System Simulation Midterm: Josh Humphrey

(1) a) Design: 2nd Order, AB-2 predictor, zero at $-6/11$

$$H_p(z) = \frac{\beta_1 z + \beta_0}{z^2 - z} \longrightarrow P(z) = z^2 - z \text{ so } \alpha_2 = 1, \alpha_1 = -1, \alpha_0 = 0$$

$$C_1 = 0 = -\ell_1\alpha_1 + \ell_2\alpha_2 - \beta_1 - \beta_0 = -1 + 2 - \beta_1 - \beta_0 = 1 - \beta_1 - \beta_0 \rightarrow \beta_1 + \beta_0 = 1$$

$$\left(-\frac{6}{11}\beta_1\right) + \beta_0 = 0 \longrightarrow 6\beta_1 = 11\beta_0 \longrightarrow \beta_0 = \frac{6}{11}\beta_1$$

$$\beta_1\left(\frac{11}{11} + \frac{6}{11}\right) = 1 \longrightarrow \beta_1\left(\frac{17}{11}\right) = 1 \Rightarrow \beta_1 = \frac{11}{17} \text{ and } \beta_0 = \frac{6}{17}$$

$$\therefore H_p(z) = \frac{\frac{11}{17}z + \frac{6}{17}}{z^2 - z}$$

b) Design: 2nd Order, AB-2 corrector, zero at $-6/11$

$$\beta_2\left(-\frac{6}{11}\right)^2 + \beta_1\left(-\frac{6}{11}\right) + \beta_0 = 0$$

$$C_1 = 0 = \alpha_1 + 2\alpha_2 - \beta_1 - \beta_2 - \beta_0 \longrightarrow \beta_0 + \beta_1 + \beta_2 = 1$$

$$C_2 = 0 = \frac{3}{2} - \beta_1 - 2\beta_0 \longrightarrow \beta_1 + 2\beta_2 = 3/2$$

$$\beta_0 = 15/289 \qquad \beta_1 = 229/578 \qquad \beta_2 = 319/578$$

$$\therefore H_c(z) = \frac{\left(\frac{15}{289}\right)z^2 + \frac{229}{578}z + \frac{319}{578}}{z^2 - z}$$

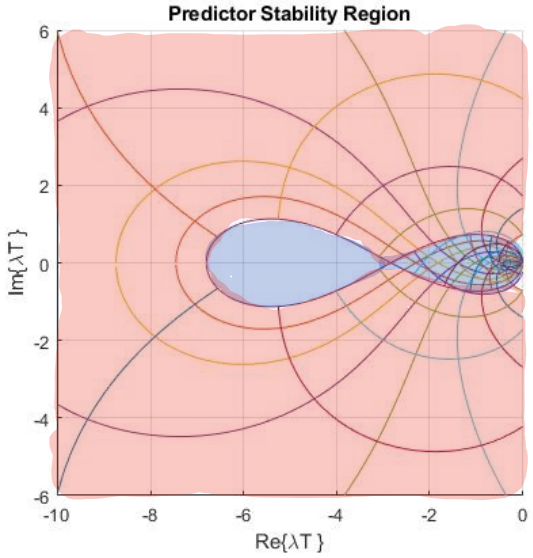
(2) a) Order of Accuracy: Highest degree : 1

↳ The error depends on our sample time

b) Local truncation error: $LTE = C_0 f^{(0)}(x_0) + C_1 f^{(1)}(x_1)$

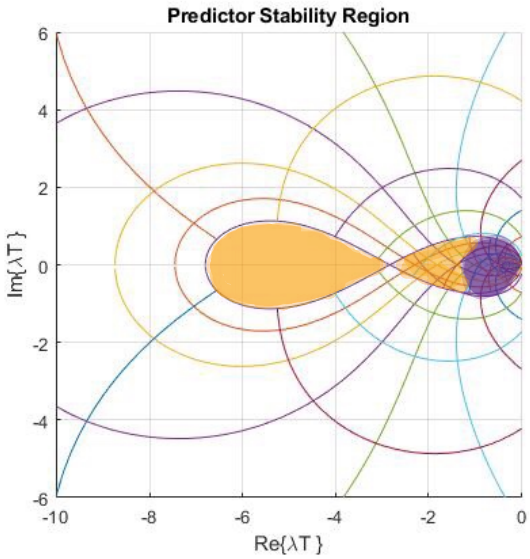
From Table 4.1 → $Error = \frac{5T^3 \cdot x^{(3)}}{12}$

c/d)



← Stable

← Unstable



← Tuned

← Detuned

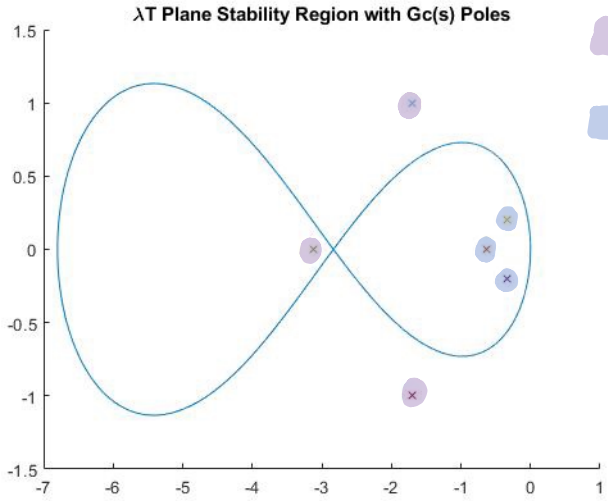
(3) $A = \begin{bmatrix} -6.5 & -14.4 & -12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$ $D = 0$

$\dot{x}_1 = -6.5x_1 - 14.4x_2 - 12x_3 + u_s$

$\dot{x}_2 = x_1$

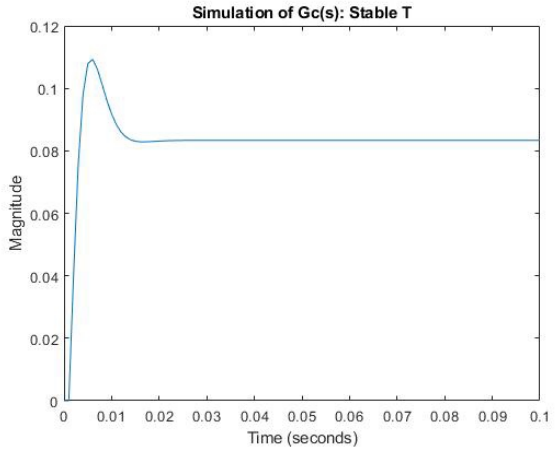
$\dot{x}_3 = x_2$

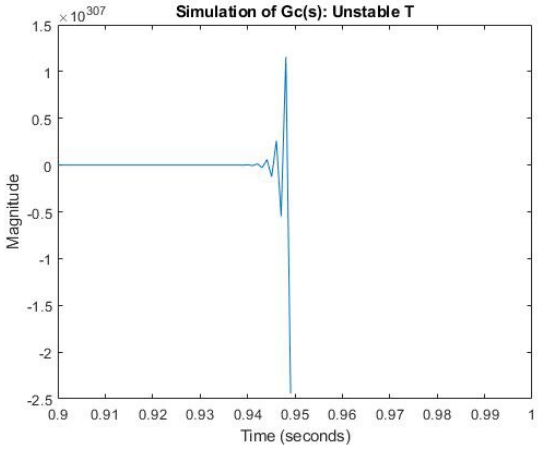
$y = x_2 + x_3$



← T = 1.0

← T = 0.2





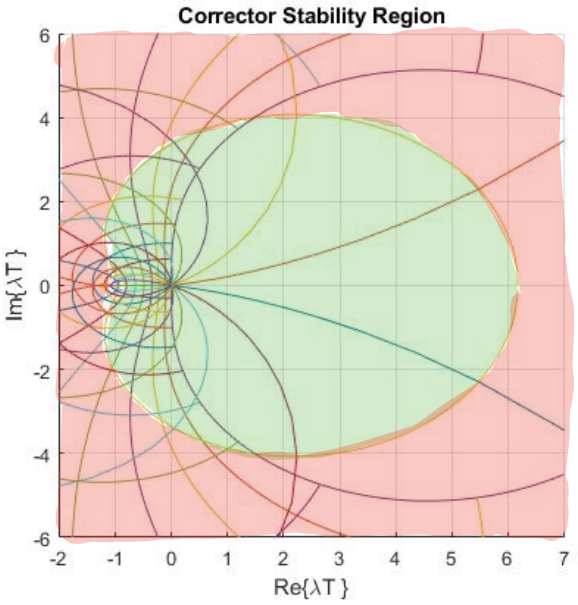
④ a) Order of Accuracy: Highest degree: 2

↳ The error depends on our sample time

b) Local truncation error: $LTE = C_0 f^{(0)}(x_0) + C_1 f^{(1)}(x_1) + C_2 f^{(2)}(x_2)$

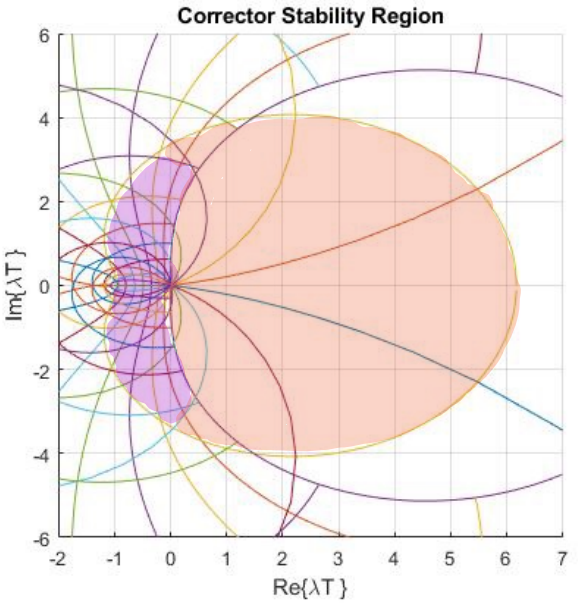
From Table 4.1 → $\text{Error} = \dfrac{-T^3 \cdot x^{(2)}}{12}$

c/d)



← Stable

← Unstable



← Tuned

← Detuned

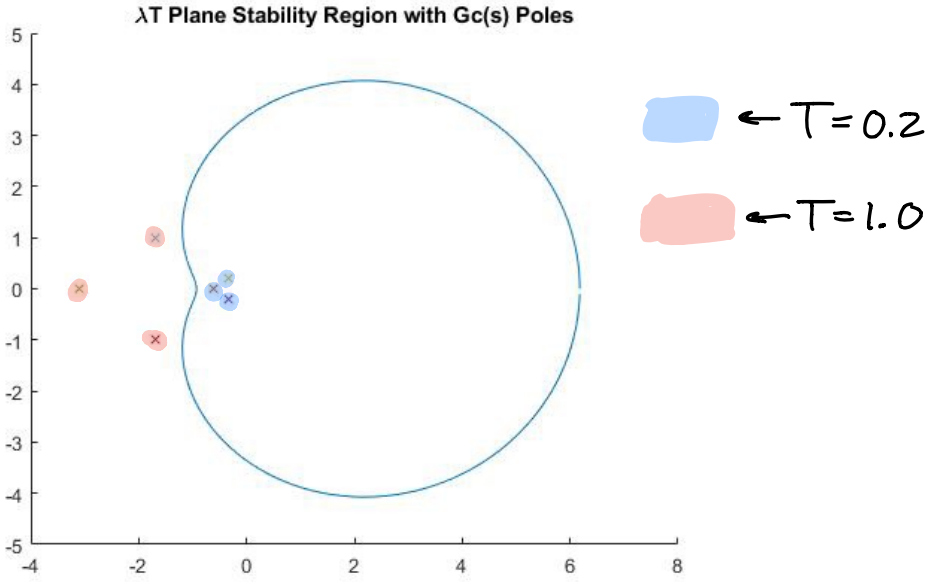
(5) $A = \begin{vmatrix} -6.5 & -14.4 & -12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$ $B = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$ $C = \begin{vmatrix} 0 & 1 & 1 \end{vmatrix}$ $D = 0$

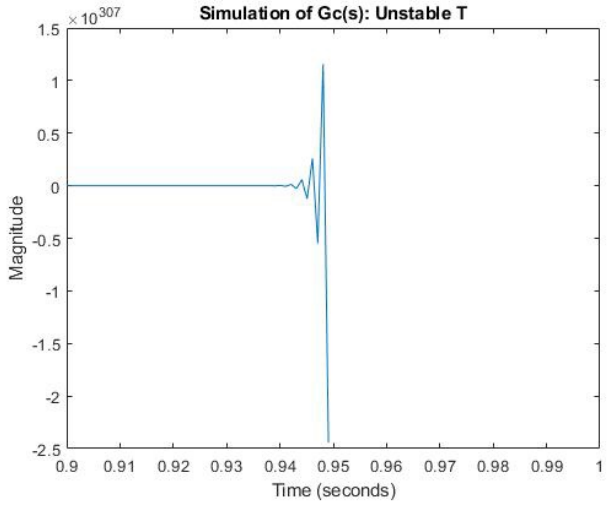
$\dot{x}_1 = -6.5x_1 - 14.4x_2 - 12x_3 + u_s$

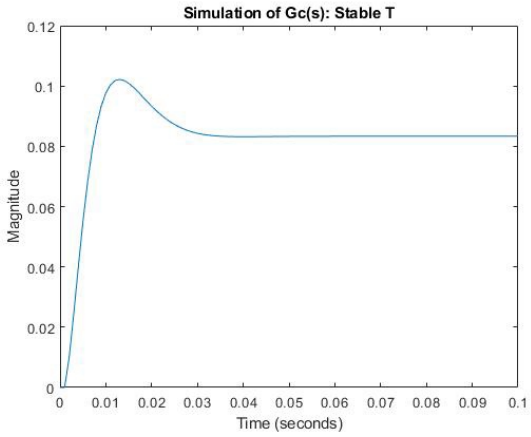
$\dot{x}_2 = x_1$

$\dot{x}_3 = x_2$

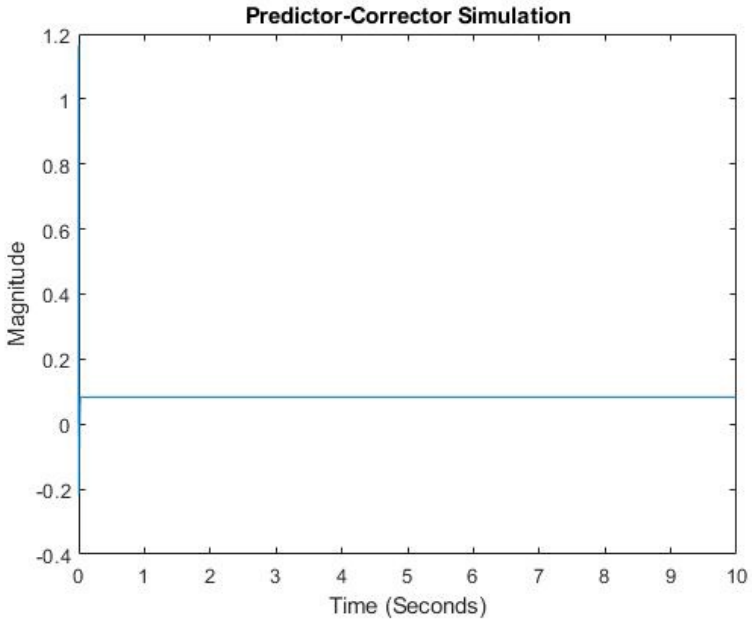
$y = x_2 + x_3$







(6)


Predictor-Corrector Simulation
Magnitude
Time (Seconds)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
1
2
3
4
5
6
7
8
9
10