

# Analysis and Verification of Software

## **Homework 2**

due by February 20, 2015

# Exercise 1

- Prove that if  $L$  is a lattice, then for each  $x, y$  in  $L$  it is true that:  $\text{lub}(x, \text{glb}(x, y)) = x$

# Exercise 2

- Let  $A$  be any set, and consider the set  $B$  whose elements are all the subsets of  $A$ , partially ordered by subset inclusion.
- Prove that  $B$  is a complete lattice.
- What are the lub and glb operators?

# Exercise 3

- Design (at your choice) a (finite) complete lattice  $A$ , with at least 15 elements.

Provide four (non-trivial) examples of  $f:A \rightarrow A$  such that  $f$  is, respectively ...

1. non monotone
2. monotone but not continuous
3. a function with no fixpoints
4. continuous

In the last case, list the set of fixpoints of  $f$ .

# Exercise 4

- Prove the following theorem:

If  $(P, <)$  is a complete lattice, and  $f$  is a monotone (increasing) function from  $P$  to  $P$ , then  $f$  has exactly one minimal fixed point.

# Exercise 5

- Compute the reaching definitions for the nodes in the following flowgraph.

