modulation scheme

|  |  |  |
| --- | --- | --- |
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The joint probability function can be shown as:

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| --- | --- | --- |
| BPSK and QPSK |  |  |

where is the Rayleigh Fade variable defined in Equation10. By setting the integration limits to and

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Rearranging the limits to show:

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If we assess the right hand side integral we get

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Substituting Equation 34 into Equation 33 we get

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Simplifying to the following equation where and .

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