

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Consider a $2 \times n$ checkerboard where the integer $n \geq 1$. Your task is to tile (cover) it with 1×1 squares and/or L-shaped figures made up by 3 such squares. The L-shapes can be rotated so the 2×3 board can be covered by two L's. In fact this can be done in two ways. Let a_n denote the number of tilings of the $2 \times n$ board. $a_1 = 1$ (2 squares) and $a_2 = 5$ (can you find them?). What is a_3 ? Here you can use the sum rule over the number of squares used. Which numbers of squares is possible in the 2×3 case? Time for you to come up with a RR for a_n . Then solve it with RSolve using the initial values above and plot a_n with DiscretePlot. Hint: Use the sum rule and focus on the three columns to the very right, that is columns $n - 2$, $n - 1$ and n . If you do not understand the output use the commands Re for real value and N for numerical value.

2. Check if you, like in the logistic map, can see a couple of period doublings for $g(x) = a \sin(\pi x)$. Starting value is still between 0 and 1 and a lies also between zero and one. Make an estimate of Feigenbaums constant from your observations. The program will protest somewhat when you use the command Solve but you can trust the output.**OP**

3. Run rule 225 for a seed of length 1000. All cells are white initially except for a black cell at position 10. Iterate 1000 times and present the figure. Write 225 in base 2, using for example the command BaseForm, and try to understand the rule in detail.

4. Run the rule 90016 starting with a wall, that is a square of black cells. Take for example a 100 times 100 grid and put the wall at rows and columns 10 and 90. Iterate 1,2,3,.. 10,100, 500 times. What happens? Express 90016 in base 2, using BaseForm, and describe in words the rule, when will birth and survival happen? Note ;; can be used in the construction of the seed. $seed[[10;;90,1]] = 1$ gives the value 1 to all elements in the column 1 from row 10 to row 90.