CS109A Introduction to Data Science:

Homework 9: ANNs

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In [1]:
```
# RUN THIS CELL FOR FORMAT
import requests
from IPython.core.display import HTML
styles = requests.get("https://raw.githubusercontent.com/Harvard-IACS/2018-C
HTML(styles)
```

Out[1]:

Import libraries:

In [2]:
```
import random
random.seed(112358)

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from sklearn.model_selection import cross_val_score
from sklearn.utils import resample
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import RandomForestClassifier
from sklearn.ensemble import AdaBoostClassifier
from sklearn.linear_model import LogisticRegressionCV

import keras
from keras.models import Sequential
from keras.layers import Dense

%matplotlib inline

import seaborn as sns
pd.set_option('display.width', 1500)
pd.set_option('display.max_columns', 100)

from keras import regularizers


from sklearn.utils import shuffle
```

```
Using TensorFlow backend.
```

Neural Networks

Neural networks are, of course, a large and complex topic that cannot be covered in a single homework. Here we'll focus on the key idea of NNs: they are able to learn a mapping from example input data (of fixed size) to example output data (of fixed size). We'll also partially explore what patterns the neural network learns and how well they generalize.

In this question we'll see if Neural Networks can learn a (limited) version of the Fourier Transform. (The Fourier Transform takes in values from some function and returns a set of sine and cosine functions which, when added together, approximate the original function.)

In symbols: $\mathcal{F}(s) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ixs}dx$. In words, the value of the transformed function at some point, s, is the value of an integral which measures, in some sense, how much the original f(x) looks like a wave with period s. As an example, with $f(x) = 4cos(x) + sin(2x)$, $\mathcal{F}(s)$ is 0 everywhere except at -2, -1, 1, and 2, mapping to the waves of period 1 and 1/2. The values at these points are linked to the magnitude of the waves, and their phases (roughly: sin waves versus cosine waves).

The only thing about the Fourier transform that matters for this pset is this: function goes in, re-write in terms of sine and cosine comes out.

In our specific problem, we'll train a network to map from 1000 sample values from a function (equally spaced along 0 to 2π) to the four features of the sine and cosine waves that make up that function. Thus, the network is attempting to learn a mapping from a 1000-entry vector down to a 4-entry vector. Our X_train dataset is thus N by 1000 and our y_train is N by 4.

Questions 1.1 and 1.2 will get you used to the format of the data.

We'll use 6 data files in this question:

- `sinewaves_X_train.npy` and `sinewaves_y_train.npy` : a (10,000 by 1,000) and (10,000 by 4) training dataset. Examples were generated by randomly selecting a,b,c,d in the interval [0,1] and building the curve $a\sin(b\,x) + c\cos(d\,x)$
- `sinewaves_X_test.npy` and `sinewaves_y_test.npy` : a (2,000 by 1,000) and (2,000 by 4) test dataset, generated in the same way as the training data
- `sinewaves_X_extended_test` and `sinewaves_y_extended_test` : a (9 by 1,000) and (9 by 4) test dataset, testing whether the network can generalize beyond the training data (e.g. to negative values of a)

These datasets are read in to their respective variables for you.

Question 1 [50pts]

1.1 Plot the first row of the `X_train` training data and visually verify that it is a sinusoidal curve.

1.2 The first row of the `y_train` data is $[0.024, 0.533, 0.018, 0.558]$. Visually or numerically verify that the first row of X_train is 1000 equally-spaced samples in $[0, 10\pi]$ from the function $f(x) = 0.024\sin(0.533\,x) + 0.018\cos(0.558\,x)$. This pattern (y_train is the true parameters of the curve in X_train) will always hold.

1.3 Use `Sequential` and `Dense` from Keras to build a fully-connected neural network. You can choose any number of layers and any number of nodes in each layer.

1.4 Compile your model via the line `model.compile(loss='mean_absolute_error', optimizer='adam')` and display the `.summary()`. Explain why the first layer in your network has the indicated number of parameters.

1.5 Fit your model to the data for 50 epochs using a batch size of 32 and a validation split of 0.2. You can train for longer if you wish- the fit tends to improve over time.

1.6 Use the `plot_predictions` function to plot the model's predictions on `X_test` to the true values in `y_test` (by default, it will only plot the first few rows). Report the model's overall loss on the test set. Comment on how well the model performs on this unseen data. Do you think it has accurately learned how to map from sample data to the coefficients that generated the data?

1.7 Examine the model's performance on the 9 train/test pairs in the `extended_test` variables. Which examples does the model do well on, and which examples does it struggle with?

1.8 Is there something that stands out about the difficult examples, especially with respect to the data the model was trained on? Did the model learn the mapping we had in mind? Would you say the model is overfit, underfit, or neither?

Hint:

- Keras's documentation and examples of a Sequential model are a good place to start.
- A strong model can achieve validation error of around 0.03 on this data and 0.02 is very good.

In [3]:
```
def plot_predictions(model, test_x, test_y, count=None):
    # Model - a Keras or SKlearn model that takes in (n,1000) training data
    # test_x - a (n,1000) input dataset
    # test_y - a (n,4) output dataset
    # This function will plot the sine curves in the training data and those
    # It will also print the predicted and actual output values.

    #helper function that takes the n by 4 output and reverse-engineers
    #the sine curves that output would create
    def y2x(y_data):
        #extract parameters
        a=y_data[:,0].reshape(-1,1)
        b=y_data[:,1].reshape(-1,1)
        c=y_data[:,2].reshape(-1,1)
        d=y_data[:,3].reshape(-1,1)

        #build the matching training data
        x_points = np.linspace(0,10*np.pi,1000)
        x_data = a*np.sin(np.outer(b,x_points)) + c*np.cos(np.outer(d,x_poin
        return x_data

    #if <20 examples, plot all. If more, just plot 5
    if count==None:
        if test_x.shape[0]>20:
            count=5
        else:
            count=test_x.shape[0]

    #build predictions
    predicted = model.predict(test_x)
    implied_x = y2x(predicted)
    for i in range(count):
        plt.plot(test_x[i,:],label='true')
        plt.plot(implied_x[i,:],label='predicted')
        plt.legend()
        plt.ylim(-2.1,2.1)
        plt.xlabel("x value")
        plt.xlabel("y value")
        plt.title("Curves using the Neural Network's Approximate Fourier Tra
        plt.show()
        print("true:", test_y[i,:])
        print("predicted:", predicted[i,:])
```

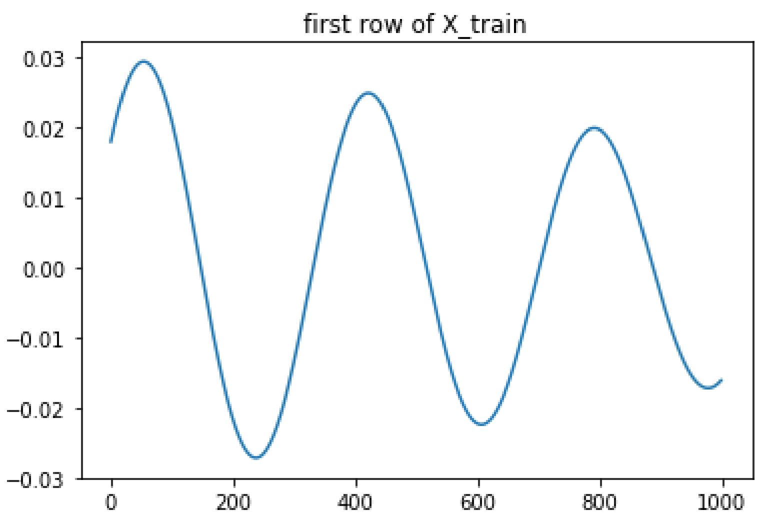
In [4]:
```
X_train = np.load('data/sinewaves_X_train.npy')
y_train = np.load('data/sinewaves_y_train.npy')

X_test = np.load('data/sinewaves_X_test.npy')
y_test = np.load('data/sinewaves_y_test.npy')

X_extended_test = np.load('data/sinewaves_X_extended_test.npy')
y_extended_test = np.load('data/sinewaves_y_extended_test.npy')
```

Answers:

1.1 Plot the first row of the `X_train` training data and visually verify that it is a sinusoidal curve

In [5]:
```
plt.plot(X_train[0,:])
plt.title("first row of X_train")
```

Out[5]: `Text(0.5,1,'first row of X_train')`

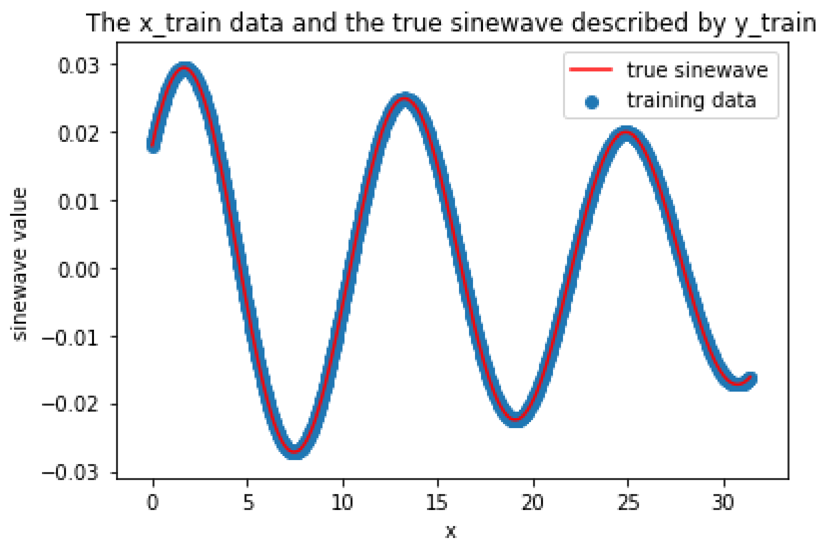


It is sinusoidal!

1.2 The first row of the `y_train` data is $[0.024, 0.533, 0.018, 0.558]$. Visually or numerically verify that the first row of X_train is 1000 equally-spaced samples in $[0, 10\pi]$ from the function $f(x) = 0.024 \sin(0.533\,x) + 0.018 \cos(0.558\,x)$. This pattern (y_train is the true parameters of the curve in X_train) will always hold.

In [6]:
```
# your code here
def reverse_fourier(y1,y2,y3,y4):
    f = lambda x: y1*np.sin(y2*x)+y3*np.cos(y4*x)
    return f
```

In [7]:
```
f = reverse_fourier(y_train[0,0],y_train[0,1],y_train[0,2],y_train[0,3])
x = np.linspace(0,10*np.pi,1000)
plt.plot(x, f(x), color = 'red', label = 'true sinewave')
plt.scatter(x, X_train[0,:], label = 'training data')
plt.xlabel('x')
plt.ylabel('sinewave value')
plt.legend()
plt.title("The x_train data and the true sinewave described by y_train")
plt.show()
```



1.3 Use `Sequential` and `Dense` from Keras to build a fully-connected neural network. You can choose any number of layers and any number of nodes in each layer.

In [8]:
```
# your code here
model = Sequential()
model.add(Dense(units=64, activation='relu', input_dim=1000))
model.add(Dense(units=16, activation='relu'))
model.add(Dense(units=4))
```

1.4 Compile your model via the line `model.compile(loss='mean_absolute_error', optimizer='adam')` and display the `.summary()`. Explain why the first layer in your network has the indicated number of parameters.

In [9]:
```
# your code here
model.compile(loss = 'mean_absolute_error', optimizer = 'adam')
```

In [10]:
```
model.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
dense_1 (Dense)              (None, 64)                64064     
_________________________________________________________________
dense_2 (Dense)              (None, 16)                1040      
_________________________________________________________________
dense_3 (Dense)              (None, 4)                 68        
=================================================================
Total params: 65,172
Trainable params: 65,172
Non-trainable params: 0
_________________________________________________________________
```

Your answer here The first layer has 64064 parameters because each node in layer 1 has 1000 parameters for each input plus 1 bias parameters.

This means that there should be $64 \times 1000 + 64 \times 1 = 64064$ parameters in the first layer, as observed.

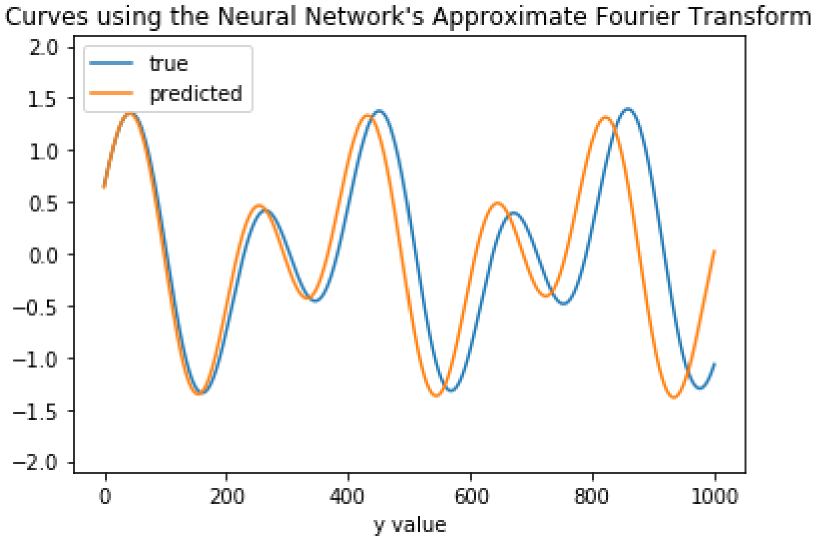
1.5 Fit your model to the data for 50 epochs using a batch size of 32 and a validation split of .2. You can train for longer if you wish- the fit tends to improve over time.

In [11]:
```
# your code here
model.fit(X_train, y_train, epochs=75, batch_size=32, validation_split=0.2)
```

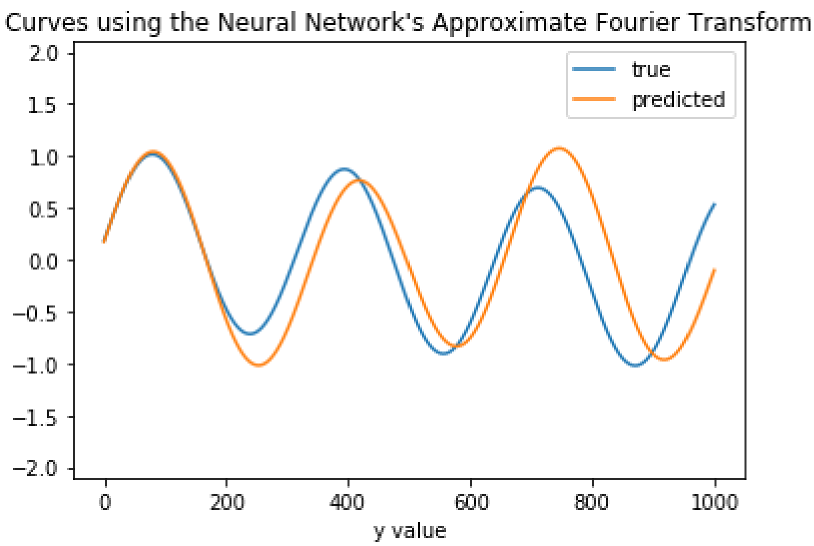
```
- val_loss: 0.0421
Epoch 45/75
8000/8000 [==============================] - 0s 52us/step - loss: 0.0401
- val_loss: 0.0399
Epoch 46/75
8000/8000 [==============================] - 0s 56us/step - loss: 0.0393

- val_loss: 0.0402
Epoch 47/75
8000/8000 [==============================] - 0s 53us/step - loss: 0.0388
- val_loss: 0.0399
Epoch 48/75
8000/8000 [==============================] - 0s 53us/step - loss: 0.0392
- val_loss: 0.0392
Epoch 49/75
8000/8000 [==============================] - 0s 51us/step - loss: 0.0385
- val_loss: 0.0377
Epoch 50/75
8000/8000 [==============================] - 0s 50us/step - loss: 0.0384
- val_loss: 0.0405
```

1.6 Use the `plot_predictions` function to plot the model's predictions on `X-test` to the true values in `y_test` (by default, it will only plot the first few rows). Report the model's overall loss on the test set. Comment on how well the model performs on this unseen data. Do you think it has accurately learned how to map from sample data to the coefecients that generated the data?

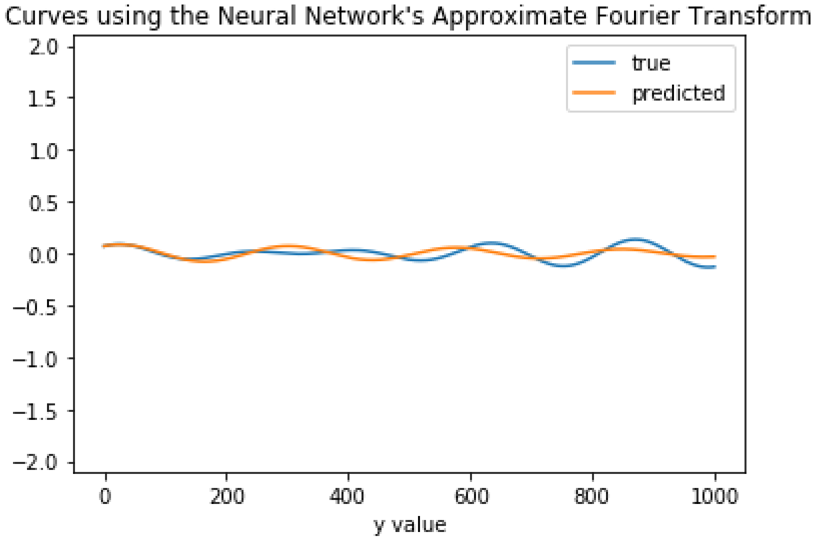
In [12]:
```
plot_predictions(model, X_test, y_test)
```



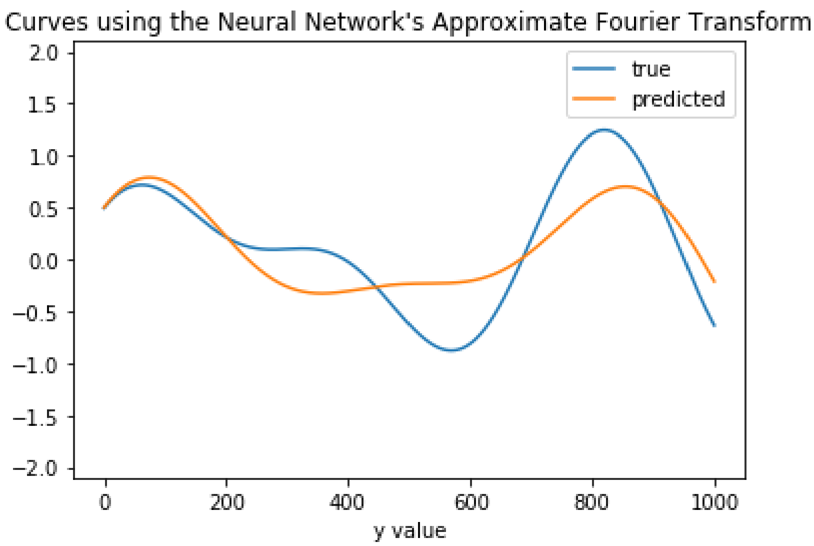

```
true: [0.86199664 0.98175913 0.65523998 0.4870337 ]
predicted: [0.86999345 1.0236945  0.64175546 0.5153728 ]
```



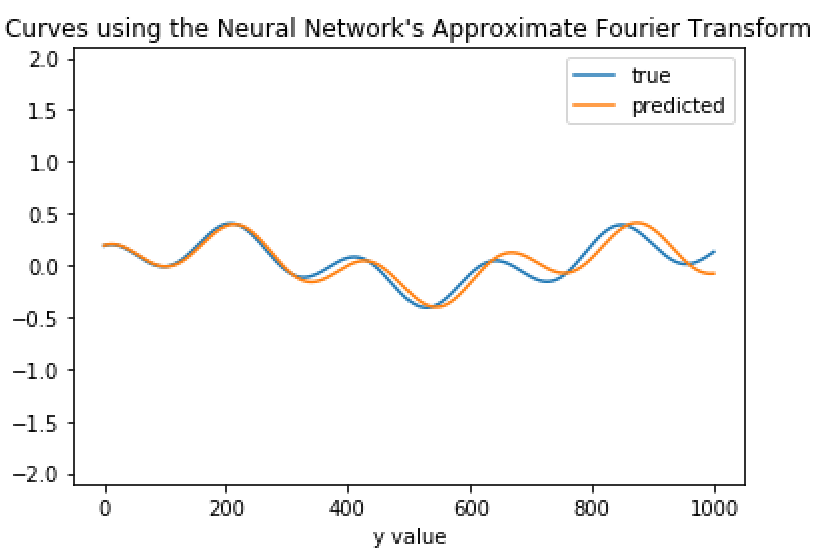

```
true: [0.8406355  0.63159555 0.18328701 0.11174618]
predicted: [0.91290563 0.6017617  0.17470305 0.283667  ]
```




```
true: [0.06591224 0.75183886 0.06986143 0.91352303]
predicted: [0.04898883 0.69550693 0.07144059 0.7319591 ]
```




```
true: [0.75610725 0.30861152 0.49522059 0.48394499]
predicted: [0.75225264 0.3256001  0.5020611  0.42715037]
```




```
true: [0.2229353  0.27885697 0.18696198 0.94846283]
```

```
predicted: [0.21779624 0.28652287 0.19066267 0.91510826]
```

In [13]:
```
model.evaluate(X_test,y_test)
```

```
2000/2000 [==============================] - 0s 18us/step
```

Out[13]:
```
0.035660766631364825
```

Report the model's overall loss on the test set.

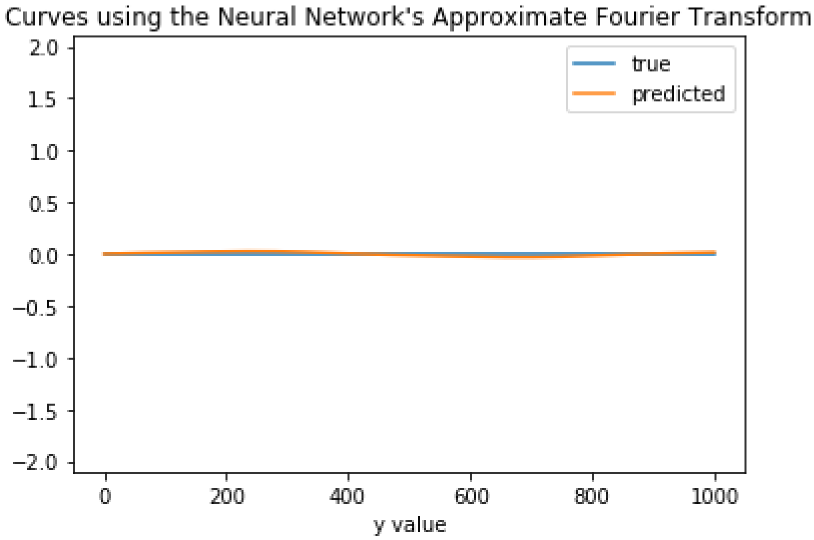
The model's loss on the test set is 0.034

Comment on how well the model performs on this unseen data. Do you think it has accurately learned how to map from sample data to the coefecients that generated the data?

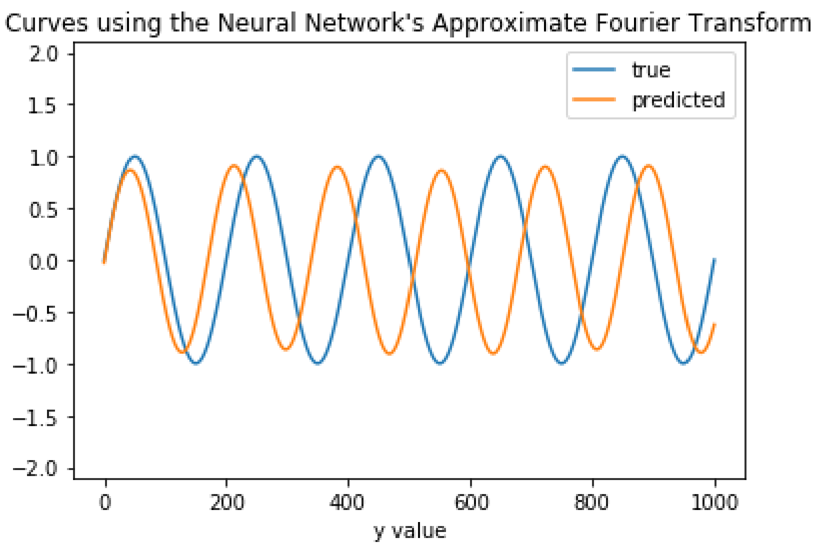
The model predicts the coefficients pretty well. However, when we recreate the sinusoidal data from the coefficients, the predicted curve gets increasingly divergent from the original curve at higher x values.

1.7 Examine the model's performance on the 9 train/test pairs in the `extended_test` variables. Which examples does the model do well on, and which examples does it struggle with?

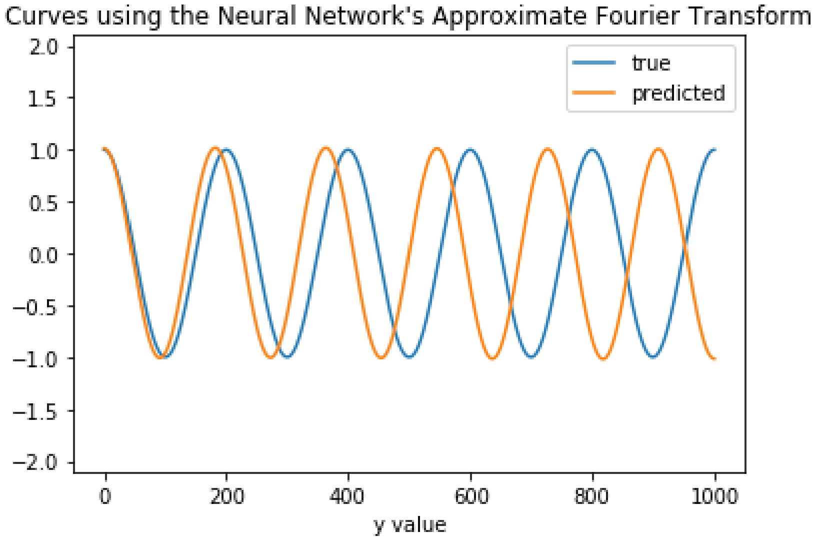
In [14]:
```
# your code here
plot_predictions(model, X_extended_test, y_extended_test, count=9)
```



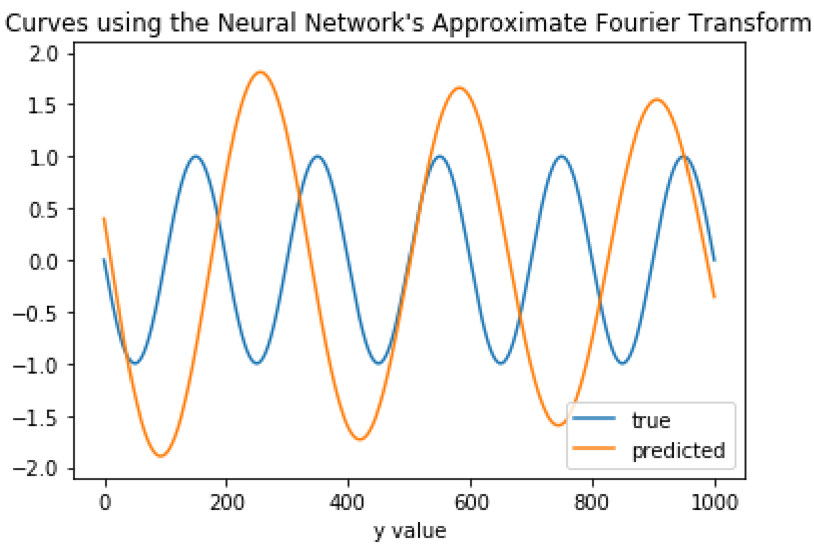

```
true: [0. 0. 0. 0.]
predicted: [0.02780331 0.22685683 0.00199215 0.69877553]
```



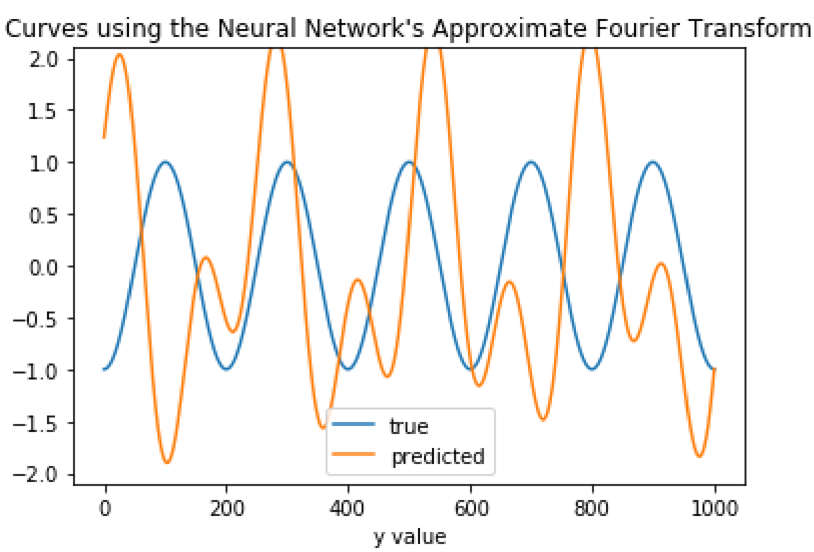

```
true: [1. 1. 0. 0.]
predicted: [ 0.8908934   1.1758093  -0.02668707  0.3642555 ]
```




```
true: [0. 0. 1. 1.]
predicted: [0.00690118 0.18466479 1.0126295  1.1000335 ]
```

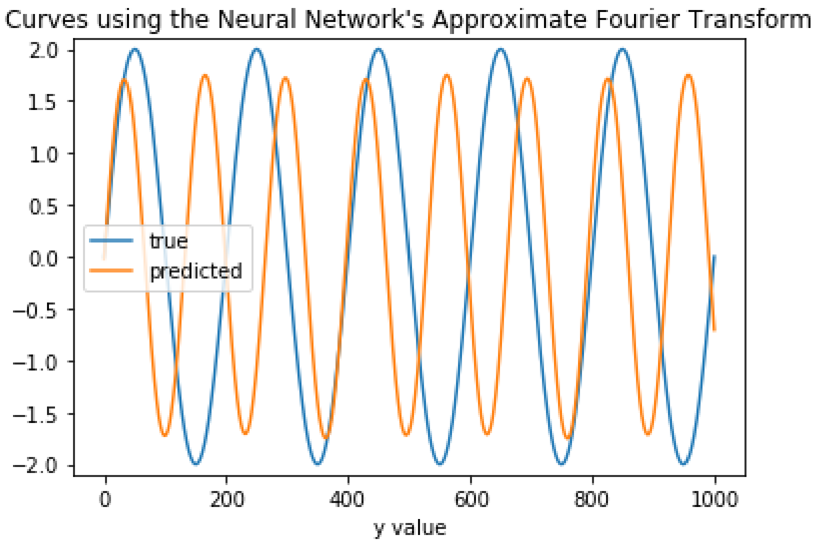



```
true: [-1.  1.  0.  0.]
predicted: [ 1.896416   -0.61016524  0.39871505  0.57040143]
```

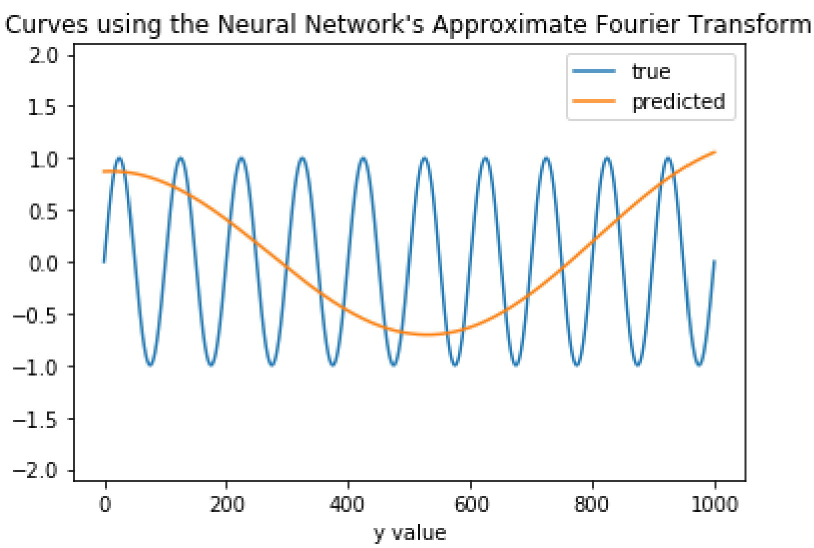



```
true: [ 0.  0. -1.  1.]
```

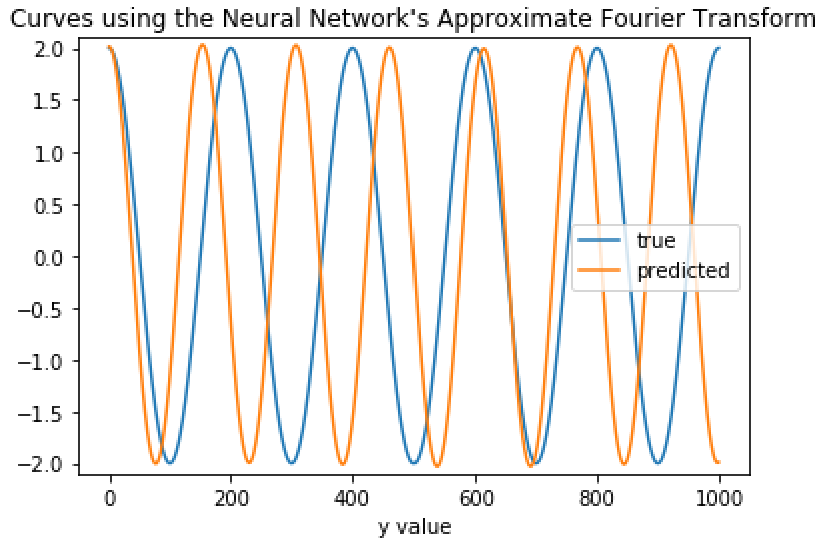
```
predicted: [ 1.0621685   1.5752645   1.2350804 -0.7434264]
```



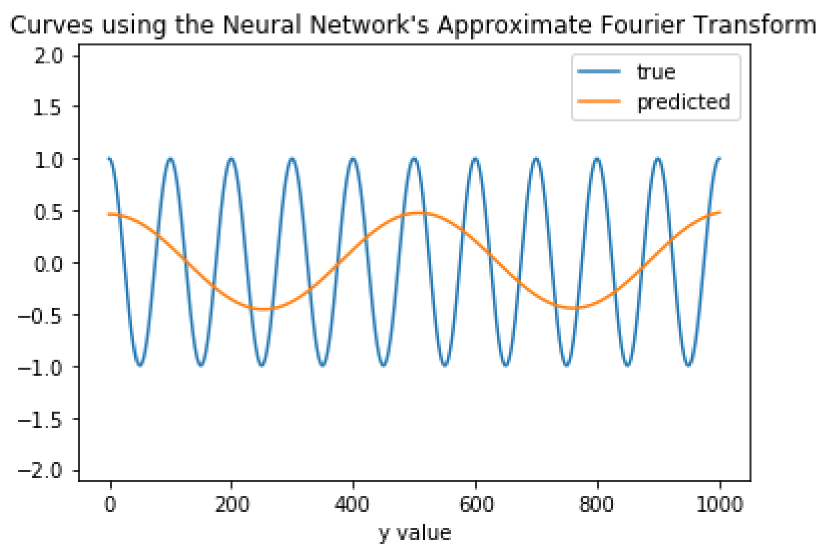

```
true: [2. 1. 0. 0.]
predicted: [ 1.7258799   1.5137789  -0.02440314  0.52439547]
```




```
true: [1. 2. 0. 0.]
predicted: [0.3234838  0.03204279 0.8715768  0.18551424]
```




```
true: [0. 0. 2. 1.]
predicted: [0.01941893 0.24667618 2.0139792  1.3024819 ]
```




```
true: [0. 0. 1. 2.]
predicted: [-0.04132552 -0.01900119  0.4632437  -0.39430016]
```

Your answer here

The model struggles when the true parameters are either greater than 1 or less than 0.

1.8 Is there something that stands out about the difficult observations, especially with respect to the data the model was trained on? Did the model learn the mapping we had in mind? Would you say the model is overfit, underfit, or neither?

The difficult observations are out of sample, since our data was generated with parameters between [0, 1]. This means that the model did not properly learn the fourier transform because if it had, it would generalize to all parameters. The model is overfit, because it fit to our specific samples rather than the underlying data generating process (the parameterized sinusoidal curve).

Regulrizing Neural Networks

In this problem set we have already explored how ANN are able to learn a mapping from example input data (of fixed size) to example output data (of fixed size), and how well the neural network can generalize. In this problem we focus on issues of overfitting and regularization in Neural Networks.

As we have explained in class, ANNs can be prone to overfitting, where they learn specific patterns present in the training data, but the patterns don't generalize to fresh data.

There are several methods used to improve ANN generalization. One approach is to use an achitecutre just barely wide/deep enough to fit the data. The idea here is that smaller networks are less expressive and thus less able to overfit the data.

However, it is difficult to know a priori the correct size of the ANN, and computationally costly to hunt for a correct size. Given this, other methodologies are used to fight overfitting and improve the ANN generalization. These, like other techniques to combat overfitting, fall under the umbrella of Regularization.

In this problem you are asked to regularize a network given to you below. The train dataset can be generated using the code also given below.

> **Question 2 [50 pts]**

2.1 ** Data Download and Exploration**: For this problem, we will be working with the MNIST dataset (Modified National Institute of Standards and Technology database) which is a large database of handwritten digits and commonly used for training various image processing systems. We will be working directly with the download from `keras MNIST dataset` of 60,000 28x28 grayscale images of the 10 digits, along with a test set of 10,000 images.

Please refer to the code below to process the data.

For pedagogical simplicity, we will only use the digits labeled `4` and `9`, and we want to use a total of 800 samples for training.

2.2 Data Exploration and Inspection: Use `imshow` to display a handwritten 4 and a handwritten 9.

2.3 Overfit an ANN: Build a fully connected network (FCN) using `keras`:

1. Nodes per Layer: 100,100,100,2 (<-the two-class 'output' layer)
2. Activation function: reLU
3. Loss function: binary_crossentropy
4. Output unit: Sigmoid
5. Optimizer: sgd (use the defaults; no other tuning)
6. Epochs: no more than 2,000

7. Batch size: 128
8. Validation size: .5

This NN trained on the dataset you built in 2.1 will overfit to the training set. Plot the training accuracy and validation accuracy as a function of epochs and explain how you can tell it is overfitting.

2.4 Explore Regularization: Your task is to regularize this FCN. You are free to explore any method or combination of methods. If you are using anything besides the methods we have covered in class, give a citation and a short explanation. You should always have an understanding of the methods you are using.

Save the model using `model.save(filename)` and submit in canvas along with your notebook.

We will evaluate your model on a test set we've kept secret.

1. Don't try to use extra data from NMIST. We will re-train your model on training set under the settings above.
2. Keep the architecture above as is. In other words keep the number of layers, number of nodes, activation function, and loss fucntion the same. You can change the number of epochs (max 2000), batch size, optimizer and of course add elements that can help to regularize (e.g. drop out, L2 norm etc). You can also do data augmentation.
3. You *may* import new modules, following the citation rule above.

Grading: Your score will be based on how much you can improve on the test score via regularization:

1. (0-1] percent will result into 10 pts
2. (1-2] percent will result into 20 pts
3. (2-3] percent will result into 30 pts
4. Above 3 percent will result in 35 pts
5. Top 15 groups or single students will be awarded an additional 10 pts
6. The overall winner(s) will be awarded an additional 5 pts

2.1 Data Download and Exploration: For this problem, we will be working with the MNIST dataset (Modified National Institute of Standards and Technology database) which is a large database of handwritten digits and commonly used for training various image processing systems. We will be working directly with the download from `keras MNIST dataset` of 60,000 28x28 grayscale images of the 10 digits, along with a test set of 10,000 images.

Please refer to the code below to process the data.

For pedagogical simplicity, we will only use the digits labeled `4` and `9` , and we want to use a total of 800 samples for training.

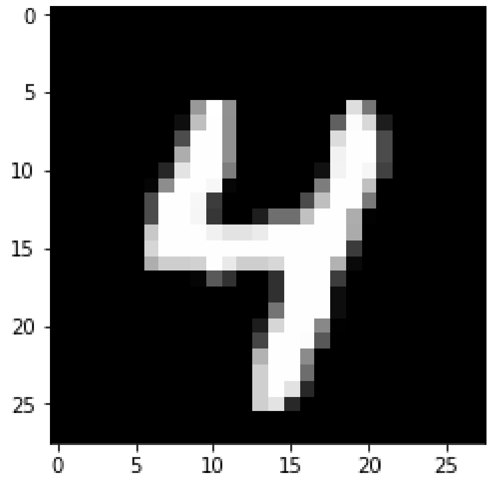
In [15]:
```
## Read and Setup train and test splits in one
from keras.datasets import mnist
from random import randint

(x_train, y_train), (x_test, y_test) = mnist.load_data()

#shuffle the data before we do anything
x_train, y_train = shuffle(x_train, y_train, random_state=1)
```

In [16]:
```
def preprocess(x, y):
    select = np.logical_or(y == 4, y == 9)
    x_subset = x[select]
    y_subset = y[select]

    idx = np.random.randint(len(y_subset), size=800)
    x_800 = x_subset[idx]
    y_800 = y_subset[idx]

    y_binary = np.array([0 if y == 4 else 1 for y in y_800])

    # Preprocess data using keras.utils.to_categorical
    # your code here
    y_final = keras.utils.to_categorical(y_binary)

    # scale the data otherwise reLU can become unstable
    # your code here
    x_scaled = x_800/255
    x_final = x_scaled.reshape(-1,28*28)

    return x_final, y_final

x_train_final, y_train_final = preprocess(x_train, y_train)

x_test_final, y_test_final = preprocess(x_test, y_test)
```

2.2 Data Exploration and Inspection: Use `imshow` to display a handwritten 4 and a handwritten 9.

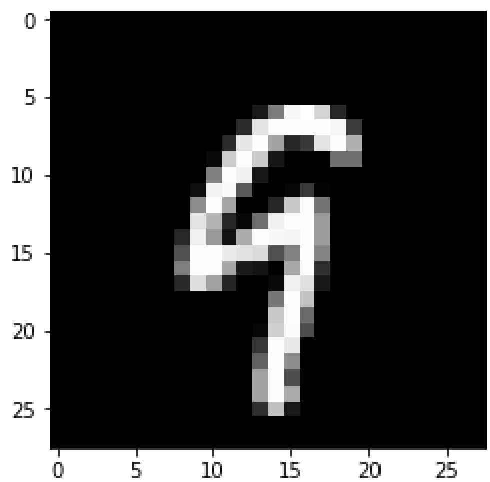
In [17]:
```
# your code here
plt.imshow(x_train_final.reshape(-1,28,28)[y_train_final[:, 0] == True][0],
```

Out[17]: <matplotlib.image.AxesImage at 0x1371cd358>



In [18]:
```
plt.imshow(x_train_final.reshape(-1,28,28)[y_train_final[:, 1] == True][0],
```

Out[18]: <matplotlib.image.AxesImage at 0x1342224a8>



2.3 Overfit an ANN: Build a fully connected network (FCN) using `keras` :

1. Nodes per Layer: 100,100,100,2 (<-the two-class 'output' layer)
2. Activation function: reLU
3. Loss function: binary_crossentropy
4. Output unit: Sigmoid
5. Optimizer: sgd (use the defaults; no other tuning)
6. Epochs: no more than 1,000
7. Batch size: 128
8. Validation size: .5

This NN trained on the dataset you built in 2.1 will overfit to the training set. Plot the training accuracy and validation accuracy as a function of epochs and explain how you can tell it is overfitting.

In [19]:
```
# your code here
model = Sequential()
model.add(Dense(units=100, activation='relu', input_dim=28*28))
model.add(Dense(units=100, activation='relu'))
model.add(Dense(units=100, activation='relu'))
model.add(Dense(units=2, activation='sigmoid'))

model.compile(loss = 'binary_crossentropy', optimizer = 'sgd', metrics=['ac

model.summary()
```

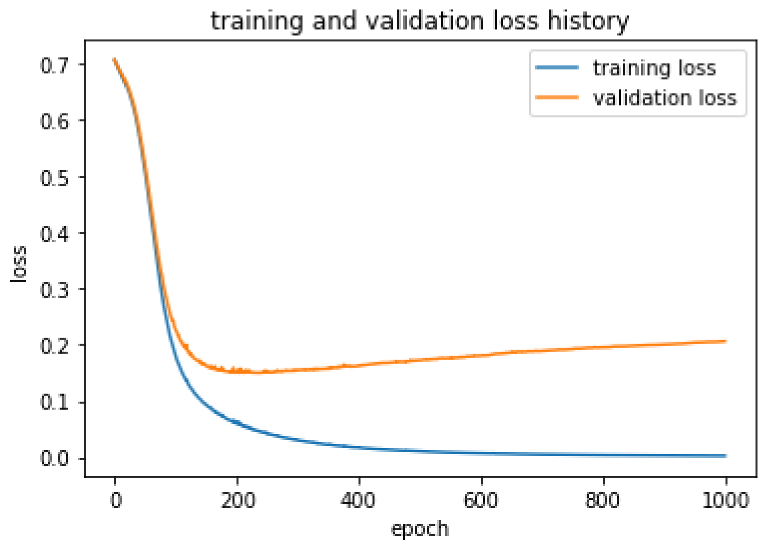
```
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
dense_4 (Dense)              (None, 100)               78500     
_________________________________________________________________
dense_5 (Dense)              (None, 100)               10100     
_________________________________________________________________
dense_6 (Dense)              (None, 100)               10100     
_________________________________________________________________
dense_7 (Dense)              (None, 2)                 202       
=================================================================
Total params: 98,902
Trainable params: 98,902
Non-trainable params: 0
_________________________________________________________________
```

In [20]:
```
#your code here
history = model.fit(x_train_final, y_train_final, epochs = 1000, batch_size
```

```
acc: 1.0000 - val_loss: 0.2061 - val_acc: 0.9525
Epoch 995/1000
400/400 [==============================] - 0s 33us/step - loss: 0.0027 -
acc: 1.0000 - val_loss: 0.2063 - val_acc: 0.9525
Epoch 996/1000
400/400 [==============================] - 0s 33us/step - loss: 0.0026 -
acc: 1.0000 - val_loss: 0.2062 - val_acc: 0.9525
Epoch 997/1000
400/400 [==============================] - 0s 34us/step - loss: 0.0026 -
acc: 1.0000 - val_loss: 0.2063 - val_acc: 0.9525

Epoch 998/1000
400/400 [==============================] - 0s 34us/step - loss: 0.0026 -
acc: 1.0000 - val_loss: 0.2063 - val_acc: 0.9525
Epoch 999/1000
400/400 [==============================] - 0s 34us/step - loss: 0.0026 -
acc: 1.0000 - val_loss: 0.2064 - val_acc: 0.9525
Epoch 1000/1000
400/400 [==============================] - 0s 33us/step - loss: 0.0026 -
acc: 1.0000 - val_loss: 0.2064 - val_acc: 0.9525
```

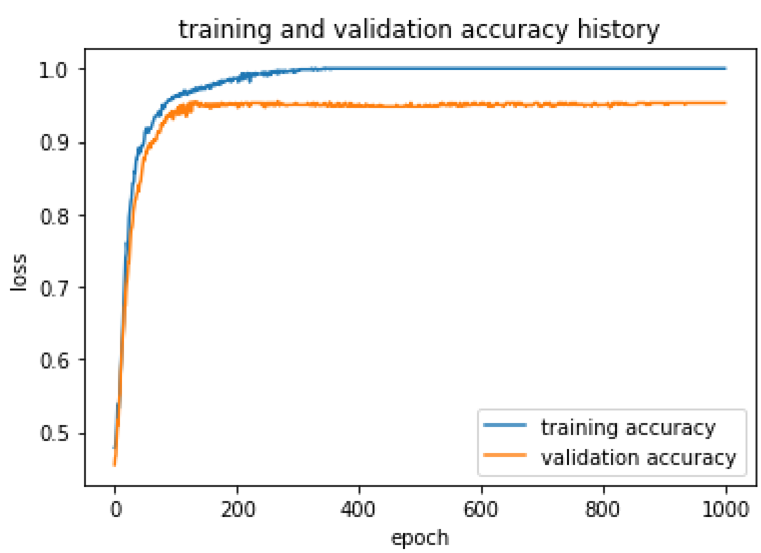
In [21]:
```
plt.plot(history.history['loss'], label = 'training loss')
plt.plot(history.history['val_loss'], label = 'validation loss')
plt.xlabel('epoch')
plt.ylabel('loss')
plt.legend()
plt.title('training and validation loss history')
```

Out[21]: Text(0.5,1,'training and validation loss history')



In [22]:
```
plt.plot(history.history['acc'], label = 'training accuracy')
plt.plot(history.history['val_acc'], label = 'validation accuracy')
plt.xlabel('epoch')
plt.ylabel('loss')
plt.legend()
plt.title('training and validation accuracy history')
```

Out[22]: Text(0.5,1,'training and validation accuracy history')



You can tell the model is overfitting because after epoch ~300, the validation loss begins to slowly increase and the validation accuracy stays constant.

2.4 Explore Regularization: Your task is to regularize this FCN. You are free to explore any method or combination of methods. If you are using anything besides the methods we have covered in class, give a citation and a short explanation. You should always have an understanding of the methods you are using.

Save the model using `model.save(filename)` and submit in canvas along with your notebook.

Note: Due to computational constraints, I won't perform full cross validation to select regularization parameters and, instead, will simply hold out a validation set.

Baseline

In [23]:
```
baseline_loss = history.history['val_loss'][-1]
baseline_acc = history.history['val_acc'][-1]
print("the baseline loss is {} and the baseline accuracy is {}".format(base
```

```
the baseline loss is 0.20637527942657471 and the baseline accuracy is 0.9
525
```

In [24]:
```
model.evaluate(x_test_final, y_test_final)
```

```
800/800 [==============================] - 0s 26us/step
```

Out[24]:
```
[0.15103098668623716, 0.95]
```

L1 Regularization

In [25]:
```
def train_model_l1(r, x, y, epochs = 1000, batch_size = 128, input_dim = 28
    model = Sequential()
    model.add(Dense(units=100, activation='relu', input_dim=input_dim, kern
    model.add(Dense(units=100, activation='relu', kernel_regularizer = regu
    model.add(Dense(units=100, activation='relu', kernel_regularizer = regu
    model.add(Dense(units=2, activation='relu', kernel_regularizer = regula
    model.compile(loss = 'binary_crossentropy', optimizer = 'sgd', metrics=
    history = model.fit(x, y, epochs = epochs, batch_size=batch_size, valid
    return history.history['val_loss'][-1], history.history['val_acc'][-1]
```

In [26]:
```
reg_param = [0.0001, 0.001, 0.01, 0.1]
loss_results_l1 = {}
acc_results_l1 = {}
for r in reg_param:
    print('regularization parameter:', r)
    loss, acc = train_model_l1(r, x_train_final, y_train_final, epochs = 20
    loss_results_l1[r] = loss
    acc_results_l1[r] = acc
```

```
regularization parameter: 0.0001
regularization parameter: 0.001
regularization parameter: 0.01
regularization parameter: 0.1
```

L2 Regularization

In [27]:
```
def train_model_l2(r, x, y, epochs = 1000, batch_size = 128, input_dim = 28
    model = Sequential()
    model.add(Dense(units=100, activation='relu', input_dim=input_dim, kern
    model.add(Dense(units=100, activation='relu', kernel_regularizer = regu
    model.add(Dense(units=100, activation='relu', kernel_regularizer = regu
    model.add(Dense(units=2, activation='relu', kernel_regularizer = regula
    model.compile(loss = 'binary_crossentropy', optimizer = 'sgd', metrics=
    history = model.fit(x, y, epochs = epochs, batch_size=batch_size, valid
    return history.history['val_loss'][-1], history.history['val_acc'][-1]
```

In [28]:
```
reg_param = [0.0001, 0.001, 0.01, 0.1]
loss_results_l2 = {}
acc_results_l2 = {}
for r in reg_param:
    print('regularization parameter:', r)
    loss, acc = train_model_l1(r, x_train_final, y_train_final, epochs = 20
    loss_results_l2[r] = loss
    acc_results_l2[r] = acc
```

```
regularization parameter: 0.0001
regularization parameter: 0.001
regularization parameter: 0.01
regularization parameter: 0.1
```

Dropout

In [29]:
```
from keras.layers import Dropout
```

In [30]:
```
def train_model_dropout(r, x, y, epochs = 1000, batch_size = 128, input_dim
    model = Sequential()
    model.add(Dense(units=100, activation='relu', input_dim=28*28))
    model.add(Dropout(r))
    model.add(Dense(units=100, activation='relu'))
    model.add(Dropout(r))
    model.add(Dense(units=100, activation='relu'))
    model.add(Dropout(r))
    model.add(Dense(units=2, activation='sigmoid'))
    model.compile(loss = 'binary_crossentropy', optimizer = 'sgd', metrics=
    history = model.fit(x, y, epochs = epochs, batch_size=batch_size, valid
    return history.history['val_loss'][-1], history.history['val_acc'][-1]
```

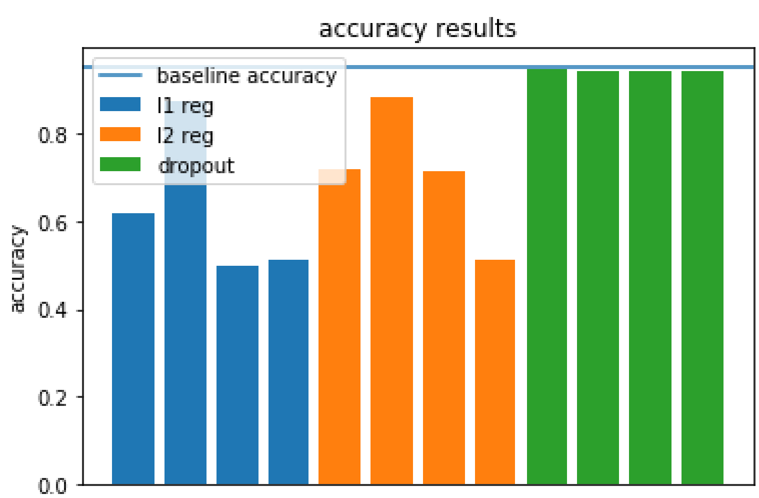
In [31]:
```
reg_param = [0.1,0.2,0.3,0.4]
loss_results_dropout = {}
acc_results_dropout = {}
for r in reg_param:
    print('regularization parameter:', r)
    loss, acc = train_model_dropout(r, x_train_final, y_train_final, epochs
    loss_results_dropout[r] = loss
    acc_results_dropout[r] = acc
```

```
regularization parameter: 0.1
regularization parameter: 0.2
regularization parameter: 0.3
regularization parameter: 0.4
```

Compare results

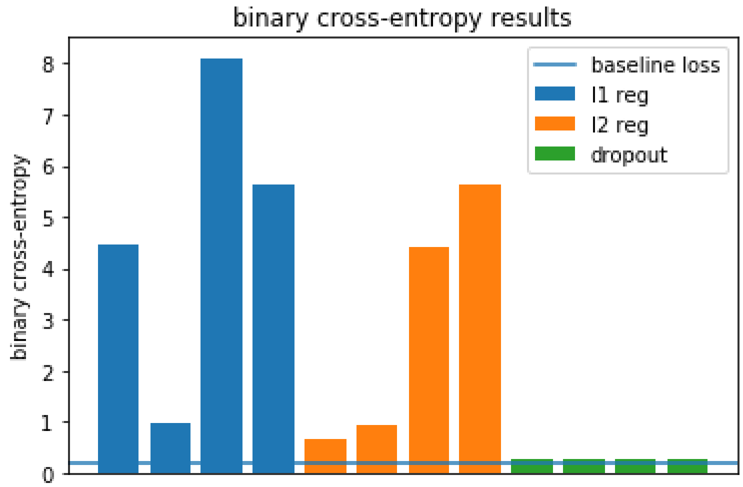
In [32]:
```
plt.bar(np.arange(0,4,1), acc_results_l1.values(), label = "l1 reg")
plt.bar(np.arange(4,8,1), acc_results_l2.values(), label = "l2 reg")
plt.bar(np.arange(8,12,1), acc_results_dropout.values(), label = 'dropout')
plt.axhline(baseline_acc, label = 'baseline accuracy')
plt.legend()
plt.title('accuracy results')
plt.xticks([])
plt.ylabel('accuracy')
```

Out[32]: `Text(0,0.5,'accuracy')`



In [33]:
```
plt.bar(np.arange(0,4,1), loss_results_l1.values(), label = "l1 reg")
plt.bar(np.arange(4,8,1), loss_results_l2.values(), label = "l2 reg")
plt.bar(np.arange(8,12,1), loss_results_dropout.values(), label = 'dropout'
plt.axhline(baseline_loss, label = 'baseline loss')
plt.legend()
plt.title('binary cross-entropy results')
plt.xticks([])
plt.ylabel('binary cross-entropy')
```

Out[33]:
```
Text(0,0.5,'binary cross-entropy')
```



The dropout model performs the best. Let's see which model performed the best.

In [34]:
```
best_dropout_param = list(acc_results_dropout.keys())[np.argmax(list(acc_re
print("The best dropout parameter is", best_dropout_param)
```

```
The best dropout parameter is 0.1
```

In [35]:
```
best_dropout_param
```

Out[35]:
```
0.1
```

Data augmentation

Let's see if we can improve performance by augmenting our data with transformed images

In [43]:
```
from keras.preprocessing.image import ImageDataGenerator
from keras.layers import Flatten, Input
from keras.models import Model
```

In [44]:
```
inp = Input(shape= (28, 28, 1))
x = Flatten()(inp)
x = Dense(100, activation='relu')(x)
x = Dropout(best_dropout_param)(x)
x = Dense(100, activation='relu')(x)
x = Dropout(best_dropout_param)(x)
x = Dense(100, activation='relu')(x)
x = Dropout(best_dropout_param)(x)
out = Dense(units=2, activation='sigmoid')(x)

model_dropout = Model(inputs=inp, outputs=out)
```

In [45]:
```
def preprocess(x, y):
    select = np.logical_or(y == 4, y == 9)
    x_subset = x[select]
    y_subset = y[select]

    idx = np.random.randint(len(y_subset), size=800)
    x_800 = x_subset[idx]
    y_800 = y_subset[idx]

    y_binary = np.array([0 if y == 4 else 1 for y in y_800])

    # Preprocess data using keras.utils.to_categorical
    # your code here
    y_final = keras.utils.to_categorical(y_binary)

    # scale the data otherwise reLU can become unstable
    # your code here
    x_reshape = x_800.reshape(x_800.shape[0], 28, 28, 1)
    x_final = x_reshape.astype('float32')

    return x_final, y_final

x_train_final, y_train_final = preprocess(x_train, y_train)

x_test_final, y_test_final = preprocess(x_test, y_test)
```

In [46]:
```
datagen = ImageDataGenerator(
    rescale=1./255,
    shear_range=0.2,
    zoom_range=0.2,
    rotation_range = 0.2,
)
# fit parameters from data
datagen.fit(x_train_final)
```

In [47]:
```
train_generator = datagen.flow(x_train_final, y_train_final, batch_size=128
model_dropout.compile(loss = 'binary_crossentropy', optimizer = 'sgd', metr
history = model_dropout.fit_generator(
    train_generator,
    steps_per_epoch=len(x_train) / 128,
    epochs=50 # need to drop down epochs due to using generator
)
```

```
Epoch 1/50
469/468 [==============================] - 9s 20ms/step - loss: 0.5837 -
acc: 0.7221
Epoch 2/50
469/468 [==============================] - 9s 19ms/step - loss: 0.2328 -
acc: 0.9224
Epoch 3/50
469/468 [==============================] - 10s 21ms/step - loss: 0.1417 -
acc: 0.9513
Epoch 4/50
469/468 [==============================] - 10s 21ms/step - loss: 0.1116 -
acc: 0.9616
Epoch 5/50
469/468 [==============================] - 11s 24ms/step - loss: 0.0879 -
acc: 0.9711
Epoch 6/50
469/468 [==============================] - 13s 28ms/step - loss: 0.0694 -
acc: 0.9786
Epoch 7/50
469/468 [==============================] - 12s 25ms/step - loss: 0.0542 -
acc: 0.9838
Epoch 8/50
469/468 [==============================] - 9s 20ms/step - loss: 0.0427 -
acc: 0.9875
Epoch 9/50
469/468 [==============================] - 10s 22ms/step - loss: 0.0344 -
acc: 0.9909
Epoch 10/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0270 -
acc: 0.9929
Epoch 11/50
469/468 [==============================] - 11s 23ms/step - loss: 0.0217 -
acc: 0.9950
Epoch 12/50
469/468 [==============================] - 10s 20ms/step - loss: 0.0179 -
acc: 0.9961
Epoch 13/50
469/468 [==============================] - 10s 20ms/step - loss: 0.0153 -
acc: 0.9968
Epoch 14/50
469/468 [==============================] - 10s 20ms/step - loss: 0.0137 -
acc: 0.9971
Epoch 15/50
469/468 [==============================] - 10s 20ms/step - loss: 0.0114 -
acc: 0.9978
Epoch 16/50
469/468 [==============================] - 11s 23ms/step - loss: 0.0094 -
acc: 0.9983
Epoch 17/50
```

```
469/468 [==============================] - 12s 25ms/step - loss: 0.0086 -
acc: 0.9983
Epoch 18/50
469/468 [==============================] - 10s 22ms/step - loss: 0.0077 -
acc: 0.9987
Epoch 19/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0071 -
acc: 0.9988
Epoch 20/50
469/468 [==============================] - 10s 21ms/step - loss: 0.0065 -
acc: 0.9989
Epoch 21/50
469/468 [==============================] - 9s 20ms/step - loss: 0.0061 -
acc: 0.9989
Epoch 22/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0055 -
acc: 0.9990
Epoch 23/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0047 -
acc: 0.9993
Epoch 24/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0047 -
acc: 0.9991
Epoch 25/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0043 -
acc: 0.9993
Epoch 26/50
469/468 [==============================] - 10s 20ms/step - loss: 0.0038 -
acc: 0.9994
Epoch 27/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0036 -
acc: 0.9995
Epoch 28/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0030 -
acc: 0.9995
Epoch 29/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0034 -
acc: 0.9993
Epoch 30/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0029 -
acc: 0.9996
Epoch 31/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0029 -
acc: 0.9996
Epoch 32/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0028 -
acc: 0.9996
Epoch 33/50
469/468 [==============================] - 9s 20ms/step - loss: 0.0024 -
acc: 0.9996
Epoch 34/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0025 -
acc: 0.9997
Epoch 35/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0023 -
acc: 0.9997
Epoch 36/50
```

```
469/468 [==============================] - 9s 18ms/step - loss: 0.0023 -
acc: 0.9996
Epoch 37/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0022 -
acc: 0.9996
Epoch 38/50
469/468 [==============================] - 9s 18ms/step - loss: 0.0022 -
acc: 0.9996
Epoch 39/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0021 -
acc: 0.9997
Epoch 40/50
469/468 [==============================] - 9s 18ms/step - loss: 0.0021 -
acc: 0.9996
Epoch 41/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0021 -
acc: 0.9996
Epoch 42/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0021 -
acc: 0.9997
Epoch 43/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0016 -
acc: 0.9998
Epoch 44/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0015 -
acc: 0.9999
Epoch 45/50
469/468 [==============================] - 9s 18ms/step - loss: 0.0017 -
acc: 0.9997
Epoch 46/50
469/468 [==============================] - 9s 19ms/step - loss: 0.0015 -
acc: 0.9997
Epoch 47/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0016 -
acc: 0.9997
Epoch 48/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0016 -
acc: 0.9997
Epoch 49/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0013 -
acc: 0.9999
Epoch 50/50
469/468 [==============================] - 8s 18ms/step - loss: 0.0014 -
acc: 0.9998
```

Assess performance on test set

In [49]:
```
model_dropout.evaluate(x_test_final, y_test_final)
```

```
800/800 [==============================] - 0s 615us/step
```

Out[49]:
```
[0.5778060766983714, 0.96375]
```

In [50]:
```
def preprocess(x, y):
    select = np.logical_or(y == 4, y == 9)
    x_subset = x[select]
    y_subset = y[select]

    idx = np.random.randint(len(y_subset), size=800)
    x_800 = x_subset[idx]
    y_800 = y_subset[idx]

    y_binary = np.array([0 if y == 4 else 1 for y in y_800])

    # Preprocess data using keras.utils.to_categorical
    # your code here
    y_final = keras.utils.to_categorical(y_binary)

    # scale the data otherwise reLU can become unstable
    # your code here
    x_scaled = x_800/255
    x_final = x_scaled.reshape(-1,28*28)

    return x_final, y_final

x_train_final, y_train_final = preprocess(x_train, y_train)

x_test_final, y_test_final = preprocess(x_test, y_test)
```

In [51]:
```
model.evaluate(x_test_final, y_test_final)
```

```
800/800 [==============================] - 0s 45us/step
```

Out[51]:
```
[0.2820719419885427, 0.92875]
```

We got a ~4% increase in test set accuracy.

Save model

In [52]:
```
model_dropout.save("regularized_model")
```

In [54]:
```
test_model = keras.models.load_model('./regularized_model')
```

In [55]:
```
test_model.summary()
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
input_2 (InputLayer)         (None, 28, 28, 1)         0         
_________________________________________________________________
flatten_2 (Flatten)          (None, 784)               0         
_________________________________________________________________
dense_60 (Dense)             (None, 100)               78500     
_________________________________________________________________
dropout_16 (Dropout)         (None, 100)               0         
_________________________________________________________________
dense_61 (Dense)             (None, 100)               10100     
_________________________________________________________________
dropout_17 (Dropout)         (None, 100)               0         
_________________________________________________________________
dense_62 (Dense)             (None, 100)               10100     
_________________________________________________________________
dropout_18 (Dropout)         (None, 100)               0         
_________________________________________________________________
dense_63 (Dense)             (None, 2)                 202       
=================================================================
Total params: 98,902
Trainable params: 98,902
Non-trainable params: 0
_________________________________________________________________
```

In [ ]: