CS 440: Final Exam, Question 1

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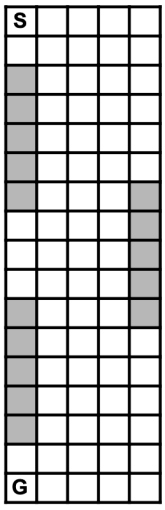
Consider the following landscape, representing an icy mountain ridge.



You begin at square **S** and your goal is to get to square **G**. You can move up/down/left/right between squares. However, because of the ice, each time you move in a given direction, there is a 0.1 probability you accidentally slide in one of the orthogonal directions (if you try to go down, and you are not at the edge of the map, you successfully go down 80% of the time, you slide left 10% of the time, and right 10% of the time). You cannot attempt to move in the direction of a wall, and if one of the orthogonal directions is blocked by a wall you slide with a (90%, 10%) split instead of (80%, 10%, 10%). You want to get to the goal as quickly as possible, as each timestep you are on the map costs you one dollar.

But beware! The gray cells represent icy ravines, and if you enter one of those cells you slide to any icy death (represented in this case by an immediate loss of $1000 and termination of the game).

Note: In general I adopt the notation that (x, y) is the cell in row x, column y.

1) **(5 points)** Let $C(x)$ be the minimal expected cost of being in cell x. What is $C(\mathbf{S})$ (in terms of its neighbors, cell $(1, 0)$ and $(0, 1)$)?

2) **(5 points)** What is $C((1, 0))$ in terms of its neighbors?

2) **(15 points)** For any cell x, give a mathematical expression for $C(x)$. Be clear on what each term or factor represents.

3) **(10 points)** If you had the value of $C(x)$ for each cell, how could you use it to find the best direction to move in cell **S**?

4) **(30 points)** Compute $C(x)$ for each x. What is $C(\mathbf{S})$?

5) **(10 points)** Determine for each cell what the best direction to move in is. Show this on the grid.

*Bonus: **(15 points)** If the cost of the icy ravine death were \$0, then you'd probably want to run to the nearest ravine to terminate the game early and minimize your overall cost. Eventually, as you start increasing the cost of falling down the ravine, you reach a point where it's preferable to start trying to avoid the ravine. At what point (cost threshold) do the best directions to take in each cell stop changing? How can you find that threshold, and what are the best directions to move when above this threshold?*