Assignment MATLAB

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1 Fit data to a circle

1.1 Applied theory in this task

The following formula is applied to solve this task:

$$2xc_1 + 2yc_2 + c_3 = x^2 + y^2 \tag{1}$$

Substituting each of the data points into this equation, we obtain the overdetermined system.

$$\begin{bmatrix} 2x_1 & 2y_1 & 1 \\ 2x_2 & 2y_2 & 1 \\ 2x_3 & 2y_3 & 1 \\ 2x_4 & 2y_4 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ x_3^2 + y_3^2 \\ x_4^2 + y_4^2 \end{bmatrix} \tag{2}$$

According to the above equation, the related numbers can be assigned to the equations and we will get the following equation.

$$\begin{bmatrix} 2 \times (-4) & 2 \times 3 & 1 \\ 2 \times 0 & 2 \times 2 & 1 \\ 2 \times 1 & 2 \times 12 & 1 \\ 2 \times 5 & 2 \times 6 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} (-4)^2 + 3^2 \\ 0^2 + 2^2 \\ 1^2 + 12^2 \\ 5^2 + 6^2 \end{bmatrix} \tag{3}$$

Based on the above equation, the following result can be found, which can be used for calculating c_1, c_2, c_3.

$$\begin{bmatrix} -8 & 6 & 1 \\ 0 & 4 & 1 \\ 2 & 24 & 1 \\ 10 & 12 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 4 \\ 145 \\ 61 \end{bmatrix} \tag{4}$$

1.2 Applied MATLAB codes in this task

```
clear
A=[-8 6 1; 0 4 1; 2 24 1; 10 12 1];          %Create matrix A
B=[25;4;145;61];                             %Create matrix B
c=A\B;                                       %According to Ac=B,c=A\B
r=sqrt(c(3)+c(1)^2+c(2)^2);                  %Calcualte 'r'
plot([-4 0 1 5], [3 2 12 6],'.r','MarkerSize',18)%Plotting the dots
hold on
th = 0:pi/50:2*pi;                           %Plotting the circle
xunit = r * cos(th) + c(1);
yunit = r * sin(th) + c(2);
plot(xunit, yunit,'Color',[0 0 1], 'LineWidth',1)
axis equal
xlabel('x')                                  %Add labels
ylabel('y')
```

1.3 Plotting the diagram

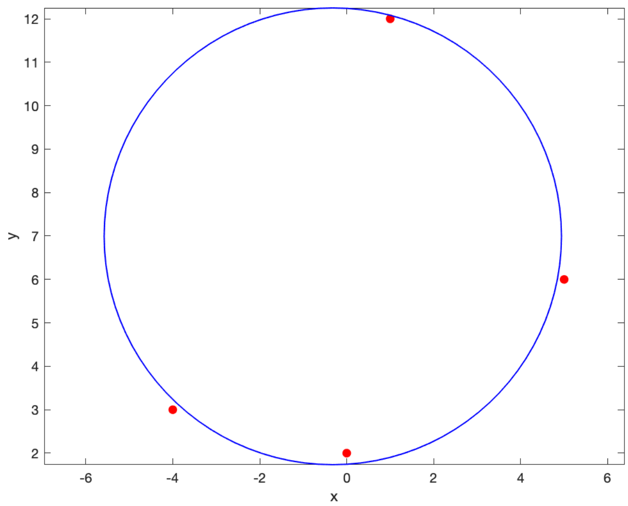

Figure 1: Fit data to a circle

2 Adapting a circle to a larger amount of data

2.1 Applied theory in this task

The following formula is applied to solve this task:

$$2xc_1 + 2yc_2 + c_3 = x^2 + y^2 \tag{5}$$

Substituting each of the data points into this equation, we obtain the over-determined system.

$$\begin{bmatrix} 2x_1 & 2y_1 & 1 \\ 2x_2 & 2y_2 & 1 \\ 2x_3 & 2y_3 & 1 \\ \vdots & \vdots & \vdots \\ 2x_{300} & 2y_{300} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ x_3^2 + y_3^2 \\ \vdots \\ x_{300}^2 + y_{300}^2 \end{bmatrix} \tag{6}$$

According to the above equation, the related numbers can be assigned to the equations and we can calculate r:

$$r = \sqrt{c_3 + c_1^2 + c_2^2} \tag{7}$$

Based on the above equation, the r equals 5.0230. The following Matlab codes are used for plotting the required figure.

2.2 Applied MATLAB codes in this task

```
clear
load ('cirkel300.mat');   %Load data
newX = X*2;               %According to formula: 2xc1+2yc2+c3=x2+y2
newY = Y*2;               %New X equals old X times 2
allOne = 1*ones(300,1);   %Create an all one column
newA = [newX newY allOne];%Create matrix new A
newB = X.^2+Y.^2;         %Create matrix new B
newC = newA\newB;         %Calculate new C
```

```
newR=sqrt(newC(3)+newC(1)^2+newC(2)^2);  %Calculate r
plot(X,Y,'.r','MarkerSize',6) %Plotting the dots
hold on
th = 0:pi/50:2*pi;          %Plotting the circle
xunit = newR * cos(th) + newC(1);
yunit = newR * sin(th) + newC(2);
plot(xunit, yunit,'Color',[0 0 1], 'LineWidth',1)
axis equal
xlabel('x')                 %add labels
ylabel('y')
```

2.3 Plotting the diagram

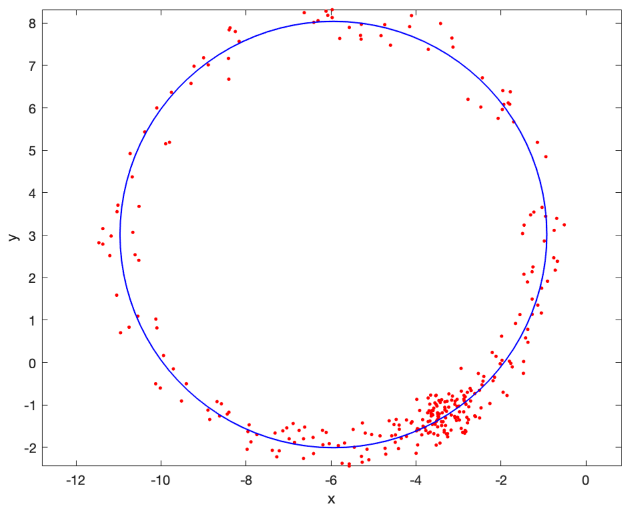

Figure 2: Adapting a circle to a larger amount of data

3 Solve system of any size

3.1 Applied MATLAB codes in this task

In this task, we applied the following code to solve the problem:

```
clear
n = input('Please input a number: ');%Receive input
x = 1/n*(1:1:n-1)'; %Create x vector
f = x.^2;           %Calculate f
v1 = -1*ones(n-2,1);%Create v1 matrix
D1 = diag(v1,-1);   %Create D1 matrix

v2 = 2*ones(n-1,1); %Create V2 matrix
D2 = diag(v2,0);    %Create D2 matrix

D3 = diag(v1,1);    %Create D3 matrix

A = n^2*(D1+D2+D3); %Matrix A is combined by D1,D2,D3
y = A\f;            %Calculate y
xExd = [0;x;1];     %Create x extend
yExd = [0;y;0];     %Create y extend
%Plot the diagram
```

```
plot(xExd,yExd,'LineStyle','-','Marker','.','MarkerSize',7)
xlabel('x');                    %add labels
ylabel('y');
axis ([0 1 0 0.05]);
set (gca, 'fontsize', 16);
grid on;
box on;
```

3.2 Plotting the diagram

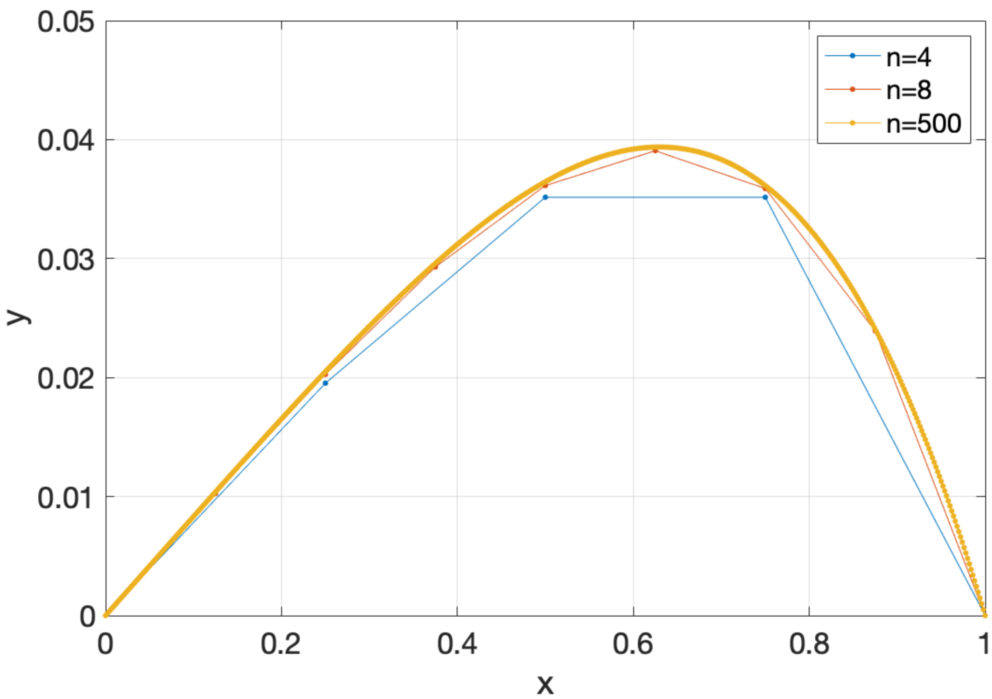

Figure 3: Solve system of any size

4 Draw a plane in 3D

4.1 Applied MATLAB codes in this task

```
clear
u = [3 3 -1];
v = [2 4 -1];
w = cross(u,v);
% w = [1 1 6], then we can get 1(x-0)+1(y-0)+6(z-0)=0
% we can get the normal equation x+y+6z=0,bring in P1(6,6,z1) P2(6,-6,z2)
% P3(-6,-6,z3) P4(-6,6,z4)
% we can get Z1=-2, Z2=0, Z3=2, Z4=0.
X = [6 6 -6 -6];
Y = [6 -6 -6 6];
Z = [-2 0 2 0];
fill3(X,Y,Z,'b','facealpha', 0.4);
hold on
DrawVector3D(u,'b');
```

```
DrawVector3D(v, 'g');
DrawVector3D(w, 'r');
xlabel ('x');
ylabel ('y');
zlabel ('z');
xticks(-10:1:10);
yticks(-10:1:10);
zticks(-10:1:10);
grid on
box on
set (gca, 'fontsize', 16);
axis equal
view ([1,0,0]) %view ([0,1,0]) %view ([0,0,1])
```

4.2 Plotting the diagram

4.2.1 Draw the diagram from "view([1,0,0])"

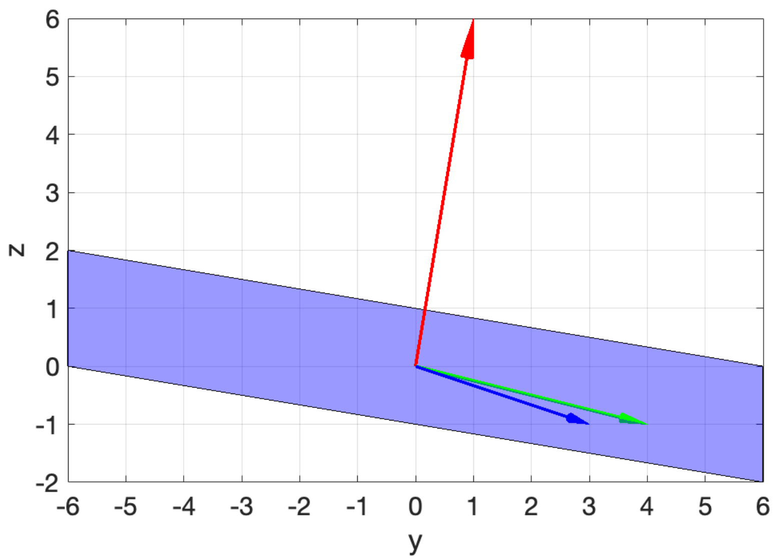

Figure 4: Draw the plane and vectors in 3D from "view ([1,0,0])"

4.2.2 Draw the diagram from "view([0,1,0])"

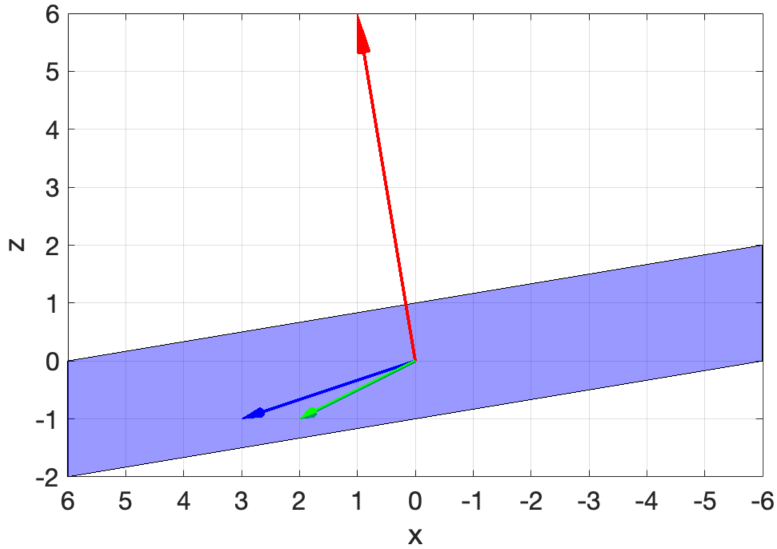

Figure 5: Draw the plane and vectors in 3D from "view ([0,1,0])"

4.2.3 Draw the diagram from "view([0,0,1])"

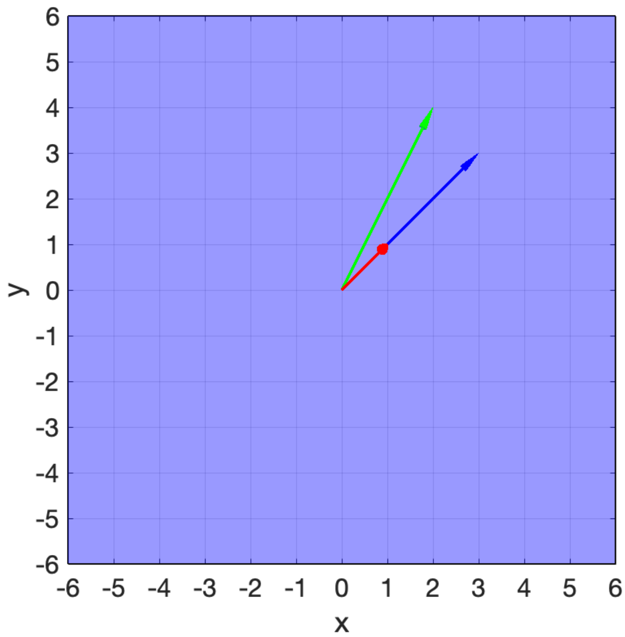

Figure 6: Draw the plane and vectors in 3D from "view ([0,0,1])"

5 Linear Transformations of a Tree

5.1 Applied MATLAB codes in this task

```
clear
load ('lnu.mat');
x = xy(1,:);
y = xy(2,:);
plot(x,y,'.k')
axis equal
hold on

%Answer for iii
S = [1 0;0 -1];
result = S*xy;
X1 = result(1,:);
Y1 = result(2,:);
% plot(X1,Y1,'.g')
% axis equal
% hold on

%Answer for iv
S1 = [cos(2/3*pi) -sin(2/3*pi); sin(2/3*pi) cos(2/3*pi)];
result2 = S1*xy;
X2 = result2(1,:);
Y2 = result2(2,:);
%plot(X2,Y2,'.r')
%hold on

%Answer for v
result3 = S1*result;
X3 = result3(1,:);
Y3 = result3(2,:);
% plot(X3,Y3,'.m')
% axis equal
% hold on
result4 = S1*xy;
result5 = S*result4;
X5 = result5(1,:);
Y5 = result5(2,:);
% plot(X5,Y5,'.c')
% axis equal

%Answer for vi
%Use the formula from the instruction of vi
%Select k = 3
k = 3;
M = 0.5*[1+k k-1; k-1 1+k];
result6 = M*xy;
X6 = result6(1,:);
Y6 = result6(2,:);
plot(X6,Y6,'.b')
axis equal
```

5.2 Plotting the diagram

5.2.1 The image of Task 5 instruction (iii):

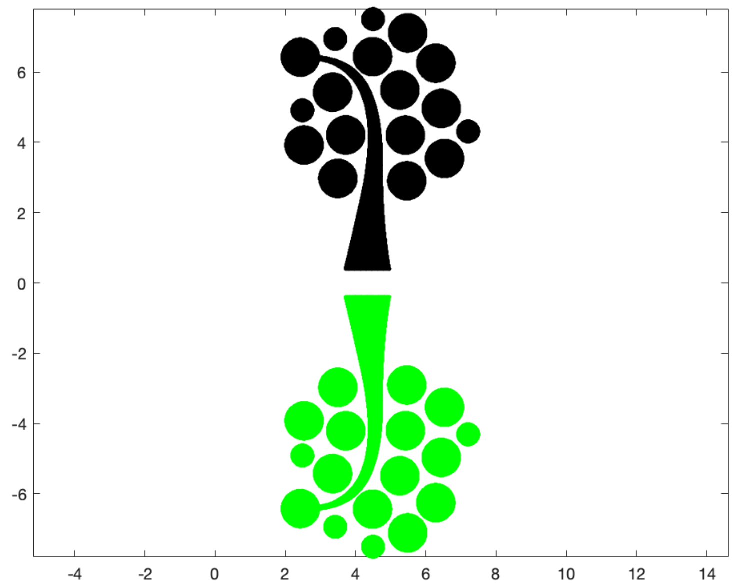

Figure 7: Transformation of xy reflected in the x-axis

5.2.2 The image of Task 5 instruction (iv):

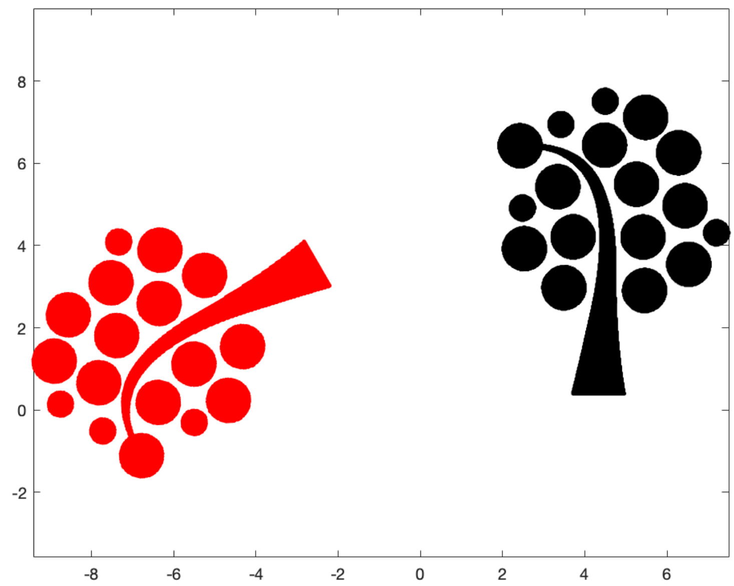

Figure 8: Transformation of rotating $2\pi/3$ counterclockwise

5.2.3 The image of Task 5 instruction (v):

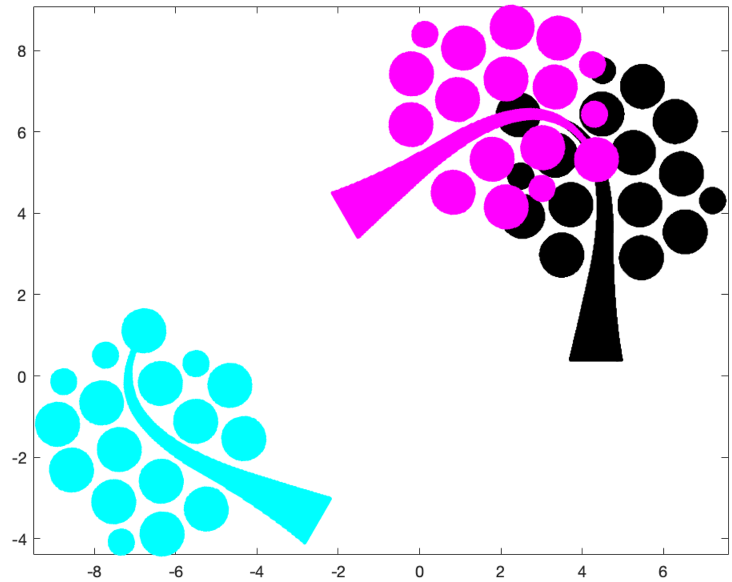

Figure 9: Composite linear transformation

5.2.4 The image of Task 5 instruction (vi):

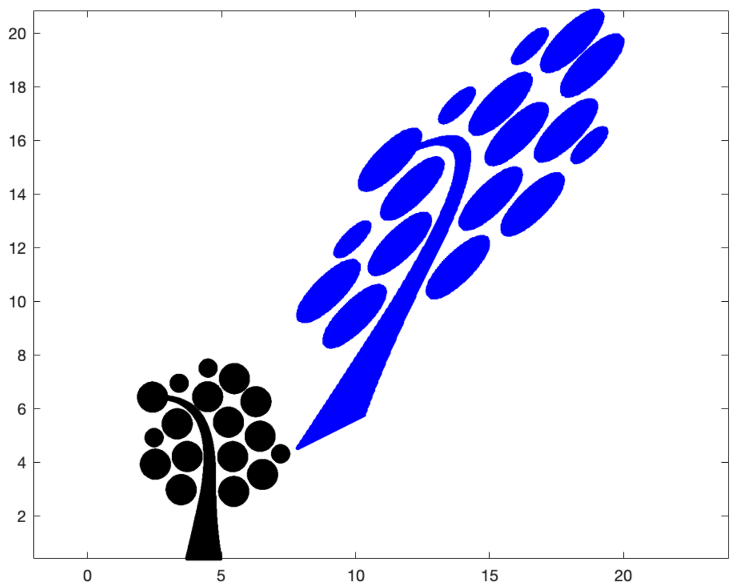

Figure 10: Stretched the image with the scale factor k=3