Dataflow analysis (ctd.)

Available expressions

- Determine which expressions have already been evaluated at each point.
- A expression $x+y$ is *available at point p* if every path from the entry to p evaluates $x+y$ and after the last such evaluation prior to reaching p, there are no assignments to x or y
- Used in :
  - global common subexpression elimination

Example

Kills x+y,
w*x, etc.

```
x = x+y
z = a+b
```

Kills z-w,
x+z, etc.

Generates
a+b

Available expressions

- What is safe?
  - To assume that an expression is **not** available at some point even if it may be.
  - The computed set of available expressions at point p will be a subset of the actual set of available expressions at p
  - The computed set of unavailable expressions at point p will be a superset of the actual set of unavailable expressions at p
  - Goal : make the set of available expressions as large as possible (i.e. as close to the actual set as possible)

Available expressions

- How are the **gen** and **kill** sets defined?
  - **gen**[B] = {expressions evaluated in B without subsequently redefining its operands}
  - **kill**[B] = {expressions whose operands are redefined in B without reevaluating the expression afterwards}
- What is the direction of the analysis?
  - forward
  - **out**[B] = **gen**[B] $\cup$ (**in**[B] - **kill**[B])

Available expressions

- What is the confluence operator?
  - intersection
  - **in**[B] = $\cap$ **out**[P], over the predecessors P of B

- How do we initialize?
  - Start with emptyset!

Avaliable Expressions: equations

$$AE_{\texttt{entry}}(p) = \begin{cases} \varnothing & \text{for initital point p} \\ \\ \cap \{ AE_{\texttt{exit}}(q) \mid (q,p) \text{ in the CFD} \} \end{cases}$$

$$AE_{\texttt{exit}}(p) = gen_{AE}(p) \cup (AE_{\texttt{entry}}(p) \setminus kill_{AE}(p))$$

Equations

n	$kill_{AE}(n)$	$gen_{AE}(n)$
1	$\varnothing$	{a+b}
2	$\varnothing$	{a*b}
3	$\varnothing$	{a+b}
4	{a+b, a*b,a+1}	$\varnothing$
5	$\varnothing$	{a+b}

$$AE_{entry}(p) = \begin{cases} \varnothing & \text{for iniitial point p} \\ \cap \{ AE_{exit}(q) \mid (q,p) \text{ in CFD} \} \end{cases}$$

$$AE_{exit}(p) = (AE_{entry}(p) \setminus kill_{AE}(p)) \cup gen_{AE}(p)$$

$AE_{entry}(1) = \varnothing$

$AE_{entry}(2) = AE_{exit}(1)$

$AE_{entry}(3) = AE_{exit}(2) \cap AE_{exit}(5)$

$AE_{entry}(4) = AE_{exit}(3)$

$AE_{entry}(5) = AE_{exit}(4)$

$AE_{exit}(1) = AE_{entry}(1) \cup \{a+b\}$

$AE_{exit}(2) = AE_{entry}(2) \cup \{a*b\}$

$AE_{exit}(3) = AE_{entry}(3) \cup \{a+b\}$

$AE_{exit}(4) = AE_{entry}(4) - \{a+b, a*b, a+1\}$

$AE_{exit}(5) = AE_{entry}(5) \cup \{a+b\}$

Solution

$AE_{entry}(1) = \varnothing$
$AE_{entry}(2) = AE_{exit}(1)$
$AE_{entry}(3) = AE_{exit}(2) \cap AE_{exit}(5)$
$AE_{entry}(4) = AE_{exit}(3)$
$AE_{entry}(5) = AE_{exit}(4)$

$AE_{exit}(1) = AE_{entry}(1) \cup \{a+b\}$
$AE_{exit}(2) = AE_{entry}(2) \cup \{a*b\}$
$AE_{exit}(3) = AE_{entry}(3) \cup \{a+b\}$
$AE_{exit}(4) = AE_{entry}(4) - \{a+b, a*b, a+1\}$
$AE_{exit}(5) = AE_{entry}(5) \cup \{a+b\}$

n	**AE_{entry}(n)**	**AE_{exit}(n)**
1	$\varnothing$	{a+b}
2	{a+b}	{a+b, a*b}
3	{a+b}	{a+b}
4	{a+b}	$\varnothing$
5	$\varnothing$	{a+b}

Result

- $[x:=a+b]^1$; $[y:=a*b]^2$; while $[y>a+b]^3$ do { $[a:=a+1]^4$; $[x:=a+b]^5$ }

n	$AE_{entry}(n)$	$AE_{exit}(n)$
1	$\emptyset$	{a+b}
2	{a+b}	{a+b, a*b}
3	{a+b}	{a+b}
4	{a+b}	$\emptyset$
5	$\emptyset$	{a+b}

- Even though the expression a is redefined in the cycle (in 4), the expression a+b is always available ai the entry of the cycle (in 3).
- Viceversa, a*b is available at the first entry of the cycle but it is killed before the next iteration (in 4).

Very Busy Expressions

- Determine whether an expression is evaluated in all paths from a point to the exit.
- An expression e is very busy at point p if no matter what path is taken from p, e will be evaluated before any of its operands are defined.
- Used in:
  - Code hoisting
    - If e is very busy at point p, we can move its evaluation at p.

Example

if $[a>b]^1$ then $([x:=b-a]^2 ; [y:=a-b]^3)$ else $([y:=b-a]^4 ; [x:=a-b]^5)$

The two expressions a-b and b-a are both very busy in program point 1.

Very Busy Expressions

- What is safe?
  - To assume that an expression is not very busy at some point even if it may be.
  - The computed set of very busy expressions at point p will be a subset of the actual set of available expressions at p
  - Goal : make the set of very busy expressions as large as possible (i.e. as close to the actual set as possible)

Very Busy Expressions

- How are the **gen** and **kill** sets defined?
  - **gen**[B] = {all expressions evaluated in B before any definitions of their operands}
  - **kill**[B] = {all expressions whose operands are defined in B before any possible re-evaluation}
- What is the direction of the analysis?
  - backward
  - **in**[B] = **gen**[B] ∪ (**out**[B] - **kill**[B])

Very Busy Expressions

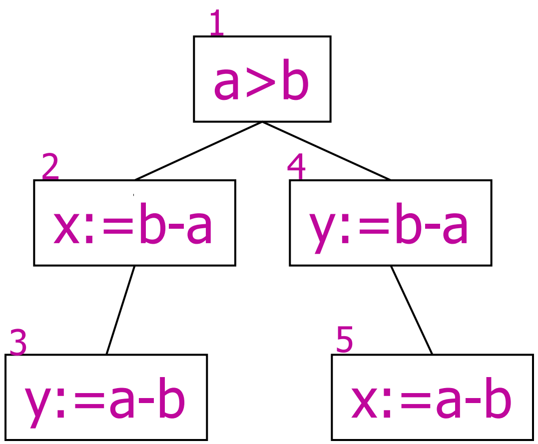
- What is the confluence operator?
  - intersection
  - **out**[B] = $\cap$ **in**[S], over the successors S of B

Very Busy Expressions: equations

$$VB_{\texttt{exit}}(p) = \begin{cases} \emptyset & \text{if p is final} \\ \cap \{ VB_{\texttt{entry}}(q) \mid (p,q) \text{ in the CFD} \} & \end{cases}$$

$$VB_{\texttt{entry}}(p) = (VB_{\texttt{exit}}(p) \setminus kill_{VB}(p)) \; U \; gen_{VB}(p)$$

n	$kill_{VB}(n)$	$gen_{VB}(n)$
1	$\emptyset$	$\emptyset$
2	$\emptyset$	{b-a}
3	$\emptyset$	{a-b}
4	$\emptyset$	{b-a}
5	$\emptyset$	{a-b}



$VB_{entry}(1)=VB_{exit}(1)$

$VB_{entry}(2)=VB_{exit}(2) \cup \{b\text{-}a\}$

$VB_{entry}(3)=\{a\text{-}b\}$

$VB_{entry}(4)=VB_{exit}(4) \cup \{b\text{-}a\}$

$VB_{entry}(5)=\{a\text{-}b\}$

$VB_{exit}(1)= VB_{entry}(2) \cap VB_{entry}(4)$

$VB_{exit}(2)= VB_{entry}(3)$

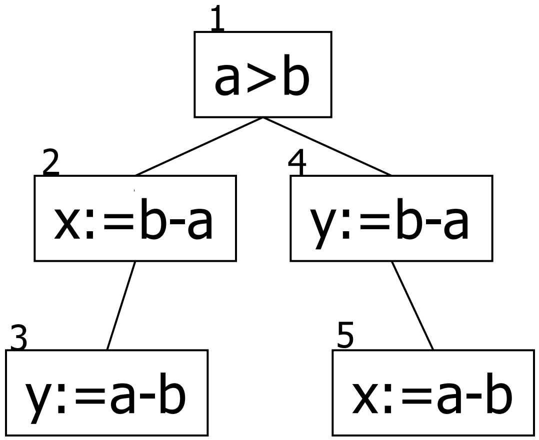
$VB_{exit}(3)= \emptyset$

$VB_{exit}(4)= VB_{entry}(5)$

$VB_{exit}(5)= \emptyset$

Result

if $[a>b]^1$ then $([x:=b-a]^2 ; [y:=a-b]^3)$ else $([y:=b-a]^4 ; [x:=a-b]^5)$



n	**$VB_{entry}(n)$**	**$VB_{exit}(n)$**
1	{a-b, b-a}	{a-b, b-a}
2	{a-b, b-a}	{a-b}
3	{a-b}	$\varnothing$
4	{a-b, b-a}	{a-b}
5	{a-b}	$\varnothing$

Dataflow analysis: a general framework

Dataflow Analysis

- Compile-time reasoning about run-time values of variables or expressions
- At different program points
  - Which assignment statements produced value of variable at this point?
  - Which variables contain values that are no longer used after this program point?
  - What is the range of possible values of variable at this program point?

Program Representation

- Control Flow Graph
  - Nodes N – statements of program
  - Edges E – flow of control
    - pred(n) = set of all predecessors of n
    - succ(n) = set of all successors of n
  - Start node n_0
  - Set of final nodes N_{final}

Program Points

- One program point before each node
- One program point after each node
- Join point – point with multiple predecessors
- Split point – point with multiple successors

Basic Idea

- Information about program represented using values from algebraic structure
- Analysis produces a value for each program point
- Two flavors of analysis
  - Forward dataflow analysis
  - Backward dataflow analysis

Forward Dataflow Analysis

- Analysis propagates values forward through control flow graph with flow of control
  - Each node n has a transfer function f_n
    - Input – value at program point before node
    - Output – new value at program point after node
  - Values flow from program points after predecessor nodes to program points before successor nodes
  - At join points, values are combined using a merge function
- Canonical Example: Reaching Definitions

Backward Dataflow Analysis

- Analysis propagates values backward through control flow graph against flow of control
  - Each node n has a transfer function f_n
    - Input – value at program point after node
    - Output – new value at program point before node
  - Values flow from program points before successor nodes to program points after predecessor nodes
  - At split points, values are combined using a merge function
- Canonical Example: Live Variables

Representing the property of interest

- Dataflow information will be lattice values
  - Transfer functions operate on lattice values
  - Solution algorithm will generate increasing sequence of values at each program point
  - Ascending chain condition will ensure termination
- Will use $\vee$ to combine values at control-flow join points

Transfer Functions

- Transfer function f_n: $P \rightarrow P$ for each node n in control flow graph
- f_n models effect of the node on the program information

Transfer Functions

Each dataflow analysis problem has a set F of transfer functions $f: P \rightarrow P$

- Identity function $i \in F$
- F must be closed under composition: $\forall f,g \in F$. the function $h(x) = f(g(x)) \in F$
- Each $f \in F$ must be monotone: $x \leq y$ implies $f(x) \leq f(y)$
- Sometimes all $f \in F$ are distributive: $f(x \vee y) = f(x) \vee f(y)$
- Distributivity implies monotonicity

Distributivity Implies Monotonicity

- Proof of distributivity implies monotonicity
- Assume $f(x \vee y) = f(x) \vee f(y)$
- Must show:
  $x \leq y$ implies $f(x) \leq f(y)$, and this is equivalent to show that $x \vee y = y$ implies $f(x) \vee f(y) = f(y)$

  $f(y) = f(x \vee y)$ (by applying f to both sides)

  $= f(x) \vee f(y)$ (by distributivity)

Forward Dataflow Analysis

- Simulates execution of program forward with flow of control
- For each node n, have
  - in_n – value at program point before n
  - out_n – value at program point after n
  - f_n – transfer function for n (given in_n, computes out_n)
- Require that solution satisfy
  - $\forall n.\ out_n = f_n(in_n)$
  - $\forall n \neq n_0.\ in_n = \vee \{ out_m\ .\ m \text{ in } pred(n) \}$
  - $in_{n0} = I$,
    where I summarizes information at start of program

Worklist Algorithm for Solving Forward Dataflow Equations

for each n do $out_n := f_n(\perp)$

$in_{n0} := I$; $out_{n0} := f_{n0}(I)$

worklist := N - { n_0 }

while worklist $\neq \varnothing$ do

    remove a node n from worklist

    $in_n := \vee \{ out_m . m \text{ in } pred(n) \}$

    $out_n := f_n(in_n)$

    if out_n changed then

        worklist := worklist $\cup$ succ(n)

Correctness Argument

- Why result satisfies dataflow equations?
- Whenever process a node n,
  the algorithm ensures that $out_n = f_n(in_n)$
- Whenever out_m changes, the algoritm puts succ(m) on worklist.
  Consider any node $n \in$ succ(m).
  It will eventually come off worklist and the algorithm will set

  $$in_n := \vee \{ out_m . m \text{ in } pred(n) \}$$

  to ensure that $in_n = \vee \{ out_m . m \text{ in } pred(n) \}$
- So final solution will satisfy dataflow equations

Termination Argument

- Why does algorithm terminate?
- Sequence of values taken on by in_n or out_n is a chain. If values stop increasing, worklist empties and algorithm terminates.
- If the lattice enjoys the ascending chain property, the algorithm terminates
  - Algorithm terminates for finite lattices
  - For lattices without ascending chain property, me may use widening operator

Widening Operators

- Detect lattice values that may be part of infinitely ascending chain
- Artificially raise value to least upper bound of chain
- Example:
  - Lattice is set of all subsets of integers
  - Could be used to collect possible values taken on by variable during execution of program
  - Widening operator might raise all sets of size n or greater to TOP (likely to be useful for loops)

Reaching Definitions

- P = powerset of set of all definitions in program (all subsets of set of definitions in program)
- $\vee = \cup$ (order is $\subseteq$)
- $\perp = \emptyset$
- $I = in_{n0} = \perp$
- F = all functions f of the form $f(x) = a \cup (x-b)$
  - b is set of definitions that node kills
  - a is set of definitions that node generates
- General pattern for many transfer functions
  - $f(x) = GEN \cup (x\text{-}KILL)$

Does Reaching Definitions Satisfy the Framework Constraints?

- $\subseteq$ satisfies conditions for $\leq$
  $x \subseteq y$ and $y \subseteq z$ implies $x \subseteq z$ (transitivity)
  $x \subseteq y$ and $y \subseteq x$ implies $y = x$ (asymmetry)
  $x \subseteq x$ (idempotence)

- F satisfies transfer function conditions
  $\lambda x.\emptyset \cup (x- \emptyset) = \lambda x.x \in F$ (identity)
  Will show $f(x \cup y) = f(x) \cup f(y)$ (distributivity)

$$
\begin{aligned}
f(x) \cup f(y) &= (a \cup (x - b)) \cup (a \cup (y - b)) \\
&= a \cup (x - b) \cup (y - b) = a \cup ((x \cup y) - b) \\
&= f(x \cup y)
\end{aligned}
$$

Does Reaching Definitions Framework Satisfy Properties?

- What about composition?

Given $f_1(x) = a_1 \cup (x\text{-}b_1)$ and $f_2(x) = a_2 \cup (x\text{-}b_2)$

Must show $f_1(f_2(x))$ can be expressed as $a \cup (x - b)$

$$
\begin{aligned}
f_1(f_2(x)) &= a_1 \cup ((a_2 \cup (x\text{-}b_2)) - b_1) \\
&= a_1 \cup ((a_2 - b_1) \cup ((x\text{-}b_2) - b_1)) \\
&= (a_1 \cup (a_2 - b_1)) \cup ((x\text{-}b_2) - b_1)) \\
&= (a_1 \cup (a_2 - b_1)) \cup (x\text{-}(b_2 \cup b_1))
\end{aligned}
$$

Let $a = (a_1 \cup (a_2 - b_1))$ and $b = b_2 \cup b_1$

Then $f_1(f_2(x)) = a \cup (x – b)$

General Result

All GEN/KILL transfer function frameworks satisfy

- Identity
- Distributivity
- Composition

properties

Available Expressions

- P = powerset of set of all expressions in program (all subsets of set of expressions)
- $\vee = \cap$ (order is $\supseteq$)
- $\perp = P$
- $I = in_{n0} = \emptyset$
- F = all functions f of the form $f(x) = a \cup (x\text{-}b)$
  - b is set of expressions that node kills
  - a is set of expressions that node generates
- Another GEN/KILL analysis

Concept of Conservatism

- Reaching definitions use $\cup$ as join
  - Optimizations must take into account all definitions that reach along ANY path
- Available expressions use $\cap$ as join
  - Optimization requires expression to reach along ALL paths
- Optimizations must conservatively take all possible executions into account. Structure of analysis varies according to way analysis used.

Backward Dataflow Analysis

- Simulates execution of program backward against the flow of control
- For each node n, have

  in_n – value at program point before n

  out_n – value at program point after n

  f_n – transfer function for n (given out_n, computes in_n)
- Require that solution satisfies

  $\forall n.\ in_n = f_n(out_n)$

  $\forall n \notin N_{final}.\ out_n = \vee \{ in_m .\ m \text{ in } succ(n) \}$

  $\forall n \in N_{final} = out_n = O$

  Where O summarizes information at end of program

Worklist Algorithm for Solving Backward Dataflow Equations

for each n do $in_n := f_n(\perp)$

for each $n \in N_{final}$ do $out_n := O$; $in_n := f_n(O)$

worklist := $N - N_{final}$

while worklist $\neq \emptyset$ do

  remove a node n from worklist

  $out_n := \vee \{ in_m . m \text{ in } succ(n) \}$

  $in_n := f_n(out_n)$

  if in_n changed then

    worklist := worklist $\cup$ pred(n)

Live Variables

- P = powerset of set of all variables in program (all subsets of set of variables in program)
- $\vee = \cup$ (order is $\subseteq$)
- $\perp = \emptyset$
- O = $\emptyset$
- F = all functions f of the form $f(x) = a \cup (x-b)$
  - b is set of variables that node kills
  - a is set of variables that node reads