

Task 3 - Oversampling

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General

## Approvals and Dates

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| --- | --- |
|  | Approval Date |
| A. E. Jones |  |

## Change Record

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| --- | --- | --- | --- | --- |
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## Acronyms

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| --- | --- |
| EPC | Evolved Packet Core |
| LTE | Long-Term Evolution |
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## References

[*example*](http://ipwuk-wss03/Shared Documents/Research and Development/LTE/Product Management/RRD/LTE RRD.xlsx) reference

# Introduction

In the previous task all simulations have assumed a T-spaced model, where the signal is sampled at the same frequency as the symbol rate. Therefore if denotes the modulated message and denotes the output waveform after Digital to Analogue Conversation (DAC). Figure 1 demonstrates a T-spaced samples, where a digital signal which is discrete in frequency and time is converted into analogue signal which is continuous in frequency and time. This methodology represents the following equation.

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Where T denotes the time spacing between symbols as shown in Figure 1, by transmitting a series of narrow pulses the synchronisation between the transmitter and receiver would have to be perfect in order to recover the transmitted symbol. In addition a narrow pulse has a wide frequency response due to the large number of frequency components, hence then bandwidth for a simple T space model is large, as bandwidth is expensive this is not ideal.

Figure 1 - T Space Model

# Sampling Theory

## Nyquist Sampling Theorem

The Nyquist rate denotes the minimum sampling rate required to avoid aliasing, where aliasing describes signals that are indistinguishable from one another, i.e. the sampled signal (discrete) does not accurately represent the change of the signal (continuous) as the peaks and troughs of the signal have not been captured.

The Nyquist rate states that the sampling frequency should be at least twice the bandwidth of a bandlimited signal or channel, which is denoted in

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Where is the sampling frequency and is the Nyquist Frequency, which is defined as the highest frequency at which the signal can have non zero energy i.e. if the frequency range is from. In time the samples are spaced at a distance of apart.

Sampling below the Nyquist rate results in an inaccurate discrete representation of the signal. It is also important to take into account the point at which sampling occurs, for example Figure 2 shows a Nyquist rate of two where the first subplot shows a discrete signal with only DC terms even thought the Nyquist criterion is met. This is because the Nyquist Criterion simple shows the requirements to avoid aliasing where the optimum sampling point is known. If the waveform is not correctly aligned then the power obtained from the waveform will be inaccurate. The fewer samples taken the lower the average power of the waveform.

Hence the second subplot shows that given the knowledge of the peak amplitude a sampling rate of two is sufficient to obtain all subsequent peaks and troughs.



Figure 2 – Sine Wave

A sine wave sampled in accordance to the Nyquist rate criterion captures the key points of the sine wave; increase the sampling rate above provides a greater resolution in the discrete domain. This is shown in Figure 3, where a Nyquist rate of are represented.



Figure 3 – Sampled Sine Wave Nyquist Rate = 4 and 8

# System Model

The system model shown in Figure 4 describes a generic communication system; this report concentrates of the oversampling and filter aspect of the system model.

Figure 4-Generic Communication System

## Oversampling

A signal that is oversampled is sampled at a frequency greater than the Nyquist Frequency at a rate of Beta, which is defined as:

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Whereis the highest frequency component in the over sampled signal. An oversampled signal is usually an integer multiple of the bandwidth i.e. 2, 4, 6, 8; however fractional oversampled ratio’s can be implemented. In order to achieve an oversampled model the sample spacing is increased in the time domain, this is achieved by introducing zero padding between each symbol that is transmitted. Figure 5 represents a signal with unit amplitude with an up sampling rate of 4 where this information transmitted is. The length of zero padding is determined by i.e. 3 in this case.

Figure 5 – 4 Times over Sampled signal

It is important to note that the up sampled signal contains no additional information and has increased the required bandwidth by a factor of Beta, i.e. in this case the bandwidth required is to represent the same signal shown in Figure 1. The signal in the time domain occupies the same space however the signal represented in the frequency domain is periodic and hence is repeated by a factor of Beta occupying a new bandwidth of where is the Nyquist Frequency of the original signal. This can be seen in Figure 7, where the second plot denotes the up sampled signal at Beta is equal to 2. In the frequency domain the signal wanted is still only that of zero to. Figure 6 shows an oversampled QPSK modulated signal at unit power, where the first subplot shows the T-space signal, the second subplot shows the over sampled signal where Beta is equal to two and the third subplot demonstrates how the oversampled signal is decimated back to a T-spaced model at the receiver. Effectively a signal is created which conforms to the Nyquist sampling criterion allowing the signal at the receiver to decimate back to T-spaced model without introducing Inter Symbol Interference.



Figure 6 – Up Sampled Signal Beta = 2



Figure 7 – Frequency Domain Beta = 2

## Pulse Shaping

In the first and second task the digital information has been converted, depending on the modulation scheme into a discrete sequence of narrow pulses with associated magnitudes and phases. In order to transmit the signal, interpolation between the symbols is required which produces an analogue wave form that is then transmitted i.e. digital to analogue conversion (DAC). Therefore a pulse is required, where the zero padded elements of the over sampled signal are assigned a value in order to produce a pulse shape for the T spaced symbols.

By defining a pulse shaping filter the values between the symbols can be interpolated where the pulse shaped signal is denoted as:

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Where is the pulse shaping filter, also known as the interpolation filter, many different type of filters maybe be used. Consideration should be taken when choosing a filter as inter symbol interference can be induced as a result of the filter design.

### Rectangular Filter

Currently the oversampled signal contains no additional information. By applying a rectangular filter the period between symbols are interpolated. The rectangular filter repeats the symbol for the oversampled values i.e. the zero padded elements between sampled signals, this can be seen if Figure 8. This is achieved by convolving the up sampled signal with the rectangular filter with unit value, where Beta defines the oversample rate of the signal and the length of the rectangular filter.



Figure 8 – Rectangular Filter Beta = 4

The frequency response of the rectangular filter with oversample ratio is 4 and 8 can be seen in Figure 9’s subplots 1 and 2 respectively. The frequency response for the filter is obtained by taking the FFT of the filter h[n], with N number of FFT points. The number of points determine the resolution of the FFT hence the greater the number of points the great the resolution. In order to obtain N FFT points, the filter is zero padded with zero padding appended to the filter. The signal is therefore interpolated in the frequency domain, if the NFFT is small the resolution can be too low to obtain any useful information in the frequency domain and hence the frequency response of the filter may look like Figure 10.



Figure 9 – Frequency Response of Rectangular Filter Beta = 4 and 8



Figure 10- Frequency Response of Rectangular Filter with Low Resolution

### Pulse shaping

By applying a rectangular filter the series of narrow impulses is shaped into a square pulse as shown in Figure 8, where the pulse length is equal to the symbol in time.

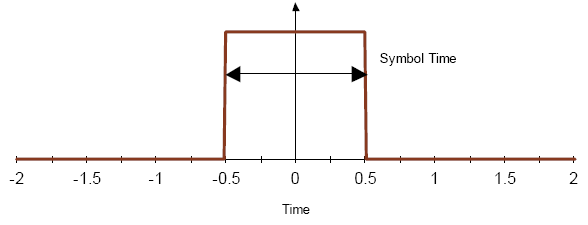


Figure 11 – Rectangular Filter Time Domain

The frequency response of a square pulse is determined by the following equation

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Where Ts is the symbol time period

Hence in the frequency domain a square pulse is a sinc wave form, where the sinc function would be infinitely long. Due to the rise and fall time required to generate a square pulse it is impossible to generate a true square pulse, in addition the frequency components of a square wave would tend to infinite. Therefore in order to implement a rectangular filter the system must be made causal by shift the rectangular and truncating to **N** samples, where Figure 11 show a casual filter design. The taps of a sinc function cross the zero amplitude at integer multiples of and the immediate sidelobes are 13 dB lower than the main lobe, Figure 12 shows the frequency response of a square pulse. It is also important to note by increasing Beta we increase the pulse width, in doing so the frequency content is reduced compared to the sampling bandwidth.

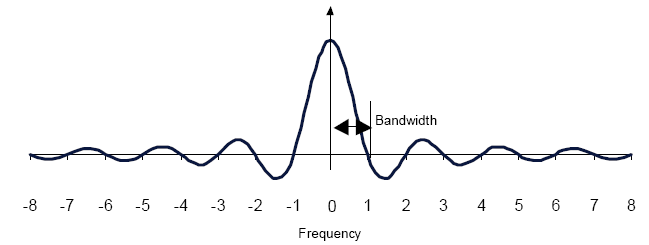


Figure 12 – Rectangular Filter Frequency Domain

Referring to Equation 4 in order to obtain the symbol from the signal can be sampled at multiples k of the symbol time T, where Equation 4 can be broken into two parts shown below.

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In order to avoid Inter symbol interference the taps of the filter should be zero for integer multiples of T and non zero at time 0. This is satisfied by the rectangular filter, as shown in Equation 7.

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For a rectangular filtered output signal we can actually sample at multiple integers of integer time periods between T and Beta as the symbol is repeated at the same magnitude. The eye diagram for a rectangular filter signal is shown in Figure 13, where the time variation of zero crossing is ideal, there is no signal variation and hence the eye opening is wide. Therefore as it stands the rectangular filter is perfect for pulse shaping, however this may not be the case when we band-limit the transmitted signal.



Figure 13 – Eye Diagram for Rectangular Filter

### Frequency Domain Analysis of Rectangular Pulse Shaped Signal

The frequency domain response of the input signal convolved with the rectangular filter, shows a greater energy than that of the input signal, where there is an approximately 5dB increase in the maximum signal value in Figure 14, this is because the interpolation filter has not been normalised and is therefore applying a gain. It is important to note that the second and third sub plots are the same signal represented differently. The second subplot depicts the Fast Fourier transform (FFT) of the over sampled signal after it has been passed through the rectangular filter i.e. subplot one convolved with rectangular filter of length four. Where the first half of the frequency range is is sufficient to identify the key component frequencies of the signal. The third subplot illustrates a shifted FFT which is often the form utilised by the analysis tool in the lab. A shifted FFT is centred at zero, which is achieved by rearranging the output from the FFT with a circular shift producing the output shown in third subplot. It can be seen that the decay in the transition band is slow for a rectangular filter.



Figure 14 – Frequency Domain of Square Pulse Shaped Signal

One of the main disadvantages of the rectangular filter is that its spectrum is of the form of a sinc and as a result has significant signal power at frequency higher than the symbol rate due to the spectral components of a square pulse. This can be seen in Figure 15 where the initial stages of generating a square wave is shown through Gibb’s effect. Where the blue signal is made up of all the other harmonics shown in multiple colours. The side lobes in Figure 14 are approximately 13dB lower than the main lobe, thus wasting bandwidth. Bandwidth is expensive commodity especially in mobile communications; therefore this is not a desirable filter design, even though this is an improvement on the narrow pulse T space model.



Figure 15 – Square wave, Gibbs Effect

# Power Spectrum Estimation

Using the FFT shown in Figure 14 to estimated the transmit spectrum is not ideal as the difference between each frequency point can be plus or minus several dB’s from the previous value, this can clearly be seen by looking at the magnitude around the centre point of the third subplot. This is because increasing the number of data points in time by the rate, increases the variance of the output signal accordingly. Hence the more a signal is over sampled the less useful the FFT is to estimate the power spectrum; one solution is Welch’s power spectrum estimate.

## Welch’s Power Spectrum Estimate

Welch’s power spectrum estimate divides the total signal length **N** into sub samples of length **NFFT**, where an overlap is defined and the standard default overlap is 50%.

Figure 16 – Welch’s Power Spectrum Estimate

Firstly a window is defined, usually from the cosine family such as a hanning window or hamming window but there are many possible window choices each with different tradeoffs.

* Hanning Window: optimised for small widening of the main lobe. Parameter choice very low aliasing vs. A slight decrease in resolution
* Hamming Window: optimised to minimise energy outside the mainlobe
* Kaiser: optimised to minimise energy outside mainlobe. Parameter choice to trade resolution vs. sidelobe suppression.
* Chebyshev: optimised to control peak sidelobe level. Parameter choice gives directly (flat) sidelobe level.

The length of the window is determined by the number of FFT points which is equal to L in Figure 16, and the offset of each segment is the overlap D. Therefore D number of points overlapped, as a general rule D is 50% of the number of FFT points. The following equation denotes the Welch Method:

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Where is the chosen window, U is the factor which ensure that the periodogram is asymptotically unbiased i.e. as the sample size tends to infinity.

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is the i-th modified periodogram

Figure 17 shows the results of a Welch Power Spectrum Estimate of a single tone sine wave with unit power. The first subplot uses a 256 point Hanning window, where the input signal (A section of the sine wave of length L) is suppressed at both ends due to the shape of the Hanning window. This therefore reduces the overall signal power, where this reduction of the signal power is denoted as the coherent power gain. The coherent power gain is the square of the DC gain, where the DC gain is the sum of magnitude in the window function. This means that the magnitude measured at the DFT bin is not the same as the real magnitude of the signals frequency component at that frequency. Where an ideal DFT bin is where each frequency component of the signal contributes to a single frequency of the DFT. By decreasing the window size and hence the number of DFT point the attenuation increases, this is confirmed in Figure 18.



Figure 17- PSD unit value waveform, NFFT = 128



Figure 18-PSD unit value waveform, NFFT = 256

However if a random signal with a PDF of is now used as the input signal to the pediogram, then power spectrum estimate for said signal is as shown in Figure 19. The white noise signal is not attenuated to the same extent as the prior signal. A Noise signal adds non-coherently , meaning that the noise sums on a power basis instead of an amplitude basis. Where as a sinusoids or other deterministic signal can be seen to be add coherently. This is important when considering the signal to noise ratio, as the coherent addition of a deterministic signal and the non coherent addition of white noise can be used to obtain the signal to noise ratio. Essentially the noise energy remains uniformly distributed but the deterministic signal all the energy adds up and is seen at the frequency of the wave form, in the case of where the wave form is complex and made of multiple sinusoid's, the energy for each sinusoid will be seen at its respected frequency. For example consider a complex sinusoid where white noise has been added:

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Where is the complex amplitude, is the white nose and the output signal is of the form . The Discrete Fourier Transform is given by:

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Splitting the into Signal and Noise terms:

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Now let for some , hence is one of the DFT frequencies , then the following is true:

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The modulus squared is therefore

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As is noise is Additive White Gaussian noise of the form , then will also have zero mean for all ,the time average is given by:

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Where the final term can be expanded to give:

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As due to being white noise.

Therefore the average square magnitude DFT of N samples of a sinusoid in white noise is given by:

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The signal to noise ratio can therefore be shown as:

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where the SNR is zero in every DFT bin except for the DFT bin as seen in Equation 14, the amplitude of the complex sinusoid is and the variance mean square of the noise is . Therefore the DFT induces a processing gain in the DFT bin where the sinusoid falls.



Figure 19 PSD AWGN, NFFT = 256

# Alternative Channel Filters

In Section 3 the T space model has been converted into an oversampled T space model where an interpolation filter has been used to interpolate the over sampled values. Hence a pulse shape has been formed for each symbol. However the pulse shaped signal still contains significant energy at frequencies greater than the symbol rate, hence more appropriate band-limiting is required.

## Band-limiting

Aliasing can be avoided by choosing a Nyquist rate greater the twice the maximum frequency of the signal to be transmitted, choosing a rate equal to is impossible unless the signal is guaranteed to have a no components above. In addition the Channel bandwidth maybe a factor which is limited, therefore to achieve or a desired sampling rate the signal must be bandlimited. In order to reduce the bandwidth back to that of the Nyquist Frequency the filter must be designed to conform to the Nyquist criteria and therefore should match the oversample parameters.

The frequency response of the model of an analogue filter provided in task 3 is shown in Figure 20; the impulse response of the analogue filter is convolved with the zero padded signal and the rectangular filter signal producing subplots one and two shown in Figure 21 respectively. The role off of the analogue filter is much sharper than the rectangular filter; this is due to the extended impulse response and smooth shape in the time domain, there are no fast changes thus there are no high frequency terms unlike the rectangular filter.



Figure 20 – Frequency Response of Analogue Filter

The zero padded signal convolved with the analogue filter shown in subplot one of Figure 21, clearly shows a similar response to that of the analogue filter. It is clear that the energy has increased in the second subplot. This is because the interpolation filter does not have unit energy unlike the model of an analogue filter shown Figure 20; hence the first subplot of Figure 21 has not increase in energy. Each symbol in the T sample model is represented by a narrow impulse of unit power. By convolving the zero padded signal with interpolation filter the symbol is now represented by a pulse of length Beta, where the pulse energy has increased accordingly. Therefore the filter is required to have unit power, as shown by the analogue filter. This is simply achieved by normalising the filter to that of oversampling rate Beta, hence:

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By applying lowpass or bandpass filter the frequency components outside the limits of the filter are attenuated, therefore the square pulse harmonics are reduced causing a smoothing effect on the pulse. These effects can clearly been seen in the second subplot of Figure 22, also the signal does not accurately represent the square pulsed signal. More importantly we have changed the pulse shape and therefore surely affected the eye diagram shown in Figure 13. The ideal sampling points represented in green show that in many cases the analogue filtered signal is no longer representative of the original signal at the point of sampling. Hence at t=16 the signal is far less than the square pulse, due to inter symbol interference. Perhaps the most apparent point is that Figure 20 shows a cut off frequency of half the oversampled rate. The filter has removed too much of the signal below the original Nyquist frequency, therefore causing too much amplitude variation, ensuring that the signal no longer is ISI free at the point of sampling. A large amount of the spectral components of the square pulse have been cut off by the filter and symbol delay and smearing has been introduced.

The filter is design to remove all frequencies above the Nyquist point of a four times oversample system. Currently part of the wanted signal is sitting outside the analogue filter and hence it is attenuated. As the sample rate is increased the wanted signal will be squashed up further until it none of the wanted signal is out of band. Increase the sample rate yet further will result in the wanted signal taking up less and less space within the filter.



Figure 21 – Analogue filter convolved with zero padded and rectangular filtered signals

Understandably the eye diagram shown in Figure 23 shows that there is no clear optimum sample point and as a result significant ISI has been introduced. Increasing the sample rate as state prior where Beta is equal to 8, as the oversample rate is increased the filter cut off point will sit further and further above the Nyquist Frequency and hence more of the original signal will be retained. The trade off is bandwidth, increase the sample rate results in a higher bandwidth. This filtered response at Beta is equal to 8 is much closer to that of the square pulse wave, which is shown in Figure 24 subplot two. The eye diagram shown in Figure 25 shows the expected response for the filtered square pulse, there are still signs of ISI as the pulse shaping is being altered by the analogue filter. The higher spectral components of the square pulse are cut off by the filter and symbol delay and smearing have been introduced. Hence at the point of sampling there is a significant amount of ISI present, the timing variation and distortion has been introduce as a result of the delay and smearing of the symbols.



Figure 22 – Analogue Filtered Square Pulse, Beta = 4



Figure 23 – Eye Diagram of Analogue Filtered Signal, Beta = 4



Figure 24 - Analogue Filtered Square Pulse, Beta = 8



Figure 25 - Eye Diagram of Analogue Filtered Signal, Beta = 8

Applying a raised cosine filter rather than a rectangular filter to perform pulse shaping, the spectral components of the pulse have decreased and hence the pulse takes up less bandwidth due to the low pass filter qualities of a raised cosine filter. Figure 26 shows the provided impulse response in red, the blue impulse response shows the model of an analogue filter.



Figure 26 – Raised Cosine Impulse Response

When looking at subplot one of Figure 27, the cut off is not at the Nyquist frequency rather there is a small roll off period, reducing the spectral components of the pulse. This results in the signal shown in Figure 28 subplot one, where it is clear that at the point of sampling the signal is representative of the original T space narrow impulse model. It is important to note that the point of interest is the sampled point and the wave shape before and after this point is of little interest. Another important note is the fact that the figure has been quantised, and hence as a result there are rounding errors, which can clearly be seen on some of the sidelobes of subplot one. The impulse response of the filter in subplot two shows that it meets the criteria to prevent ISI at the point of sampling. This is shown by the fact that the taps cross zero at integer multiples of T and has a non zero component at time zero.

The eye diagram for this signal is shown in Figure 29 demonstrates that there is no ISI at the point of sampling; however the signal there after shows a wide time variation of zero crossing denoted by the black arrow and a considerable amount of amplitude variation denoted by the red arrow.



Figure 27-Interpolation Filter



Figure 28-Raised Cosine Pulse Shaped Waveform



Figure 29-Eye Diagram for Raised Cosine Filter

Applying a filter to this signal has no effect to the ISI as the pulse shaping filter has already bandlimited, where the spectral components that are not at the point of sampling have been attenuated. Hence the filter removes some of the spectral components below the Nyquist Frequency, only ensuring that the T-spaced sample has all its spectral energy. This is the effect of the role off factor alpha of the filter design, this also determines the number of taps the filter has. Therefore applying the analogue filter which is bandlimited to the Nyquist Frequency has no effect on shape of the pulse will as the higher spectral components have already been removed by the pulse shaping filter. The Frequency response before and after the analogue filter implementation is shown in Figure 30. It can be seen that the transition band has a much steeper roll off than that of the rectangular filter, by applying the analogue filter roll off rate has decreased slightly. This is because the analogue filter has a higher roll off rate the interpolation filter where alpha is 0.22. Figure 31 shows the effects of changing the variable alpha on the impulse response of the filter, where the number of taps and hence the complexity of the filter is increased as alpha decreases. Figure 32 shows the variation in roll off and transmission band size, where the roll off becomes steeper and the transmission band size decreases as alpha decreases. The frequency response shown in Figure 32 clear shows that it is undesirable to have a value of alpha tending towards one as the frequency content of the pulse shaped signal will be increase. However an eye diagram for alpha is equal to one would show a wider eye, with very little timing variation and distortion due to the increase in frequency content.



Figure 30-Frequency Domain of RC and Analogue Filtered Signals



Figure 31-Impulse Response for RC Filter with varying alpha



Figure 32-Frequency Response for RC Filter with varying alpha

# Root Raised Cosine

To implement a Raised Cosine filter requires splitting the filter into two parts, where a set of matched Root Raise Cosine filters are designed. The transfer function of a RRC filter is the square root of the Raised Cosine filter; hence the combination of the two filters, one in the transmitter and one in the receiver yields a Raised Cosine transfer function.

Figure 33 – RRC Filter Design

## Raised Cosine Transfer Function

The frequency response for a raised cosine transfer function is determined in Equation 25

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The impulse response for Equation 25 is given in Equation 14 where the taps of the filter cross the zero amplitude at integer multiples of .

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Applying l'hopital's rule the following equations can be determined to defined the filters characteristics.

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The filter length for the raised cosine and the root raised cosine is determined by

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where is half the number of taps required

## Root Raised Cosine Transfer Function

Similarly the characteristics of the Root Raised Cosine filter can be obtained by taking the root of the frequency response of the raised cosine filter.

|  |  |  |
| --- | --- | --- |
|  |  |  |

From this the impulse response of the filter can be obtained, it is important to note that the impulse response of the RRC does not meet the Nyquist Criterion and therefore on its own will introduce ISI. However using this design in the transmitter and receiver yields a raised cosine filter which meets the requirements of Nyquist Criterion.

|  |  |  |
| --- | --- | --- |
|  |  |  |

Applying l'hopital's rule the following equations can be determined to defined the filters characteristics.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

As the taps for a RRC filter cannot tend to infinity in time, the filter design is truncated and a roll off period is defined which is denoted as alpha. As stated in the previous section as alpha is reduced then the speed of roll of in the frequency domain is increased. This results in a larger number of taps and therefore a more complex filter design, hence it is desirable to design a filter with the minimal length which meets the requirements of the communication system. Figure 34 shows a RRC filter with alpha set to 0.22, which produces a reasonable roll off rate in the frequency domain shown in subplot two. Subplot three shows the pulse shaped signal, where it is clear that ISI will occur as the pulse is not representative of the sampled signal. This is to be expected as the RRC filter does not meet the Nyquist criterion for suppressing spectral distortion at integer multiples of the sample rate, i.e. at integer multiples of T the RRC filter does not cross zero, this is only achieved when the two RRC filters are combined to yield a raised cosine filter.

Applying the analogue filter and the second RRC filter yields Figure 35 where is it is clear that no ISI has been introduced and the signal has been bandlimited to the Nyquist Frequency of the T space model. It is important to note that the sampling points are known, where the point of sampling is determined by the index of peak value of the of the impulse response of each filter i.e. the index of peak value of the raised cosine plus the index of peak value of analogue filter. The signal is then sampled from this point at intervals of Beta to Beta times by number of T spaced samples. This methodology only works if the channel is not time varying, therefore the current model only represent noise seen at the receiver.



Figure 34-RRC Filtered Waveform



Figure 35 Full RRC Filtered Wavefor

# QPSK Modulation Scheme with Over Sampling

Updating the simulation model with the oversampled T spaced model, pulse shaped by a root raised cosine filter the effect on the bit error rate can be seen in Figure 36. It is clear that the interpolation filter for each oversampled rate isn't introducing ISI and hence effecting the expected bit error rate for said modulation scheme. However this system requires inherent knowledge of the system in order to decimate the oversample signal back to a T-spaced model. In practise the receiver will have to determine the timing parameters by performing a channel estimate, which is cover in the next report.



Figure 36-Bit Error Rate for QPSK modulation scheme

# Appendix



Figure 37-Time Domain 4 times oversampled



Figure 38-Frequency Domain 4 times oversampled



Figure 39-Time Domain 8 times oversampled



Figure 40-Frequency Domain 8 times oversampled