cs109a_hw4_209_submit

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1 CS109A Introduction to Data Science:

1.1 Homework 4 AC 209 : Regularization

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Names of people you have worked with goes here:

```
In [2]: # RUN THIS CELL FOR FORMAT
        import requests
        from IPython.core.display import HTML
        styles = requests.get("https://raw.githubusercontent.com/Harvard-IACS/2018-CS109A/maste
        HTML(styles)

Out[2]: <IPython.core.display.HTML object>

In [3]: # Imports
        import numpy as np
        import matplotlib.pyplot as plt
        from sklearn.datasets import make_regression
        from sklearn.linear_model import LinearRegression, Ridge, Lasso, ElasticNet, RidgeCV, L
        from sklearn.model_selection import train_test_split
        from sklearn.metrics import r2_score

        %matplotlib inline
```

Question 1 [12 pts]

Ridge and LASSO regularizations are powerful tools that not only increase generalization, but also expand the range of problems that we can solve. We will study this statement in this question.

5.1 Let $X \in \mathbb{R}^{n \times p}$ be a matrix of observations, where each row corresponds an observation and each column corresponds to a predictor. Now consider the case $p > n$: explain why there is no unique solution to the OLS estimator.

5.2 Now consider the Ridge formulation. Show that finding the ridge estimator is equivalent to solving an OLS problem after adding p dummy observations with their X value equal to λ at the j-th component and zero everywhere else, and their Y value set to zero. In a nutshell, show that the ridge estimator can be found by getting the least squares estimator for the augmented problem:

$$X^* = \begin{bmatrix} X \\ \sqrt{\lambda} I \end{bmatrix}$$