Vertical Mean

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1 Notations

Note here I adopt slightly different notations compared to Lam's thesis. Lam's usage of $\bar{X}$ is a little bit different from the common mathematical way of writing.

I. $X_k(s_n)$: energy of processor k at s_n. Notice this emphasizes three things: A. at processor k B. at step (n-1); C. at temperature s_n. If a stationary setting temperature is hold unchanged so that $s_{n-1} = s_n$, then it only emphasizes step.

II. $\mathbf{E}[X_k(s_{n-1})]$: the expected energy of processor k at step (n-1) with temperature s_{n-1}. Note that $\mathbf{E}[X(s_{n-1})]$ has nothing to do with the mean under the stationarity condition. Also, it emphasizes the same things as item (I) defined above.

III. $\underline{X}_k(s_n)$: denotes the stationary process as $n \to \infty$ for $X_k(s_n)$. Note it does not emphasize step n any ore. Instead, it stands for the mean of the whole stationary process.

IV. $\mu_k(s_n)$ and $\sigma_k^2(s_n)$: denote the mean and variance of the stationary process $\underline{X}_k(s_n)$.

An important thing to think about: if all the N processors starts at the same energy, then all such $\mu_k(s_n)$ and $\sigma_k^2(s_n)$ will be equal for all $k \in \{1, 2, ..., N\}$. But what about they start at different energy? (I think they will still be the same but I need to think carefully).

V. $Y(s_n)$: vertical mean defined as $Y(s_n) = \frac{\sum_{k=1}^{N} X_k(s_n)}{N}$.

2 Quasi-stationarity and AR

2.1 Quasi-stationarity

$\forall k \in \{1, 2, ..., N\}$, if $|\mathbf{E}(X_k(s_{n-1})) - \mu(s_n)| \leq \lambda_k \sigma(s_n)$ (quai-stationarity for each processor), then we have

$$
\begin{aligned}
|\mathbf{E}(Y(s_{n-1})) - \mu(s_n)| &= |\mathbf{E}(\frac{\sum_{k=1}^{N} X_k(s_n)}{N}) - \mu(s_n)| \\
&= \frac{1}{N}|\mathbf{E}(\sum_{k=1}^{N} X_k(s_n)) - N\mu(s_n)| \\
&= \frac{1}{N}|\sum_{k=1}^{N}[\mathbf{E}(X_k(s_n)) - \mu(s_n)]| \\
&\leq \frac{1}{N}\sum_{k=1}^{N}|[\mathbf{E}(X_k(s_n)) - \mu(s_n)]| \\
&\leq \frac{1}{N}\sum_{k=1}^{N}\lambda_k\sigma(s_n) \\
&= \bar{\lambda}\sigma(s_n)
\end{aligned} \tag{1}
$$

where $\bar{\lambda} = \frac{\sum_{k=1}^{N}\lambda_k}{N}$. Therefore it is also quasi-stationary with $\bar{\lambda}$.

2.2 AR

$X_k(s_n) = r_k(s_n)(X_k(s_{n-1}) - \mu(s_n)) + \mu(s_n) + (N_k)_n$. If all the processors. I tend to think that if all the processors start at the same energy state, then $r_k(s_n)$ will be the same among the processors since $r_k(s_n) = \frac{Cov(X_k(s_{n-1}), X_k(s_n))}{Var(X_k(s_n))}$ assuming weak stationarity (it is not necessarily true though). **However, what will happen if the processes start at different energy state?** This requires further thinking. If $r_k(s_n)$ are the same our lives will be much easier.